

Introduction to Nelson Mathematics 11

Nelson Mathematics 11 is designed to help you develop your skill at solving real problems using mathematical skills and logical reasoning. There are questions that give you a chance to practise familiar mathematical skills like solving equations, graphing data, and modelling. You will also have opportunities to tackle problems that require you to develop your own strategy for solving them. Throughout the book, you will be encouraged to use a variety of methods to communicate what you have learned to others.

Variety of Approaches

The lessons in *Nelson Mathematics 11* provide the opportunity to explore concepts on your own or through working with others. Some lessons provide several solved examples to help you build an understanding of a concept; others guide you through the investigation of a concept.

There are five kinds of lessons in *Nelson Mathematics 11*:

2.10 **Equivalent Rates and General Annuities**

In previous sections, you have worked with **ordinary simple annuities**. In an ordinary simple annuity, the payment interval corresponds to the interest conversion period. In this section, you will investigate **annuities** where this is not the case. An annuity where the payment interval does not correspond to the interest conversion period is called a **general annuity**.

When working through this section, remember to change the decimal mode on the TI-83 Plus calculator, as necessary.

Part 1: Equivalent Rates

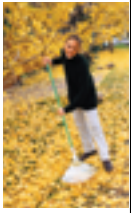
Two interest rates are said to be **equivalent** if they yield the same amounts at the end of one year, or at the end of any number of years. The **nominal** rate of interest is the annual rate.

Arithmetic Sequences **1.6**

Part 1: The Days of a Month

Here is a typical calendar for October. Do the dates have any predictable pattern?

October						
Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		



Think, Do, Discuss

- Write the sequence of dates for the last week of October. Start with Sunday's date.
- How many terms are in this sequence? Describe the pattern in this sequence.
- Extend the pattern and write the next five terms of this sequence. (Assume that the dates may be more than 31.)

Simplifying Expressions Involving Exponents **1.10** **SKILL BUILDER**

Quite often, you may have to apply several laws of exponents to simplify numerical and algebraic expressions involving exponents.

Rule	Algebraic Example	Description
product	$2^3 \times 2^5 = 2^8$	$a^m \times a^n = a^{m+n}$
quotient	$9^5 \div 3^2 = 3^7$	$a^m \div a^n = a^{m-n}$, $a \neq 0$, $m > n$
power of a power	$(2^3)^4 = 2^{12}$	$(a^m)^n = a^{mn}$
power of a product	$(2 \times 3)^4 = 2^4 \times 3^4$	$(ab)^n = a^n b^n$
power of a quotient	$\left(\frac{4}{5}\right)^3 = \frac{4^3}{5^3}$	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$, $b \neq 0$
zero as an exponent	$4^0 = 1$	$a^0 = 1$
negative exponents	$8^{-2} = \frac{1}{8^2}$	$a^{-n} = \frac{1}{a^n}$, $a \neq 0$
rational exponents	$8^{\frac{1}{3}} = (\sqrt[3]{8})$	$a^{\frac{1}{n}} = \sqrt[n]{a}$

The following examples illustrate how to simplify numerical expressions using these laws of exponents.

Example 1

Use the laws of exponents to simplify each expression.

(a) $(4^2 \times 9^3)^{\frac{1}{2}}$ (b) $\frac{5^3}{5^2 \times 5^{\frac{1}{2}}}$

Solution

(a) $(4^2 \times 9^3)^{\frac{1}{2}}$
 $= (4^2)^{\frac{1}{2}} \times (9^3)^{\frac{1}{2}}$ Apply the power of a product law.
 $= 4^1 \times 9^{\frac{3}{2}}$ Apply the power of a power law.
 $= 4 \times 81$ Evaluate $9^{\frac{3}{2}}$.
 $= 324$ Evaluate the product.

Concept Lessons

Concepts and ideas are presented through the use of Solved Examples and Key Ideas. Practise, Apply, Solve questions provide practice and an opportunity to consolidate your understanding.

