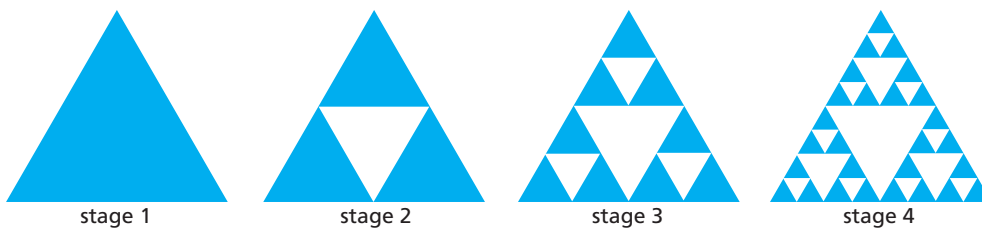


Here are six different relationships. Some of them contain a hidden pattern. Try to uncover the hidden pattern.

- A. Harry sells vacuum cleaners and earns \$400 each month. For each vacuum cleaner he sells, he earns an additional \$110.
- B. Marka deposits \$500 only once in a savings account that pays 5% interest at the end of each year.
- C. Andrew drops a rubber ball from a height of 2 m. After each bounce, the height of the ball's bounce is one-half the height of the ball's previous bounce.
- D. David tosses a coin repeatedly. Each time he tosses the coin, he records 1 if the coin comes up heads and 0 if the coin comes up tails.
- E. A square number can be represented by an arrangement of dots. Each square number is the total number of dots in each diagram. Here are the first four square numbers:



- F. In a fractal, a pattern repeats itself in the same shape. The Sierpinski triangle is an example of a **fractal**. Divide one equilateral triangle into four, congruent triangles by joining the midpoints of the sides of the larger triangle and remove the centre triangle. You can repeat these steps many times. Here are the first four stages of the Sierpinski triangle:



Think, Do, Discuss

- Copy and complete each table. List the five numbers that you enter for each table separately.

A.

Number of Vacuum Cleaners Sold	1	2	3	4	5
Total Earnings (\$)					

_____, _____, _____, _____, _____

B.

At End of Each Year	1	2	3	4	5
Money in Savings Account (\$)					

_____, _____, _____, _____, _____

C.

Bounce	1	2	3	4	5
Height of Ball After Each Bounce (m)					

_____, _____, _____, _____, _____

D.

Toss	1	2	3	4	5
Heads = 1, Tails = 0					

_____, _____, _____, _____, _____

E.

Diagram	1	2	3	4	5
Square Number					

_____, _____, _____, _____, _____

F.

Stage	1	2	3	4	5
Total Number of Triangles					

_____, _____, _____, _____, _____

- Which of the relationships are predictable? unpredictable?
- If possible, describe in words the pattern in each relationship.
- If possible, use the pattern to find the next three numbers in each list.
- Do any of these relationships come to an end? Explain.
- For each relationship and if possible, find an algebraic expression that describes the value of each term according to its position in the list. Let n represent the position of the term in the list, where n is a natural number, $\{1, 2, 3, \dots\}$.
- Use the algebraic expression you found in step 6 to determine the tenth number in each list.
- If n is the independent variable, then graph each predictable relationship. Can you join the points with a line or a curve? Explain.

Key Ideas

- A **sequence** is an ordered list of numbers. Each number is called a **term**.
- When a sequence follows a specific pattern, you can describe each term of the sequence by its position in the list. For example, in the sequence, 3, 6, 9, 12, 15, 18, ... , each term is a multiple of 3. The sixth term of the sequence is 18.

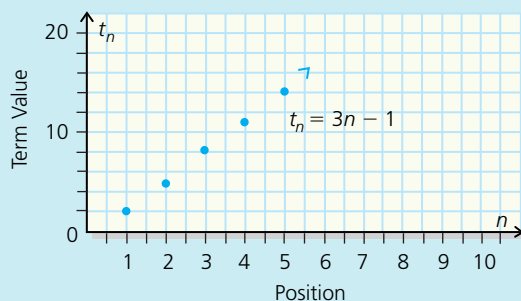
position	1	2	3	4	5	6
term	3	6	9	12	15	18

- The general form of a sequence is $t_1, t_2, t_3, \dots, t_n, \dots$. The subscript of each term identifies the position of the term in the sequence. As a result, n belongs to the set of natural numbers, $\{1, 2, 3, 4, 5, \dots\}$.
- A sequence that has a last term is a **finite sequence**. A sequence that has no last term, but continues on indefinitely, is an **infinite sequence**.

