

Chapter 1 Review

Patterns of Growth: Sequences

Check Your Understanding

1. Must a sequence have a recognizable pattern? Explain.
2. Can any sequence be represented by a formula or by a general term?
3. Explain why the set of natural numbers is used in the general term of a sequence.
4. How do you decide whether a list of numbers represents an arithmetic sequence or a geometric sequence? Use examples in your explanation.
5. In any arithmetic sequence,
 - (a) what is true about the rate of change between consecutive terms of the sequence?
 - (b) what type of relationship exists between n and t_n ?
6. In any geometric sequence,
 - (a) what is true about the rate of change between consecutive terms of the sequence?
 - (b) what type of relationship exists between n and t_n ?
7. Write the recursive formula for
 - (a) any arithmetic sequence
 - (b) any geometric sequence
8. Explain why an investment of \$1000 will be worth more in ten years if the interest is compounded annually, but is not simple.
9. What is the difference between the amount of an investment and the present value of an investment? Use an example in your explanation.
10. In your own words, define the n th root of a number a . Express the n th root of a using exponents.
11. Without using a calculator, evaluate $25^{-\frac{3}{2}}$ using two different methods. Verify both answers by using a calculator.
12. List and describe the steps for simplifying the expression $\left(\frac{1}{c}\right)^{n+m} (c^3)^m$.
13. How can you decide whether an exponential equation should be solved algebraically or by graphing?
14. Solve $6(3^{2x-1}) = 486$ algebraically. Verify your answer by solving this equation using graphing technology.

Review and Practice

1.1 Exploring Patterns and Sequences

1. What is a sequence? What is a term in a sequence? Use examples to illustrate your answers.
2. What is the general term of a sequence? How can you use the general term to generate the terms of a sequence?
3. Describe a situation that could be represented by
 - (a) a finite sequence
 - (b) an infinite sequence
4. Determine the first five terms of each sequence. Start with $n = 1$.
 - (a) $t_n = 4n + 1$
 - (b) $t_n = 2n^2 - 2n$
 - (c) $t_n = 3^{n-1}$
 - (d) $t_n = -3n + 2$
 - (e) $t_n = (2n + 1)(2n - 1)$
 - (f) $t_n = \frac{10}{n}$
5. Graph the first six terms of each sequence in question 4.
6. Determine the general term of each sequence.
 - (a) 2, 7, 12, 17, ...
 - (b) 30, 24, 18, 12, ...
 - (c) 9, 27, 81, 243, ...
 - (d) $\frac{5}{2}, \frac{5}{4}, \frac{5}{8}, \frac{5}{16}, \dots$
 - (e) 2, 5, 10, 17, ...
 - (f) $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \dots$
7. A car's purchase price is \$24 000. At the end of each year, the value of the car is three-quarters of the value at the beginning of the year.
 - (a) Write the first four terms of the sequence of the car's value at the end of each year.
 - (b) Determine the general term of this sequence.
 - (c) Find the value of the car at the end of the seventh year.

1.2–1.3U Sequences and Recursive Formulas

8. Compare how to generate the terms of a sequence from a recursive formula and from the general term of a sequence.
9. A recursive formula consists of two parts. Describe them.
10. Write the first five terms of each sequence.
 - (a) $t_1 = 7, t_n = (t_{n-1}) - 10$
 - (b) $t_1 = -3, t_n = 5(t_{n-1}) + 6$
 - (c) $t_1 = -4, t_n = (t_{n-1}) \times 2$
 - (d) $t_1 = 3, t_n = 4(t_{n-1})^2 - 2$

- 11.** Write a recursive formula for each sequence.
- (a) 2, 5, 8, 11, ... (b) 18, 11, 4, -3, ... (c) 3, 9, 81, 6561, ...
 (d) 2, 4, 6, 10, ... (e) $1, \frac{2}{3}, \frac{4}{9}, \frac{8}{27}, \dots$ (f) 4, 6, 12, 36, ...
- 12.** Graph the first five terms of each recursive sequence.
- (a) $t_1 = 6, t_n = (t_{n-1}) + 3$ (b) $t_1 = -4, t_n = 3(t_{n-1})$
 (c) $t_1 = 8, t_n = t_{n-1} \div 2$ (d) $t_1 = 2, t_n = 10 - t_{n-1}$
- 13.** Dennis must take 250 mg of sinus medication every 12 h. Only 25% of the medication remains in his body by the time he takes the next dose.
- (a) At what level will the mass of medication become constant?
 (b) How long will it take for the medication to reach this level?

