

Financial Applications of Sequences and Series

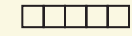
Pattern Recognition

When the independent variable in a relation changes by a steady increment, the dependent variable often changes according to a pattern.

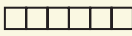
Type of Pattern	Description	Example																												
Linear	The first differences between dependent variables are constant.	<table><tr><td>Number of Books ($\times 1000$)</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>Cost of Books (\$) ($\times 1000$)</td><td>60</td><td>70</td><td>80</td><td>90</td><td>100</td></tr><tr><td>First Differences ($\times 1000$)</td><td>10</td><td>10</td><td>10</td><td>10</td><td></td></tr></table> In a linear relation, each first difference is always the same. Here, each first difference is \$10 000.	Number of Books ($\times 1000$)	1	2	3	4	5	Cost of Books (\$) ($\times 1000$)	60	70	80	90	100	First Differences ($\times 1000$)	10	10	10	10											
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First Differences ($\times 1000$)	10	10	10	10																										
Quadratic	The first differences between dependent variables change, but the second differences are constant.	<table><tr><td>Time (s)</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>Height of Ball (m)</td><td>5</td><td>30.1</td><td>45.4</td><td>50.9</td><td>46.6</td><td>32.5</td></tr><tr><td>First Differences</td><td>25.1</td><td>15.3</td><td>5.5</td><td>-4.3</td><td>-14.1</td><td></td></tr><tr><td>Second Differences</td><td>-9.8</td><td>-9.8</td><td>-9.8</td><td>-9.8</td><td></td><td></td></tr></table> In this case, the first differences are not the same, so the relation is nonlinear. In a quadratic relation, each second difference is always the same. Here, each second difference is -9.8.	Time (s)	0	1	2	3	4	5	Height of Ball (m)	5	30.1	45.4	50.9	46.6	32.5	First Differences	25.1	15.3	5.5	-4.3	-14.1		Second Differences	-9.8	-9.8	-9.8	-9.8		
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Exponential	There is a common multiplier between dependent variables.	<table><tr><td>Time (h)</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>Number of Bacteria</td><td>1000</td><td>2000</td><td>4000</td><td>8000</td><td>16 000</td></tr><tr><td>Common Multiplier</td><td>$\times 2$</td><td>$\times 2$</td><td>$\times 2$</td><td>$\times 2$</td><td></td></tr></table>	Time (h)	0	1	2	3	4	Number of Bacteria	1000	2000	4000	8000	16 000	Common Multiplier	$\times 2$	$\times 2$	$\times 2$	$\times 2$											
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Practice

1. Examine each pattern.

(a) , , , ?

(b) , , , ?

(c) , , , ?

(d) , , , ?

(e) , , , ?

(f) , , , ?

- Draw the next diagram.
 - Write the four numbers that represent the four diagrams, in terms of the number of squares or triangles.
 - Determine the fifth number.
2. Suppose each diagram in question 1 were made from toothpicks.
- What numbers represent each pattern of four diagrams, in terms of the number of toothpicks?
 - How many toothpicks will be in the fifth diagram?
3. Examine the data in each table.

- Is the pattern of the dependent variable linear, quadratic, or exponential?
- State the next value of y .

(a)

x	3	4	5	6
y	9	11	13	15

(b)

x	0	1	2	3
y	1	0.5	0.25	0.125

(c)

x	2	4	6	8
y	-13	-49	-109	-193

(d)

x	0	2	4	6
y	1	4	16	64

4. Neville has 49 m of fencing to build a dog run. Here are some dimensions he considered to maximize the area.
- Describe the type of pattern displayed by
- the width and
 - the area.

Length (m)	1	2	3	4	5
Width (m)	23.5	22.5	21.5	20.5	19.5
Area (m ²)	23.5	45.0	64.5	82.0	97.5

5. What pattern does the change in mass of a radioactive material each year show?

Time (a)	0	1	2	3
Mass (g)	500	250	125	62.5

7. What pattern does the change in cost of renting a pickup truck show?

Distance (km)	0	100	200	300	400
Rental Cost (\$)	35.00	41.00	47.00	53.00	59.00

8. Predict the next number in each pattern.

- 3, 5, -7, 9, -11, $\blacksquare$
- 0.3, -0.09, 0.027, -0.0081, $\blacksquare$
- 4, -16, -36, -64, $\blacksquare$
- $\frac{1}{8}$, $\frac{11}{24}$, $\frac{19}{24}$, $\frac{27}{24}$, $\frac{35}{24}$, $\blacksquare$

Operations with Rational Numbers

Set of rational numbers $\mathbf{Q} = \left\{ \frac{a}{b} \mid a, b \in \mathbf{I}, b \neq 0 \right\}$. For all operations, begin by changing all mixed numbers to improper fractions.