The sequence 2, 6, 18, 54, 162 is finite.

The sequence 1, 3, 4, 7, 11, ... is infinite. The ellipsis, ... , shows that the sequence continues on with the same pattern indefinitely.

- The most interesting sequences have patterns. This type of sequence can be described by a rule, or an algebraic expression. You can use the rule to generate one, several, or all of the terms of the sequence. This rule is called the **general term** of the sequence and it is represented by t_n . For example, the general term of the sequence 3, 6, 9, 12, 15, ... is $t_n = 3 \times n$.
- You can graph a sequence by plotting points, (position, term), on the x - y plane. You cannot join the plotted points with a straight line or a curve because the first element of each ordered pair is always a natural number. The graph of a sequence is then called **discrete**.



Example 1

Find, if possible, the next four terms of each sequence. If you cannot find the next four terms, then explain why.

- (a) The sequence of the number of seats in each of the first four rows in an auditorium, beginning with the first row, is 17, 22, 27, and 32.
- (b) The sequence of points in the last four football games that the Hamilton Tiger Cats played is 13, 21, 35, and 17.

Solution

- (a) The numbers increase constantly by 5. The next four terms are then 37, 42, 47, and 52.
- (b) The number of points scored in any game depends on many factors. Because the terms do not have a predictable pattern, the next four terms of this sequence cannot be determined.

Example 2

Determine the first four terms of each sequence.

- (a) $t_n = 3n^2 - 4$
- (b) $t_n = 5^{n-1}$
- (c) $t_n = \frac{n-2}{n+2}$

Solution

To find the first four terms of each sequence, substitute 1, 2, 3, and 4 for n .

(a) $t_n = 3n^2 - 4$

$t_1 = 3(1)^2 - 4$	$t_2 = 3(2)^2 - 4$	$t_3 = 3(3)^2 - 4$	$t_4 = 3(4)^2 - 4$
$= -1$	$= 8$	$= 23$	$= 44$

The first four terms are -1 , 8 , 23 , and 44 .

(b) $t_n = 5^{n-1}$

$t_1 = 5^{1-1}$	$t_2 = 5^{2-1}$	$t_3 = 5^{3-1}$	$t_4 = 5^{4-1}$
$= 5^0$	$= 5^1$	$= 5^2$	$= 5^3$
$= 1$	$= 5$	$= 25$	$= 125$

The first four terms are 1 , 5 , 25 , and 125 .

(c) $t_n = \frac{n-2}{n+2}$

$t_1 = \frac{1-2}{1+2}$	$t_2 = \frac{2-2}{2+2}$	$t_3 = \frac{3-2}{3+2}$	$t_4 = \frac{4-2}{4+2}$
$= -\frac{1}{3}$	$= \frac{0}{4}$	$= \frac{1}{5}$	$= \frac{2}{6}$
	$= 0$		$= \frac{1}{3}$

The first four terms are $-\frac{1}{3}$, 0 , $\frac{1}{5}$, and $\frac{1}{3}$.

Example 3

Determine whether each sequence is finite or infinite.

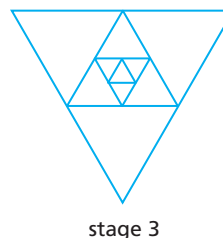
- (a) all positive integers divisible by 5
- (b) the leap years from 2000 to the end of the twenty-first century

Solution

- (a) The sequence is 5, 10, 15, 20, ... You can multiply each term by 5 to get the next term, and you could do this repeatedly, without end. This sequence is then infinite.
- (b) The year 2000 is a leap year. Then every fourth year after 2000 is a leap year. The last leap year in the century is 2096. The sequence is 2000, 2004, 2008, 2012, ... , 2096. Because the sequence has a last term, it is finite.

