

Chapter 1

Patterns of Growth: Sequences

Patterns are fundamental to mathematics. Identifying patterns will allow you to make accurate predictions about the future and to analyze past events. You may see patterns in nature, music, art, architecture, and, of course, mathematics. You can model some patterns using algebraic relationships. For example, in this chapter, you will calculate interest, which grows in a predictable pattern, using formulas.

In this chapter, you will

- determine mathematical relationships for representing predictable patterns
- examine recursive relationships
- investigate and use arithmetic sequences to solve problems
- investigate and use geometric sequences to solve problems
- solve problems involving simple interest and compound interest
- revisit the laws of exponents and extend them to include rational exponents
- solve exponential equations

Visit the Nelson Mathematics student Web site at www.math.nelson.com. You'll find links to interactive examples, online tutorials, and more.

Connections



knapweed



purple loosestrife

The Chapter Problem

Controlling Non-Native Plant Populations

The first Canadian settlers brought with them plants from their native lands, for example, the lilac. When other non-native plants, seeds, animals, and birds are introduced to our natural environment, the results can be unpredictable.

Invasive or noxious non-native plants can destroy and disrupt wildlife habitat, threaten the existence of endangered species, and disrupt migratory bird flight patterns.

In Ontario, purple loosestrife, which was brought to North America in the ballasts of sailing ships in the nineteenth century, is overtaking much of the wetlands. Knapweed, another non-native plant, is spreading at an alarming rate through cultivated fields, pastures, and roadsides. Biologists look for ways to naturally control the rapid growth of these weeds without using harmful chemicals and pesticides.

Knapweed Facts

- Ontario is home to three varieties of knapweed: spotted knapweed, brown knapweed, and Russian knapweed.
- It takes one year for knapweed to germinate, or sprout, and produce seed.
- Four percent of the seeds in the seedbank, or cache of seeds, germinate each year. The remaining 96% of the seeds produced by each plant are added to the seedbank.
- About 25% of the seedlings grow to maturity in one year.
- One knapweed plant produces 1000 seeds per plant.
- Knapweed seeds remain viable in soil for eight years.
- Knapweed plants live for five years.

Source: Ontario Ministry of Agriculture and Food.

Non-Native Plant Problem

Suppose 100 knapweed seeds are accidentally carried into a field on hikers' clothing or shoes. If there are no other factors that will affect their growth, then how many knapweed plants and seeds will be produced in the field over ten years? When will the number of plants exceed five hundred trillion?

For help with this problem, see pages 25, 35, 50, 60, 82, and 96.

Challenge 1

Chess is a fascinating game, with a long history. There are many stories about the origin of chess. One ancient story suggests that chess was invented for a wealthy ruler in India. The ruler was so pleased with the game that he offered the inventor whatever he wanted as a reward. The inventor thought for awhile and then asked for grains of wheat, to be given to him as follows:

- one grain on the first square of the chessboard
- two grains on the second square
- four grains on the third square, and so on, until there were grains of wheat on each of the 64 squares of the chessboard

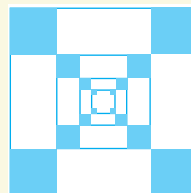
For each successive square, the number of grains of wheat is double.

1. The mass of a typical wheat grain is 0.0648 g. Calculate the total mass of wheat for the first ten squares.
2. One tonne (t) is 1 000 000 g. In 1996, the total world wheat production was 582 500 000 t. At this rate, how many years would it take to fill the chessboard completely?
3. Research to determine the selling price of 1 t of wheat in today's market. Use this information to determine the cash value of the inventor's reward.



Challenge 2

Jane draws a square and shades four corners. One side of each new square is one-quarter the length of the original square. Using the corners of the shaded squares Jane draws another square, and then shades its four corners in the same manner. If she repeats this pattern over and over again, what portion of the original square will be shaded?



Web Challenge

For a related Internet activity, go to www.math.nelson.com.