Addition and Subtraction

Write equivalent fractions with a common denominator. Add or subtract numerators.

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

Multiplication

Reduce to lowest terms and multiply.

$$\left(\frac{a}{b} \right) \left(\frac{c}{d} \right) = \frac{ac}{bd}$$

Example 1

Evaluate $-1\frac{2}{5} + 3\frac{7}{10}$.

Solution

$$\begin{aligned} -1\frac{2}{5} + 3\frac{7}{10} &= \frac{-7}{5} + \frac{37}{10} \\ &= \frac{(-7)(2) + (37)(1)}{10} \\ &= \frac{-14 + 37}{10} \\ &= \frac{23}{10} \\ &= 2\frac{3}{10} \end{aligned}$$

Division

Multiply by the reciprocal of the divisor.

$$\begin{aligned} \frac{\frac{a}{b}}{\frac{c}{d}} &= \left(\frac{a}{b} \right) \left(\frac{d}{c} \right) \\ &= \frac{ad}{bc} \end{aligned}$$

Example 2

Evaluate $\frac{\left(\frac{2}{3} \right) \left(\frac{-3}{4} \right)}{2\frac{1}{2}}$.

Solution

$$\begin{aligned} \frac{\left(\frac{2}{3} \right) \left(\frac{-3}{4} \right)}{2\frac{1}{2}} &= \frac{\cancel{1}^1 \cancel{2}^1 \left(\frac{-\cancel{3}^1}{\cancel{4}^1} \right)}{\cancel{1}^1 \cancel{2}^1 \left(\frac{\cancel{2}^1}{\cancel{5}^1} \right)} \\ &= \frac{-1}{2} \\ &= \frac{-1}{2} \cdot \frac{5}{5} \\ &= \left(\frac{-1}{\cancel{2}^1} \right) \left(\frac{\cancel{2}^1}{5} \right) \\ &= -\frac{1}{5} \end{aligned}$$

Practice

1. Evaluate.

(a) $\frac{1}{2} - \frac{-2}{3}$

(b) $\frac{3}{5} - \frac{2}{3}$

(c) $\frac{-7}{11} + \frac{1}{3}$

(d) $\frac{5}{-6} + \frac{2}{-4}$

(e) $\frac{2}{-3} - 2\frac{5}{6}$

(f) $\frac{-3}{4} + 3\frac{2}{5}$

(g) $-8\frac{1}{4} + 3\frac{1}{2}$

(h) $\frac{7}{10} - \frac{-3}{4} + \frac{3}{5}$

(i) $1\frac{3}{4} + 2\frac{1}{5} - 4\frac{2}{3}$

(j) $\frac{-5}{7} + \frac{2}{3} - \frac{-1}{2}$

2. Evaluate.

(a) $\left(\frac{-2}{-3}\right)\left(\frac{9}{-10}\right)$

(b) $\left(\frac{1}{-2}\right)\left(\frac{2}{-5}\right)$

(c) $\left(\frac{-5}{12}\right)\left(\frac{4}{-15}\right)$

(d) $\left(2\frac{1}{2}\right)\left(-5\frac{2}{3}\right)$

(e) $\left(\frac{9}{4}\right)\left(\frac{-8}{27}\right)$

(f) $\left(-3\frac{1}{11}\right)\left(1\frac{1}{10}\right)$

(g) $\left(5\frac{3}{7}\right)\left(\frac{-7}{19}\right)$

(h) $\left(\frac{-2}{-9}\right)\left(-3\frac{1}{4}\right)$

(i) $\left(-4\frac{1}{3}\right)\left(-2\frac{3}{4}\right)$

(j) $\left(\frac{-2}{3}\right)\left(\frac{15}{-18}\right)\left(\frac{9}{5}\right)\left(\frac{-7}{8}\right)$