1.4–1.6 Arithmetic Sequences

- 14.** Explain how to use the first differences between pairs of consecutive terms of a sequence to determine if the sequence is arithmetic.
- 15.** (a) Write the general term of any arithmetic sequence.
 (b) Explain the meaning of each variable in the formula in (a).
- 16.** Graph an arithmetic sequence in which
- (a) the first term is 4 and the terms increase by 3
 (b) the first term is 10 and the terms decrease by 2
- 17.** For each of the following arithmetic sequences, find
- i. the common difference ii. the general term iii. t_{15}
- (a) 6, 12, 18, 24, ... (b) -40, -25, -10, 5, ... (c) 36, 31, 26, 21, ...
 (d) $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1\frac{1}{4}, \dots$ (e) 1.2, 1.9, 2.6, 3.3, ... (f) 4, 36, 68, 100, ...
- 18.** Graph each sequence.
- (a) $t_n = -n + 5$ (b) $t_n = 7n - 4$ (c) $t_n = -3n + 7$ (d) $t_n = \frac{n+4}{2}$
- 19.** Ralph deposits \$5000 in a savings account that pays simple interest at 3.25%/a. He makes no other deposits.
- (a) By what amount will the year-end balance increase each year?
 (b) Write the sequence of the year-end balances over five years.
 (c) Determine the general term.
 (d) Graph this sequence.
 (e) At the end of one year, the balance in the account is \$5975. For how long has the original deposit earned interest?

20. Two terms have been given for each arithmetic sequence. Determine a , d , and t_n for each sequence.
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|---------------------------------|-------------------------------|
| (a) $t_5 = 24, t_9 = 36$ | (b) $t_6 = -41, t_{15} = -95$ |
| (c) $t_{11} = 91, t_{20} = 172$ | (d) $t_2 = -8, t_{12} = -158$ |

1.7 Geometric Sequences

21. Explain how to determine whether a sequence is geometric.
22. Graph a geometric sequence whose terms are
- | | |
|----------------|----------------|
| (a) increasing | (b) decreasing |
|----------------|----------------|
23. (a) Write the general term of any geometric sequence.
(b) Explain the meaning of each variable in the formula in (a).
24. For each of the following geometric sequences, determine
- | | | |
|-----------------------------------|-----------------------------------|---------------|
| i. the common ratio | ii. the general term | iii. t_{12} |
| (a) 2, 14, 98, 686, ... | (b) $-3, -12, -48, -192, \dots$ | |
| (c) 160, 80, 40, 20, ... | (d) $7, -21, 63, -189, \dots$ | |
| (e) 0.4, 0.08, 0.016, 0.0032, ... | (f) $6561, 2187, 729, 243, \dots$ | |
25. A transport truck that was bought for \$149 500 depreciates in value by 15% each year.
- (a) Starting with the purchase price, write the sequence of the truck's value at the end of each year for the next five years.
- (b) Determine the general term of this sequence.
- (c) Determine the truck's value at the end of the seventh year.

1.8 Compound Interest: Amount and Present Value

26. Explain why more money accumulates if the interest is compounded than if the interest is simple.
27. (a) Write the compound interest formula for determining the amount of an investment.
(b) Explain the meaning of each variable in the formula in (a).
28. Show how to use the formula for amount to determine present value.

- 29.** For each situation, determine
- the amount
 - the interest earned
- \$5000 borrowed for 7 years at 6%/a, compounded annually
 - \$10 500 invested for 4 years, at 4.8%/a, compounded monthly
- 30.** For each situation, determine
- the present value of
 - the interest earned on
- a loan of \$21 500 that will be due in 6 years. The interest rate is 8%/a, compounded quarterly.
 - a loan of \$100 000 that will be due in 5 years. The interest rate is 5%/a, compounded semiannually.
- 31.** Linh deposits \$12 000 in an account that pays 6%/a, compounded semiannually. After five years, the interest rate changes to 6%/a, compounded monthly. Calculate the value of the money eight months after the change in the interest rate.
- 32.** Mandy and her fiancé plan to buy a new house three years from now. They intend to make a down payment of \$20 000. The Eastern Bank offers an account with an interest rate of 8%/a, compounded monthly. How much money must they invest today to reach their goal?