Concept Lessons with Think, Do, Discuss Questions

Concepts and ideas are explored through real-life contexts. Your teacher may act as a coach and guide your class through the questions to build your understanding of new concepts and skills or you may be asked to answers these questions in groups or on your own. The Key Ideas provide a summary of the important ideas discussed. To clarify your understanding, several solved examples are presented, along with questions from the Practise, Apply, Solve questions.

Skill Builder Lessons

These lessons focus on the development of mathematical skills. The purpose of these lessons is to extend and improve your skills in mathematics involving familiar and unfamiliar concepts. Skills are taught through the use of solved examples and then reinforced through the Key Ideas and the Practise, Apply, Solve questions.

Technology Lessons

Technology lessons help you develop your skills with the use of technology, including graphing calculators, digital probes, spreadsheets, and graphing software. Step by step instructions and screen shots are provided.

Exploration Lessons

Exploration lessons often introduce and explore a new concept. These activities provide a hands-on experience that can be done in class or on your own at home.

Performance Tasks

These tasks range from short problem-solving questions to longer experiments and investigations. Your teacher may assign some of these activities so both of you can see how well you understand the concepts of the chapters. These can be done in class or as take-home tasks. You could also do these activities on your own for extra practice and review.

For some of these activities, you will work on your own, while for others you will likely work in a group. Along with quizzes, tests, and exams these activities provide an opportunity for you to demonstrate your understanding of the most important ideas in the grade 11 Functions and Relations course and Functions course.

TECHNOLOGY

1.2

TI-83 Plus Calculator: Generating the Terms of a Sequence

On the TI-83 Plus calculator, you can create a list from the general term that displays the specific terms of the sequence.

Generate or list the first five terms of the sequence defined by $t_n = n^2$.

1. Select sequence from the List OPS menu.

Press $\boxed{2\text{nd}} \boxed{\text{LIST}} \boxed{\text{OPS}}$. Scroll down to sequence and press $\boxed{\text{ENTER}}$.

2. Enter the information for sequence.

You will need to enter the following:

- the expression of the general term
- the variable n — for $\boxed{\text{X,T,Y,Z}}$ represent n
- the first position number
- the last position number

As an increment — the increment is 1, because the difference between each pair of consecutive natural numbers is always 1.

Press $\boxed{\text{X,T,Y,Z}} \boxed{\text{X,T,Y,Z}} \boxed{\text{X,T,Y,Z}} \boxed{\text{X,T,Y,Z}} \boxed{\text{X,T,Y,Z}} \boxed{\text{X,T,Y,Z}}$.

3. Generate the first five terms of the sequence.

Press $\boxed{\text{ENTER}}$.

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Continued

Exploration

Investigating Ways of Cutting "Vegetables"

1.5

When experienced cooks prepare dishes, they usually cut up a vegetable using different techniques. Often the cook cuts the vegetable in long thin strips. Then he or she stacks the pieces and then cuts in a perpendicular direction to the strips. Why does a cook do this?



Think, Do, Discuss

Use two rectangular pieces of paper and a pair of scissors. The paper will act as your "vegetable." You will cut each piece of paper in strips in two different ways.

Method 1

1. (a) Take one piece of paper and cut a thin strip (about 2 cm wide) along the width.



(b) Repeat cutting the entire sheet into strips, all of which should be about the same size.

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Performance Tasks for Part 1

Financial Applications of Sequences and Series

THE FOLLOWING ACTIVITIES SHOULD EACH TAKE LESS THAN A PERIOD TO COMPLETE.

1. Building a Pyramid

Build a model of a square pyramid using sugar cubes. Build the model so that it has at least six layers. Suppose you were to continue to build the model.

(a) How many sugar cubes would be in the 75th layer from the top?

(b) How many sugar cubes would you need to construct a model with 100 layers?

(c) Analyze this problem and create an algebraic model to describe this situation. Explain how you arrived at the algebraic model and how the algebraic model can be used to make predictions. Use an example to illustrate.

2. Population Growth

The People's Republic of China has the largest population of any country in the world. According to the 1994 World Population Prospects, approximately 1.1 billion people lived in China in 1990. In 1990, the population of China was growing at a rate of 1.5% per year.