3. Evaluate.

(a) $\frac{2}{-3} \div \frac{-4}{3}$

(b) $\frac{-3}{8} \div \frac{-2}{3}$

(c) $-1\frac{1}{2} \div \frac{9}{4}$

(d) $2\frac{5}{7} \div \frac{-38}{21}$

(e) $-6\frac{1}{8} \div \frac{7}{-16}$

(f) $\frac{\frac{-2}{3}}{\frac{4}{9}}$

(g) $\frac{\frac{3}{-5}}{\frac{21}{-20}}$

(h) $\frac{\frac{5}{9}}{\frac{-10}{27}}$

(i) $\frac{\frac{-4}{5}}{\frac{8}{-15}}$

(j) $\left(2\frac{1}{3}\right) \div \left(-3\frac{2}{3}\right)$

4. Evaluate.

(a) $\frac{2}{-5} - \left(\frac{-1}{10} + \frac{-1}{2}\right)$

(b) $\frac{3}{5} - \left(2\frac{1}{2} - \frac{2}{3}\right)$

(c) $-3\frac{4}{7} - \left(\frac{2}{5} - 2\frac{2}{3}\right)$

(d) $\frac{-4}{9} \left(\frac{3}{8} - \frac{1}{12}\right)$

(e) $-2\frac{5}{8} \left(-1\frac{2}{3} + 4\frac{3}{4}\right)$

(f) $3\frac{4}{5} \div \left(2\frac{1}{4} - \frac{2}{3}\right)$

(g) $\left(2\frac{1}{8}\right)\left(-1\frac{3}{4}\right)\left(3\frac{2}{3}\right)$

(h) $\left(-1\frac{1}{8}\right)\left(2\frac{3}{4}\right) \div \left(3\frac{2}{3}\right)$

(i) $\left(\frac{5}{12}\right) \div \left(-1\frac{3}{4}\right)\left(3\frac{2}{3}\right)$

(j) $\left(1\frac{3}{8}\right)\left(-1\frac{3}{4}\right) \div \left(-3\frac{2}{3}\right)\left(2\frac{3}{4}\right)$

5. Evaluate the expression.

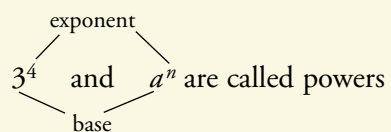
(a) $\frac{\frac{-4}{5} - \frac{3}{5}}{\frac{1}{5} - \frac{-1}{3}}$

(b) $\frac{\frac{1}{4} - \frac{-1}{3}}{\frac{-5}{12} - \frac{3}{-4}}$

(c) $\frac{1\frac{2}{3} - 2\frac{3}{4}}{-2\frac{3}{5} - 1\frac{1}{4}}$

(d) $\frac{6\frac{2}{5} - 8\frac{2}{3}}{-3\frac{5}{7} - 2\frac{3}{14}}$

Exponent Laws



4 factors of 3
 $3^4 = (3)(3)(3)(3)$

n factors of a
 $a^n = (a)(a)(a) \dots (a)$

Operations with powers follow a set of procedures or rules.

Rule	Description	Algebraic Description	Example
Zero as an Exponent	When an exponent is zero, the value of the power is 1.	$a^0 = 1$	$120^0 = 1$
Negative Exponents	A negative exponent is the reciprocal of the power with a positive exponent.	$a^{-n} = \frac{1}{a^n}$	$3^{-2} = \frac{1}{3^2}$ $= \frac{1}{9}$
Multiplication	When the bases are the same, keep the base and add exponents.	$(a^m)(a^n) = a^{m+n}$	$(5^4)(5^{-3}) = 5^{4+(-3)}$ $= 5^{4-3}$ $= 5^1$ $= 5$
Division	When the bases are the same, keep the base and subtract exponents.	$\frac{a^m}{a^n} = a^{m-n}$	$\frac{4^6}{4^{-2}} = 4^{6-(-2)}$ $= 4^{6+2}$ $= 4^8$
Power of a Power	Keep the base and multiply exponents.	$(a^m)^n = a^{mn}$	$(3^2)^4 = 3^{(2)(4)}$ $= 3^8$

Example

Simplify $\frac{(x^5y^7)^{-2}(x^2y)}{(x^{-2}y^{-4})^3}$ and evaluate for $x = 7$ and $y = -3$.

Solution

$$\begin{aligned}
 \frac{(x^5y^7)^{-2}(x^2y)}{(x^{-2}y^{-4})^3} &= \frac{x^{(5)(-2)}y^{(7)(-2)}x^2y}{x^{(-2)(3)}y^{(-4)(3)}} \\
 &= \frac{x^{-10}y^{-14}x^2y}{x^{-6}y^{-12}} \\
 &= x^{-10+2-(-6)}y^{-14+1-(-12)} \\
 &= x^{-10+2+6}y^{-14+1+12} \\
 &= x^{-2}y^{-1} \\
 &= (7)^{-2}(-3)^{-1} \\
 &= \left(\frac{1}{7^2}\right)\left(\frac{1}{-3}\right) \\
 &= -\frac{1}{147}
 \end{aligned}$$

Substitute $x = 7$ and $y = -3$.