1.9 Rational Exponents

- 33.** Copy and complete each exponent law.
- $x^{\frac{1}{n}} = \blacksquare$
 - $x^{\frac{m}{n}} = \blacksquare$
 - $x^{-\frac{m}{n}} = \blacksquare$
- 34.** Evaluate each power.
- $9^{\frac{1}{2}}$
 - $(-27)^{\frac{1}{3}}$
 - $-16^{\frac{1}{4}}$
 - $8^{\frac{1}{3}}$
 - $32^{\frac{1}{5}}$
 - $\left(\frac{1}{64}\right)^{\frac{2}{3}}$
 - $(27)^{-\frac{1}{3}}$
 - $(8)^{-\frac{4}{3}}$
 - $\left(\frac{4}{9}\right)^{\frac{3}{2}}$
 - $(32)^{-\frac{4}{5}}$
- 35.** Use a calculator to evaluate each power to two decimal places.
- $26^{\frac{1}{2}}$
 - $(-20)^{\frac{1}{3}}$
 - $-88^{\frac{1}{4}}$
 - $35^{\frac{1}{3}}$
 - $200^{\frac{1}{5}}$
 - $\left(\frac{3}{4}\right)^{\frac{1}{3}}$
 - $(945)^{-\frac{2}{3}}$
 - $(38)^{-\frac{4}{3}}$
 - $(1.35)^{\frac{3}{2}}$
 - $(26.55)^{-\frac{2}{5}}$
- 36.** Determine each amount.
- \$5000 borrowed for 10 months at 5%/a, compounded annually
 - \$5500 invested for 2 years 9 months, at 6%/a, compounded semiannually

- 37.** Determine the present value
- (a) of a loan of \$1500 that is due in 20 weeks. The interest rate is 6%/a, compounded quarterly.
 - (b) of a loan of \$100 000 that is due in 3 years 8 months. The interest rate is 7%/a, compounded semiannually.
- 38.** A rare coin was bought for \$1200; its value increases by 5% each year. Determine
- (a) an algebraic model for the coin's value over time
 - (b) the coin's value ten years after it was bought
 - (c) the coin's value 28 months after it was bought

1.10 Simplifying Expressions Involving Exponents

- 39.** State each of the eight exponents laws, then illustrate each with an example.
- 40.** Evaluate.
- (a) $25^{\frac{1}{2}} + 16^{\frac{3}{4}}$ (b) $8^{\frac{2}{3}} - 81^{\frac{3}{4}} + 4^2$ (c) $81^{-\frac{3}{4}} + 16^{-\frac{3}{4}} - 16^{-\frac{1}{2}}$
- 41.** Evaluate each expression for $x = -3$ and $y = 1$.
- (a) $\frac{2x^2y^{-2}}{x^3}$ (b) $(3x^2y^{-3})^{-3}$ (c) $\frac{3x^{-3}y^2}{x^3y^{-1}}$ (d) $\frac{x^{-2} - y}{y^{-1} + x^2}$
- 42.** Simplify.
- (a) $(2a^2b^2)^2(3a^4b)^{-2}$ (b) $(-4x^2y^{-3}z^4)^2$ (c) $(5x^3y^{-2})^3 \div (5x^{-2}y^3)^{-1}$
- 43.** Simplify.
- (a) $(a^{10} + 2p)(a^{-p} - 8)$ (b) $(2x^2)^3 - 2m\left(\frac{1}{x}\right)^{2m}$
- (c) $[(c)^{2n-3m}](c^3)^m \div (c^2)^n$ (d) $(x^{4n-m})\left(\frac{1}{x^3}\right)^{m+n}$