Example 4

The area of the largest equilateral triangle in the diagram, triangle 1, is 4096 cm^2 . The second triangle is the result of joining the midpoints of the sides of the first triangle. The third and other triangles were also drawn in this way.



- (a) Create a sequence by determining the area of each of the first four triangles.
- (b) Find the general term of this sequence.
- (c) What is the area of the eighth triangle?
- (d) Graph this sequence.

Solution

- (a) When you join the midpoints of any equilateral triangle, you divide the triangle into four, congruent, equilateral triangles. Therefore, the area of each of the four triangles is one-quarter the area of the larger triangle. Find the area of *each* smaller triangle by dividing the area of the larger triangle by 4, starting with the area 4096 cm^2 .

Triangle	1	2	3	4
Area (cm^2)	4096	$\frac{4096}{4} = 1024$	$\frac{1024}{4} = 256$	$\frac{256}{4} = 64$

- (b) The sequence is 4096, 1024, 256, 64, ... You could also generate each term by dividing the first term, 4096, by a multiple of 4, or a power of 4. Each exponent is 1 less than the position, n , of the term.

$$4096, \frac{4096}{4}, \frac{4096}{16}, \frac{4096}{64}, \dots$$

$$\frac{4096}{4^0}, \frac{4096}{4^1}, \frac{4096}{4^2}, \frac{4096}{4^3}, \dots$$

$$\text{The general term is } t_n = \frac{4096}{4^{n-1}}.$$

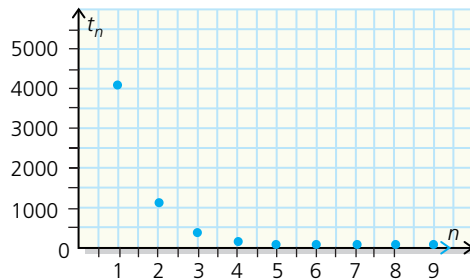
- (c) Find the area of the eighth triangle by determining t_8 .

$$\begin{aligned} t_8 &= \frac{4096}{4^8 - 1} \\ &= \frac{4096}{4^7} \\ &= \frac{4096}{16\,384} \\ &= 0.25 \end{aligned}$$

The area of the eighth triangle is 0.25 cm^2 .

- (d) The independent variable in this relationship is n , the term's position, and the dependent variable is t_n , the term's value. In any sequence, n is a natural number. Therefore, the points in the graph cannot be connected by a line or by a curve.

n	t_n
1	4096
2	1024
3	256
4	64
5	16



Practise, Apply, Solve 1.1

A

- Find the next four terms of each sequence.
 - 1, 5, 25, 125, ...
 - 16, 12, 8, 4, ...
 - 8, 14, 20, 26, ...
 - $1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \dots$
 - 9, 98, 987, 9876, ...
 - $-3, 6, -12, 24, \dots$
 - 5, 4, 9, 13, 22, ...
 - 6, 16, 36, 66, ...
 - 1, 2, 2, 4, 8, ...
 - 1, 8, 27, 64, ...
 - 0.3, 0.5, 0.7, 0.9, ...
 - 12, 6, 3, 1.5, ...
- Find the next four terms of each sequence. If you cannot find the next four terms, then explain why.
 - the dates in January when it snows: 2, 3, 8, 12, ...
 - the house numbers on the right side of a street: 1024, 1026, 1028, 1030, ...
 - the number of goals scored by the Toronto Maple Leafs in each of their home games: 1, 5, 4, 2, ...
 - the number of bacteria in a petri dish each hour: 48, 96, 192, 384, ...
 - Paula's annual salary: \$42 000.00, \$43 680.00, \$45 427.20, \$47 244.29, ...
 - the number of students waiting at your bus stop each day: 16, 12, 17, 17, ...