Practice

1. Evaluate to three decimal places where necessary.

(a) 4^2	(b) 5^0
(c) 3^{-2}	(d) -3^2
(e) $(-3)^2$	(f) 2^{-3}
(g) $\left(\frac{1}{2}\right)^3$	(h) $\left(\frac{2}{3}\right)^{-2}$
(i) $(-1)^{75}$	(j) $(-1)^{92}$
(k) $(0.5)^2$	(l) $(-0.2)^{-3}$
(m) $(0.5)^4$	(n) $(3.1)^{-3}$
(o) $(-3.62)^{-5}$	(p) $1200(1.05)^9$

2. Express each power with a positive exponent and then simplify to an exact value.

(a) $3^0 + 5^0$	(b) $2^{-1} + 3^{-1}$
(c) $5 - 4^{-1}$	(d) $\left(\frac{1}{2}\right)^{-1} \left(\frac{2}{3}\right)^{-1}$
(e) $2^{-2} + 2^{-3}$	(f) $\left(\frac{1}{2}\right)^{-1} + \left(\frac{1}{3}\right)^{-1}$
(g) $12(2^{-1} - 3^{-1})$	(h) $\frac{2^{-1} + 3^{-1}}{\left(\frac{1}{3}\right)^2}$

3. Evaluate to an exact answer.

(a) $\frac{81^3}{9^7}$	(b) $\frac{-125^5}{625^3}$
(c) $(64^{-5})(8^9)$	(d) $\frac{(343^2)(243^2)}{(27^4)(49^4)}$

4. Simplify.

(a) $(x)^5(x)^3$	(b) $(m)^{-2}(m)^4(m)^{-3}$
(c) $(y)^{-5}(y)^2$	(d) $(a^b)^c$
(e) $(x^3)^4$	(f) $\frac{c^8}{c^7}$
(g) $\frac{t^5}{t^{-2}}$	(h) $\frac{(x^5)(x^{-3})}{x^{-2}}$
(i) $(n^{-5})^{-1}$	(j) $(m^2n^3)^4$
(k) $((m^{-3})^4)^{-2}$	(l) $\left(\frac{x^2}{y^{-3}}\right)^3$

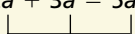
5. Simplify.

(a) $(x^2y^4)(x^{-3}y^2)$
(b) $(-2m^3)^2(3m^2)^3$
(c) $\frac{(5x^2)^{-2}}{(5x^2)^{-3}}$
(d) $(4u^3v^2)^{-2} \div (-2u^2v^3)^{-3}$

6. Simplify and then evaluate.

(a) $\frac{[(m^3)(n^{-2})]^{-1}}{m^{-5}n}$, $m = 5$ and $n = -2$
(b) $\frac{(27^{-m})^{-3}}{(9^n)^{-2}}$, $m = 1$, $n = 2$

Expanding, Simplifying, and Factoring Algebraic Expressions

Type	Description	Example
Collecting Like Terms $2a + 3a = 5a$ 	Add or subtract the coefficients of the like terms.	$3a - 2b - 5a + b$ $= 3a - 5a - 2b + b$ $= -2a - b$
Distributive Property $a(b + c) = ab + ac$	Multiply each term of the binomial by the monomial.	$-4a(2a - 3b)$ $= -8a^2 + 12ab$
Product of Two Binomials $(a + b)(c + d)$ $= ac + ad + bc + bd$	Multiply the first term of the first binomial by the second binomial and then multiply the second term of the first binomial by the second binomial. Collect like terms if possible.	$(2x^2 - 3)(5x^2 + 2)$ $= 10x^4 + 4x^2 - 15x^2 - 6$ $= 10x^4 - 11x^2 - 6$
Common Factoring $ab + ac = a(b + c)$	Find the largest common factor of each term.	$10x^4 - 8x^3 + 6x^5$ $= 2x^3(5x - 4 + 3x^2)$
Factoring Trinomials $ax^2 + bx + c$, when $a = 1$	Write as the product of two binomials. Determine two numbers whose sum is b and whose product is c .	$x^2 + 4x - 21$ $= (x + 7)(x - 3)$
Factoring Trinomials $ax^2 + bx + c$, when $a \neq 1$	Look for a common factor. If none exists, write as the product of two binomials using trial and error. Check by expanding and simplifying.	$3x^2 + 4x - 4$ $= (3x - 2)(x + 2)$ Check: $(3x)(x) + (3x)(2)$ $+ (-2)(x) + (-2)(2)$

Example

Factor $12w^2 - 26w + 12$.