1.11 Solving Exponential Equations

- 44.** Give an example of an exponential equation.
- 45.** When may you solve an exponential equation algebraically?
- 46.** List the steps for solving an exponential equation using graphing technology.
- 47.** Solve each expression algebraically.
- (a) $3^{2x-1} = 27$ (b) $(2^{2x})(2^{x-1}) = 32$
- (c) $5^{3x+2} = \frac{1}{5}$ (d) $6^{2x-3} = \sqrt{6}$

48. Solve each expression using graphing technology. Round to two decimal places.

(a) $5^x = 50$

(b) $6^z = 12$

(c) $2.5^c = 100$

(d) $1.05^x = 1.5$

49. Solve.

(a) $(2)^{x^2} = (2)^{-12 - 7x}$

(b) $(3^x + 2)^x = \frac{1}{3}$

(c) $9^{x^2 + 1} = (3^{3x - 1})(3^{2x})$

(d) $4^{x^2} = (16^{1-x})\left[\left(\frac{1}{4}\right)^x\right]$

50. Determine the number of terms in each geometric sequence.

(a) 3, 12, 48, ..., 49 152

(b) -7, -14, -28, ..., -3584

51. How long will it take for \$1000 invested at 8%/a, compounded quarterly, to triple in value?

Chapter 1 Summary

In this chapter, you have used the mathematical relationships arising from patterns to solve problems about investment, growth, and half-life. You have found the general term of an arithmetic sequence, $t_n = a + (n - 1)d$; a geometric sequence, $t_n = ar^{n-1}$; and a recursive sequence.

You have calculated simple interest and compound interest, and used time lines, spreadsheets, and calculators to calculate the present value of invested money, $P = A(1 + i)^{-n}$, and the future value of invested money, $A = P(1 + i)^n$.

In addition, you have applied the exponent laws and rules to solve exponential equations involving rates of growth and decay.

Chapter 1 Review Test

Patterns of Growth: Sequences

1. i. Determine the first four terms of each sequence.
ii. Is each sequence arithmetic, geometric, or neither?
 - (a) $t_n = 6n + 5$
 - (b) $t_n = \frac{2}{n(n+1)}$
 - (c) $t_n = 3(4)^{n-1}$
 - (d) $t_1 = 5, t_2 = 1, t_{n+2} = 3t_{n+1} - 2t_n$, and $n \geq 1$
2. **Knowledge and Understanding**
Determine the number of terms in each sequence.
 - (a) $-6, -11, -16, \dots, -156$
 - (b) $4, 12, 36, \dots, 972$
3. For an arithmetic sequence, $t_{13} = -177$ and $t_{22} = -207$. Find a , d , and t_n .
4. Evaluate.
 - (a) $\left(\frac{-8}{27}\right)^{-\frac{2}{3}}$
 - (b) $(36^{-\frac{1}{2}} + 8^{-\frac{2}{3}}) \div (12^{-2})$
5. Simplify.
 - (a) $\frac{(16x^4y^2)^{\frac{1}{2}}(4y^6)^{\frac{1}{2}}}{(x^3y^6)^{\frac{1}{3}}}$
 - (b) $(a^{2m+3n})(a^{m-2n}) \div [(a^m)(a^{2m})]$
6. **Communication:** Explain how you can determine whether a given sequence is arithmetic or geometric. Use an example of each type of sequence in your explanation.
7. Solve.
 - (a) $2^{3(x-2)} = 64$
 - (b) $3^{x^2+4x} = \frac{1}{81}$
8. **Application:** Monica drops a rubber ball from a height of 60 m. After each bounce, the height of the ball's bounce is $\frac{3}{5}$ the height of the ball's previous bounce. How high does the ball bounce after the tenth bounce?
9. Jacques wants to invest now so that he will have \$20 000 in five years for a down payment on a new car. How much must Jacques invest now, if the interest rate is 8%/a, compounded semiannually?
10. The town of Lancaster is growing at a rate of 3% each year. The current population is 12 400. Determine when the population will reach 13 250.
11. In a small community college lecture hall, there are 15 rows of seats. For the first four rows, the number of seats are 14, 18, 22, 26, $\dots$. How many seats does the last row have?
12. **Thinking, Inquiry, Problem Solving**
Bernie invests \$5000 in a savings account that pays 6.75%/a, compounded semiannually. Three years later, he deposits another \$2000 in the account. What will be the balance in the account at the end of five years?