3. Determine the first five terms of each sequence. Start with $n = 1$.
- (a) $t_n = n + 4$ (b) $t_n = n^2 - 2$ (c) $t_n = 2^n$
 (d) $t_n = 5n - 2$ (e) $t_n = (n + 2)(n - 2)$ (f) $t_n = \frac{5}{n}$
 (g) $t_n = 3(-2)^{n+1}$ (h) $t_n = (-1)^n$ (i) $t_n = -3n + 7$

4. Determine t_{10} of each sequence.

- (a) $t_n = 5n - 12$ (b) $t_n = 3(2^{n-5})$ (c) $t_n = \frac{n}{n+1}$
 (d) $t_n = n^2 - 10n$ (e) $t_n = \sqrt{4.9n}$ (f) $t_n = \frac{2^{n-6}}{n-2}$

B

5. Determine the general term of each sequence.

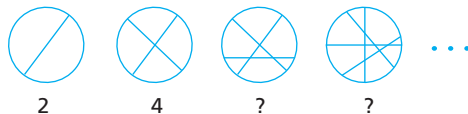
- (a) 3, 7, 11, 15, ... (b) 36, 42, 48, 54, ... (c) 4, 12, 36, 108, ...
 (d) $\frac{2}{3}, \frac{2}{9}, \frac{2}{27}, \frac{2}{81}, \dots$ (e) 5, -5, 5, -5, 5, ... (f) $1, \sqrt{2}, \sqrt{3}, 2, \dots$
 (g) 65, 61, 57, 53, ... (h) $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \dots$ (i) $1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \dots$

6. **Knowledge and Understanding:** Determine the first five terms, as well as t_{10} , for the sequence defined by $t_n = (5 - n)^n$.

7. Graph the first five terms of each sequence.

- (a) $t_n = 3n - 1$ (b) $t_n = 3^{5-n}$ (c) $t_n = -2n + 4$
 (d) $t_n = \sqrt{n+5}$ (e) $t_n = \frac{3}{n+1}$ (f) $t_n = (2n - 1)(3n)$

8. A circle can be cut into pieces, not necessarily equal, using straight lines. The circle can be cut into two pieces with one line, four pieces with two lines, and so on.



number of pieces

2

4

?

?

- (a) Determine the next three terms of this sequence.
 (b) Find t_{10} .
 (c) Describe your method for finding t_{10} .
9. (a) Use a calculator to determine the first ten terms of the sequence
 $\sqrt{2}, \sqrt{\sqrt{2}}, \sqrt{\sqrt{\sqrt{2}}}, \sqrt{\sqrt{\sqrt{\sqrt{2}}}}, \dots$
 (b) Describe the pattern of the sequence.
 (c) Predict t_{1000} of this sequence.
10. **Communication:** Explain why the graph of a sequence cannot be a line or a curve.
11. (a) Write four different sequences that each begin with 3, 6, ...
 (b) Describe the pattern in each sequence.
 (c) Determine the general term for each sequence.

12. Determine t_2 , t_5 , and t_{10} of each sequence.

(a) $t_n = 5n - 15$

(b) $t_n = n^4$

(c) $t_n = \frac{5}{n}$

(d) $t_n = (3n + 4)^2$

(e) $t_n = 2^{n+1}$

(f) $t_n = (-1)^{n+3}$

(g) $t_n = \frac{n-1}{2n+2}$

(h) $t_n = 3n^2 + 2n - 1$

(i) $t_n = (n-1)^3$

13. Determine whether each sequence is finite or infinite. Explain your answer.

(a) a pattern around the rim of a drinking glass

(b) the digits in the decimal equivalent of π

(c) the positive even numbers

(d) the square roots of all counting numbers

(e) the positive integer powers of 5, between 0 and 1 000 000

14. Application: Nikki invests \$1000 in a money market account that earns 4.5% of \$1000 each year.

(a) Determine the balance in her account at the end of the first, second, and third years.

(b) Find the general term for the balance in the account. Let n represent the year.

(c) Find the balance in the account after ten years.

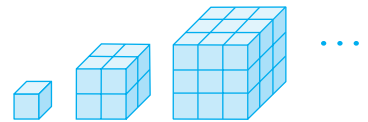
15. Sarah builds each cube with unit cubes.

(a) Write the sequence of the unit cubes in each cube.

(b) Determine the next three terms of the sequence.

(c) Write the general term of this sequence.

(d) How many unit cubes would Sarah need to build the 15th cube?