Solution

$$\begin{aligned}
 &12w^2 - 26w + 12 && \text{There is a common factor of 2.} \\
 &= 2(6w^2 - 13w + 6) && \text{To have } +6, \text{ both factors must be the same sign.} \\
 & && \text{To have } -13w, \text{ both factors must be negative.} \\
 &= 2(3w - 2)(2w - 3)
 \end{aligned}$$

Practice

1. Simplify.

- (a) $3x + 2y - 5x - 7y$
- (b) $-6m + 5n - 3p - 3m + n$
- (c) $5x^2 - 4x^3 + 6x^2$
- (d) $6 - 5x + 3y - 8x + 2 - 7y$
- (e) $(4x - 5y) - (6x + 3y) - (7x + 2y)$
- (f) $(4x^3 + 3y^2) - (7y^2 + 5x^3) + (2x^3 - 5y^2)$
- (g) $m^2n + p - (2p - 3m^2n)$
- (h) $-(x^3 - y^3) + (3x^3 + 2y^3)$

2. Expand.

- (a) $3(2x + 5y - 2)$
- (b) $-5(x - y - 2z)$
- (c) $-2m(m - n)$
- (d) $5x(x^2 - x + y)$
- (e) $-2a(5a - 4a^2 - 3a^3)$
- (f) $-7y(2x^2 - 3y^2)$
- (g) $m^2(3m^2 - 2n)$
- (h) $-3u^3v^2(-6u^{-2}v^4 + 5u^4v^{-3})$
- (i) $x^5y^3(4x^2y^4 - 2xy^5)$

3. Expand and simplify.

- (a) $3x(x + 2) + 5x(x - 2)$
- (b) $-7h(2h + 5) - 4h(5h - 3)$
- (c) $2m^2n(m^3 - n) - 5m^2n(3m^3 + 4n)$
- (d) $-3xy^3(5x + 2y + 1) + 2xy^3(-3y - 2 + 7x)$

4. Expand and simplify.

- (a) $(3x - 2)(4x + 5)$
- (b) $(7 - 3y)(2 + 4y)$
- (c) $(5x - 7y)(4x + y)$

(d) $(3x^3 - 4y^2)(5x^3 + 2y^2)$

(e) $(3mn^2 - 2m)(5mn^2 + 4m)$

(f) $(4m^5n^3 - 3m^2n)(3m^5n^3 + 5m^2n)$

5. Factor.

- (a) $4 - 8x$
- (b) $9mn - 12n$
- (c) $6x^2 - 5x$
- (d) $3m^2n^3 - 9m^3n^4$
- (e) $28x^2 - 14xy$
- (f) $5mn - 7mn^3$

6. Factor each expression.

- (a) $x^2 - x - 6$
- (b) $x^2 + 7x + 10$
- (c) $x^2 - 9x + 20$
- (d) $x^2 - 3x - 28$
- (e) $3y^2 + 18y + 24$
- (f) $2t^2 - 16t - 30$
- (g) $m^2 - 9$
- (h) $25n^2 - 64p^2$

7. Factor.

- (a) $6y^2 - y - 2$
 - (b) $12x^2 + x - 1$
 - (c) $5a^2 + 7a - 6$
 - (d) $6m^2 - 7m + 2$
 - (e) $10x^2 + 9x - 9$
 - (f) $3x^2 - 13x - 10$
 - (g) $25n^2 - 30n + 9$
 - (h) $12x^2 - 18x - 12$
 - (i) $60x^2 + 8x - 4$
-

Solving Linear and Quadratic Equations Algebraically

Linear Equations

To solve a linear equation, first eliminate any fraction by multiplying all terms by the lowest common denominator. Eliminate any brackets using the distributive property, then isolate the variable. A linear equation has only one solution.

Example 1

Solve $-3(x + 2) - 3x = 4(2 - 5x)$.

Solution

$$\begin{aligned}-3(x + 2) - 3x &= 4(2 - 5x) \\ -3x - 6 - 3x &= 8 - 20x \\ -3x - 3x + 20x &= 8 + 6 \\ 14x &= 14 \\ x &= \frac{14}{14} \\ x &= 1\end{aligned}$$

Example 2

Solve $\frac{y-7}{3} = \frac{y-2}{4}$.