(a) Suppose the population rate continued to grow at 1.5% per year. Create an algebraic model that relates the population, P , to the number of years, n , after 1990.

(b) Predict when the population will reach the two billion mark.

(c) Suppose that seven million people leave China each year for other countries. Adjust your model for the growth of the population of China to take this new information into account.

3. A Picture Is Worth a Thousand Words

Perhaps a picture could help save someone a thousand dollars. Imagine that you are a financial consultant and you want to provide your client with graphical representations of the effects of varying different options on a mortgage. Choose a current mortgage rate to work with and provide graphs showing the effects of changing payment periods and lengths of amortization. You may decide to look at several aspects to compare the mortgages—amount of interest paid, length of time to pay off the mortgage, principal owing at various time intervals, and so on.

PERFORMANCE TASKS FOR PART 1: FINANCIAL APPLICATIONS OF SEQUENCES AND SERIES 207

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Connections



The Chapter Problem
Controlling Non-Native Plant Populations
 The first Canadian settlers brought with them plants from their native lands, for example, the blue. When other non-native plants, seeds, animals, and birds are introduced to our natural environments, the results can be unpredictable.

Invasive or noxious non-native plants can destroy and displace wildlife habitat, threaten the existence of endangered species, and disrupt migratory bird flight patterns.

In Ontario, purple loosestrife, which was brought to North America in the hulls of sailing ships in the nineteenth century, is overrunning much of the wetlands. Knapweed, another non-native plant, is spreading at an alarming rate through cultivated fields, pastures, and meadows. Biologists look for ways to naturally control the rapid growth of these weeds without using harmful chemicals and pesticides.

Knapweed Facts

- Ontario is home to three varieties of knapweed: spotted knapweed, brown knapweed, and Russian knapweed.
- It takes one year for knapweed to germinate, or sprout, and produce seed.
- Four percent of the seeds in the seedbank, or cache of seeds, germinate each year. The remaining 96% of the seeds produced by each plant are added to the seedbank.
- About 25% of the seedlings grow to maturity in one year.
- One knapweed plant produces 1000 seeds per plant.
- Knapweed seeds remain viable in soil for eight years.
- Knapweed plants live for five years.

Source: Ontario Ministry of Agriculture and Food.

Challenge 1

Chess is a fascinating game, with a long history. There are many stories about the origin of chess. One ancient story suggests that chess was invented for a wealthy ruler to look. The ruler was so pleased with the game that he offered the inventor whatever he wanted as a reward. The inventor thought for awhile and then asked for grains of wheat, to be given to him as follows:

- one grain on the first square of the chessboard
- two grains on the second square
- four grains on the third square, and so on, until there were grains of wheat on each of the 64 squares of the chessboard

For each successive square, the number of grains of wheat is double.

1. The mass of a typical wheat grain is 0.0048 g. Calculate the total mass of wheat for the first six squares.
2. One tonne (t) is 1 000 000 g. In 1996, the total world wheat production was 582 500 000 t. At this rate, how many years would it take to fill the chessboard completely?
3. Research to determine the selling price of 1 t of wheat in today's market. Use this information to determine the cash value of the inventor's reward.

Challenge 2

Jane draws a square and shades four corners. One side of each new square is one-quarter the length of the original square. Using the corners of the shaded squares Jane draws another square, and then shades in four corners in the same manner. If she repeats this pattern over and over again, what portion of the original square will be shaded?



Web Challenge

Review of Essential Skills and Knowledge — Part 2

Functions

Using Properties of Relations to Sketch Their Graphs
 If the algebraic expression of a relation can be identified as linear or quadratic, its graph can be sketched without making a table or using graphing technology.

Linear Relations
 The general form of a linear relation is $ax + by + c = 0$. Once the relation is identified as linear, it can be graphed using the x - and y -intercepts or the slope, m , and y -intercept b , when $y = mx + b$.

Example 1
 Sketch the graph of $3x + 5y + 15 = 0$.