16. Mark drops a ball from a height of 20 m. After each bounce, the height of the ball's bounce is one-half the height of the ball's previous bounce.

(a) Write the sequence that shows the height of the ball's bounce after each of its first five bounces.

(b) Determine the general term of this sequence.

(c) What is the height of the ball's bounce after the tenth bounce?

(d) Is this sequence finite or infinite? Explain your choice.



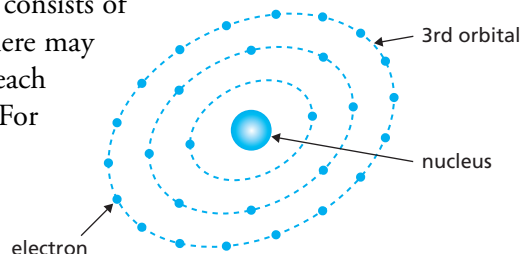
17. Marty's grandmother gives him 25¢ every time he visits. He visits her every Saturday.

(a) Write the sequence that shows the accumulated money that Marty receives for the first five Saturday visits.

(b) Determine the general term of this sequence.

(c) How much will Marty receive annually from his grandmother?

- 18. Thinking, Inquiry, Problem Solving:** An atom consists of electrons that orbit the nucleus of the atom. There may be from one to seven orbitals of electrons, and each orbital holds a maximum number of electrons. For the first four orbitals, these numbers are 2, 8, 18, and 32, respectively.



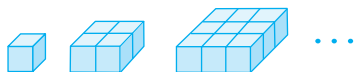
- Determine the maximum number of electrons in the outermost orbital.
 - Determine the general term of this sequence.
- 19. Check Your Understanding:** The number of bacteria in a culture doubles at the end of each hour. Ten bacteria are introduced into a petri dish.
- Create a table of the number of bacteria in the petri dish at the end of each hour for the first 5 h.
 - Extend the pattern in the table to determine the number of bacteria in the petri dish at the end of the eighth hour.
 - Find an algebraic expression that represents the number of bacteria in the petri dish at the end of the n th hour.
 - Use the algebraic expression to determine the number of bacteria at the end of the tenth hour.

C

- 20.** Consider the sequence 6, 12, 18, 24, ... Which of the following numbers are terms of this sequence?

216, 575, 492, 750, 824

- 21.** Erica builds each individual shape with unit cubes.



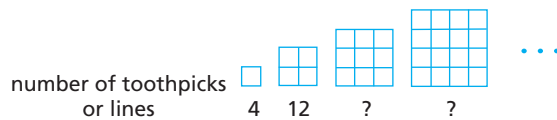
- Write the sequence that represents the surface area of each shape.
- Find the next three terms of the sequence.
- Determine the general term of the sequence.
- What is t_{20} ?

- 22.** Paul builds each shape with unit cubes.



- Write the sequence that represents the volume of each shape.
- Find the next three terms of the sequence.
- Determine the general term of the sequence.
- What is t_{20} ?

23. (a) Build or draw each stage of the diagram by starting with the first square and then adding to it. Use one toothpick or one short line for each side of the smallest square. Determine the next two terms of the sequence.



- (b) As you build or draw each stage, describe any patterns.
(c) How many toothpicks or short lines will the tenth diagram have?



The Chapter Problem—Controlling Non-Native Plant Populations

In this section, you studied sequences. Apply what you learned to answer these questions about the Chapter Problem on page 12.

- CP1.** How many of the original 100 knapweed seeds will germinate?
CP2. How many knapweed seeds will be added to the seedbank?
CP3. Of the seeds that do germinate, how many will grow to maturity in the first year?
CP4. How many new seeds will the mature plants add to the seedbank?
CP5. Use your answers for questions 1 to 4 to complete the table.

Year	Number of Seeds in the Seedbank	Number of Seeds that Germinate	Number of Remaining Seeds	Total Number of Plants	Number of New Seeds Produced
0	100	4	96	1	1000
1	1096				
2					
3					
4					
5					

- CP6.** Describe any pattern in the Total Number of Plants column.
CP7. Describe any pattern in the Number of Seeds in the Seedbank column.