Solution

$$\begin{aligned}\frac{y-7}{3} &= \frac{y-2}{4} \\ 12\left(\frac{y-7}{3}\right) &= 12\left(\frac{y-2}{4}\right) \\ 4(y-7) &= 3(y-2) \\ 4y-28 &= 3y-6 \\ 4y-3y &= -6+28 \\ y &= 22\end{aligned}$$

Quadratic Equations

To solve a quadratic equation, first rewrite it in the form $ax^2 + bx + c = 0$. Then factor the left side, if possible. Set each factor equal to zero and solve. A quadratic equation can have no roots, one root, or two roots. Not all quadratic equations can be solved this way.

Example 3

Solve $x^2 + 3x = 10$.

Solution

$$\begin{aligned}x^2 + 3x &= 10 \\ x^2 + 3x - 10 &= 0 \\ (x+5)(x-2) &= 0 \\ \text{Then } x+5 &= 0 \quad \text{or} \quad x-2 = 0 \\ x &= -5 \quad \text{or} \quad x = 2\end{aligned}$$

Example 4

Solve $-x = 3 - 2x^2$.

Solution

$$\begin{aligned}-x &= 3 - 2x^2 \\ 2x^2 - x - 3 &= 0 \\ (2x-3)(x+1) &= 0 \\ \text{Then } 2x-3 &= 0 \quad \text{or} \quad x+1 = 0 \\ 2x &= 3 \quad \text{or} \quad x = -1 \\ x &= \frac{3}{2}\end{aligned}$$

Practice

1. Solve.

- (a) $6y - 8 = 4x + 10$
- (b) $2x + 7.8 = 9.4$
- (c) $13 = 5m - 2$
- (d) $13.5 - 2m = 5m + 41.5$
- (e) $35 = -1 - 6m$
- (f) $8(y - 1) = 4(y + 4)$
- (g) $4(5 - r) = 3(2r - 1)$
- (h) $12(n - 3) - 5(n + 2)$
 $= 2(n - 4) + 7$

2. Determine the root of each equation.

- (a) $\frac{x}{5} = 20$
- (b) $\frac{2}{5}x = 8$
- (c) $4 = \frac{3}{2}m + 3$
- (d) $\frac{5}{7}y = 3 + 12$
- (e) $3y - \frac{1}{2} = \frac{2}{3}$
- (f) $4 - \frac{m}{3} = 5 + \frac{m}{2}$
- (g) $\frac{3}{5}x + \frac{2}{3} = 2 - \frac{1}{3}x$
- (h) $\frac{1}{5}(3x + 1) = 2$
- (i) $\frac{1}{2}(x + 1) = \frac{1}{4}(x - 1)$
- (j) $\frac{5m - 1}{6} = \frac{2m + 3}{3}$

3. Solve.

- (a) $(x - 3)(x + 2) = 0$
- (b) $(2x + 5)(3x - 1) = 0$
- (c) $(m + 4)(m + 3) = 0$
- (d) $(3 - 2x)(4 + 3x) = 0$
- (e) $(2y + 5)(3y - 7) = 0$
- (f) $(5n + 3)(4 - 3n) = 0$

4. Determine the roots.

- (a) $x^2 - x - 2 = 0$
- (b) $x^2 + x - 20 = 0$
- (c) $m^2 + 2m - 15 = 0$
- (d) $6x^2 - x - 2 = 0$
- (e) $6t^2 + 5t - 4 = 0$
- (f) $2x^2 + 4x - 30 = 0$

5. Solve.

- (a) $4x^2 = 8x - 1$
- (b) $4x^2 = 9$
- (c) $6x^2 - x = 1$
- (d) $5x^2 - 6 = -7x$
- (e) $12x^2 = 2 + 5x$
- (f) $-4x + 1 = -2x^2$
- (g) $3x^2 + 5x - 1 = 2x^2 + 6x + 5$
- (h) $7x^2 + 2(2x + 3) = 2(3x^2 - 4) + 13x$

6. (a) What is the height of a triangle with an area of 15 cm^2 and a base of 5 cm ?

(b) A rectangular lot has a perimeter of 58 m and is 13 m wide. How long is the lot?

7. A model rocket is shot straight into the air. Its height in metres at t seconds is given by $h = -4.9t^2 + 29.4t$. When does the rocket reach the ground?

8. The population of a city is modelled by $P = 0.5t^2 + 10t + 200$, where P is the population in thousands and t is the time in years, with $t = 0$ corresponding to the year 2000. When is the population 350 000?