Solution
 The graph is linear. Use the x - and y -intercepts to draw the graph.
 Determine the x -intercept by letting $y = 0$.
 $3x + 5(0) + 15 = 0$
 $3x = -15$
 $x = -5$
 Determine the y -intercept by letting $x = 0$.
 $3(0) + 5y + 15 = 0$
 $5y = -15$

Example 2
 Sketch the graph of $3y = -2x - 6$.

Solution
 Rewrite the relation in the form $y = mx + b$.
 Then, $y = -\frac{2}{3}x - 2$. Mark the y -intercept as -2 and use the slope to locate a second point.
 The slope is $m = -\frac{2}{3}$. Then $\frac{\Delta y}{\Delta x} = -\frac{2}{3}$.



Getting Ready

In this chapter, you will be working with percents, decimals, rational numbers, exponents and their laws, linear and quadratic equations, series of relationships, and graphs of relationships. These exercises will help you warm up for the work ahead.

1. Extend the pattern to determine the next three numbers in each sequence.
 - (a) 3, 6, 9, 12, ...
 - (b) 5, 6, 8, 11, ...
 - (c) 1, 4, 9, 16, ...
 - (d) -4, -5, -1, 6, 7, ...
 - (e) -5, -10, -20, -40, ...
 - (f) $\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{8}{3}, \dots$
2. Express each percent as a decimal.
 - (a) 45%
 - (b) 8%
 - (c) 98%
 - (d) 5.25%
 - (e) 125%
3. Evaluate.
 - (a) 5% of 20
 - (b) 18% of 240
 - (c) 15.0% of 50
 - (d) 21% of 36
 - (e) 120% of 80
 - (f) 135% of 150
 - (g) 80% of 324
 - (h) 75% of 60
4. Evaluate.
 - (a) $\frac{1}{2} + \frac{3}{4}$
 - (b) $\frac{1}{2} - \frac{3}{4}$
 - (c) $\frac{3}{4} \times \frac{1}{2}$
 - (d) $\frac{3}{4} \div \frac{1}{2}$
 - (e) $(\frac{1}{2} + \frac{3}{4}) \times \frac{1}{2}$
 - (f) $\frac{1}{2} \times (\frac{3}{4} + \frac{1}{2})$
 - (g) $(\frac{1}{2} + \frac{3}{4}) - 24$
 - (h) $(\frac{1}{2} + \frac{3}{4}) \times \frac{1}{2}$
 - (i) $\frac{1}{2} + \frac{3}{4}$
 - (j) $\frac{1}{2} + \frac{3}{4}$
 - (k) $\frac{1}{2} + \frac{3}{4}$
 - (l) $\frac{1}{2} + \frac{3}{4}$
 - (m) $(\frac{1}{2} + \frac{3}{4}) \times \frac{1}{2}$
 - (n) $(\frac{1}{2} + \frac{3}{4}) \times \frac{1}{2}$
5. Evaluate.
 - (a) 2^3
 - (b) $(-3)^2$
 - (c) -5^3
 - (d) 6^{-2}
 - (e) $(\frac{1}{2})^3$
 - (f) $(\frac{1}{2})^{-2}$
 - (g) 13^0
 - (h) $(1.5)^0$
 - (i) $(-7)^{-2}$
 - (j) 4^0
 - (k) $(2^3)^3$
 - (l) $(4^{-2})^2$
6. Evaluate.
 - (a) $4x^2 - 3x + 1$, if $x = 2$
 - (b) $2x + 3$, if $x = 10$
 - (c) $2(3)^2 + 5$, if $x = 1$
 - (d) Px , if $P = 2000$, $x = 0.08$, and $t = 2$
 - (e) $P(1 + i)^t$, if $P = 1000$, $i = 0.05$, and $t = 10$
 - (f) $P(1 + i)^t$, if $P = 5000$, $i = 0.02$, and $t = 5$
7. Simplify.
 - (a) $(x^2y)^3$
 - (b) $(x^2y)(x^2y)$
 - (c) $(x^2)^3$
 - (d) $x^2 + x^2$
 - (e) $(5x^2y)(-2xy^2)$
 - (f) $(4x^2)^3$
 - (g) $(3x^2y)^3$
 - (h) $20x^2y^3 = 5x^2y^3$
 - (i) $(-2x^2y^3)^4$
 - (j) $\frac{(x^2)^3}{(x^2)^2}$
 - (k) $\frac{(2x^2y^3)(3x^2y^3)}{(4x^2y^3)^2}$
 - (l) $\frac{(2x^2y^3)^2}{(4x^2y^3)^2}$

Special Features of *Nelson Math 11*

Connections

Mathematics is not just something you do in mathematics class. You use it in other classes and in your daily life. It will also be important as you make a career choice. At the start of each chapter, there are Chapter Problems that apply math to various careers and situations. There are also two Challenge questions that require you to use your problem solving skills combined with what you learned in the chapter. A third Web Challenge requires you to use the Internet.

Features That Help You Prepare

Review of Essential Skills and Knowledge

This feature reviews concepts from previous grades to help you prepare for the work ahead.

1. Cover up the solution to each example.
2. Try to answer the example on your own. Show all of your work.
3. Compare your solution with the one in the book.
4. If you need to, ask another student or your teacher for help.
5. Once you understand the examples, do the Practice questions. Compare your answers to those in the book.

Getting Ready

These pages review important ideas from previous grades and chapters. Use this section to determine if you are ready to start the new work for this chapter. If you cannot do these questions, be sure to ask the teacher or another student for help.

Features That Help You Review

Check Your Understanding

These questions will help you decide, on your own, whether you understand the important ideas of a chapter. Think about each question and write the answer in your notebook. As you do this, you will be creating your own summary of the chapter. You can use your answers to these questions, along with the Chapter Review, to prepare for quizzes, tests, and exams.

Review and Practice

This feature lets you reinforce your understanding of the concepts and skills you have developed. Refer to the Key Ideas and Solved Examples when answering these questions. You can use these questions for review and study.

Chapter Review Test

Use this test to review and find out if you are ready for a class test or exam.

Cumulative Review Test

There is a Cumulative Review Test at the end of Chapters 2, 4, and 6. Each test incorporates concepts and ideas from the chapters that precede it. At the end of Chapter 7, there is also a year-end test that covers material from the entire book. You can use these tests as another source of review and as extra practice for exams.

Icons

These two icons identify an opportunity to use technology.



You will use a graphing calculator or a data probe.



You will use graphing software, *The Geometer's Sketchpad*, or a computer to develop a spreadsheet.

Chapter 1 Review

Patterns of Growth: Sequences Check Your Understanding

1. Must a sequence have a recognizable pattern? Explain.
2. Can any sequence be represented by a formula or by a general term?
3. Explain why the set of natural numbers is used in the general term of a sequence.
4. How do you decide whether a list of numbers represents an arithmetic sequence or a geometric sequence? Use examples in your explanation.
5. In any arithmetic sequence,
 - (a) what is true about the rate of change between consecutive terms of the sequence?
 - (b) what type of relationship exists between n and t_n ?
6. In any geometric sequence,
 - (a) what is true about the rate of change between consecutive terms of the sequence?
 - (b) what type of relationship exists between n and t_n ?
7. Write the recursive formula for
 - (a) any arithmetic sequence
 - (b) any geometric sequence
8. Explain why an investment of \$1000 will be worth more in ten years if the interest is compounded annually, but is not simple.
9. What is the difference between the amount of an investment and the present value of an investment? Use an example in your explanation.
10. In your own words, define the n th root of a number a . Express the n th root of a using exponents.
11. Without using a calculator, evaluate $25^{-\frac{3}{2}}$ using two different methods. Verify both answers by using a calculator.

Review and Practice

1.1 Exploring Patterns and Sequences

1. What is a sequence? What is a term in a sequence? Use examples to illustrate your answers.
2. What is the general term of a sequence? How can you use the general term to generate the terms of a sequence?
3. Describe a situation that could be represented by
 - (a) a finite sequence
 - (b) an infinite sequence
4. Determine the first five terms of each sequence. Start with $n = 1$.
 - (a) $t_n = 4n + 1$
 - (b) $t_n = 2n^2 - 2n$
 - (c) $t_n = 3^{n-1}$
 - (d) $t_n = -3n + 2$
 - (e) $t_n = (2n + 1)(2n - 1)$
 - (f) $t_n = \frac{1}{n}$
5. Graph the first six terms of each sequence in question 4.
6. Determine the general term of each sequence.
 - (a) 2, 7, 12, 17, ...
 - (b) 30, 24, 18, 12, ...
 - (c) 9, 27, 81, 243, ...
 - (d) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots$
 - (e) 2.5, 10, 17, ...
 - (f) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots$
7. A car's purchase price is \$24,000. At the end of each year, the value of the car is three-quarters of the value at the beginning of the year.
 - (a) Write the first four terms of the sequence of the car's value at the end of each year.
 - (b) Determine the general term of this sequence.
 - (c) Find the value of the car at the end of the seventh year.

1.2–1.3U Sequences and Recursive Formulas

8. Compare how to generate the terms of a sequence from a recursive formula and from the general term of a sequence.
9. A recursive formula consists of two parts. Describe them.
10. Write the first five terms of each sequence.
 - (a) $t_1 = 7, t_n = (t_{n-1})^2 - 10$
 - (b) $t_1 = -3, t_n = 5(t_{n-1}) + 6$
 - (c) $t_1 = -4, t_n = (t_{n-1}) \times 2$
 - (d) $t_1 = 3, t_n = 4(t_{n-1})^2 - 2$

Chapter 1 Review Test

Patterns of Growth: Sequences

1. i. Determine the first four terms of each sequence.
 - (a) $2^{n+1} - 3 = 64$
 - (b) $3^{n+1} + n = \frac{1}{2}$
- ii. Is each sequence arithmetic, geometric, or neither?
 - (a) $t_n = 6n + 5$
 - (b) $t_n = \frac{-2}{6n+1}$
 - (c) $t_n = 3(4)^{n-1}$
 - (d) $t_1 = 5, t_2 = 1, t_{n+2} = 3t_{n+1} - 2t_n$, and so on
2. **Knowledge and Understanding**
Determine the number of terms in each sequence.
 - (a) $-6, -11, -16, \dots, -156$
 - (b) 4, 12, 36, ..., 972
3. For an arithmetic sequence, $t_1 = -177$ and $t_{10} = -207$. Find a_n and t_n .
4. Evaluate.
 - (a) $(\frac{2}{3})^{\frac{1}{2}}$
 - (b) $(36^{-\frac{1}{2}} + 8^{-\frac{1}{3}}) + (12^{-\frac{1}{2}})$
5. Simplify.
 - (a) $(\frac{16a^2y^3}{z})^{\frac{1}{2}} (\frac{1}{4}yz)^{\frac{1}{2}}$
 - (b) $(a^{\frac{1}{2}}b + \frac{1}{2}a^{\frac{1}{2}}b^{-1})^2 + [(a^{\frac{1}{2}}b)^{-1}]^2$
6. **Communication:** Explain how you can determine whether a given sequence is arithmetic or geometric. Use an example of each type of sequence in your explanation.
7. Solve.
 - (a) $2^{n+1} - 3 = 64$
 - (b) $3^{n+1} + n = \frac{1}{2}$
8. **Application:** Monica drops a rubber ball from a height of 60 m. After each bounce, the height of the ball's bounce is $\frac{2}{3}$ the height of the ball's previous bounce. How high does the ball bounce after the tenth bounce?
9. Jacques wants to invest now so that he will have \$20,000 in five years for a down payment on a new car. How much must Jacques invest now, if the interest rate is 8%*a*, compounded semiannually?
10. The town of Lancaster is growing at a rate of 3% each year. The current population is 12,400. Determine when the population will reach 13,250.
11. In a small community college lecture hall, there are 15 rows of seats. For the first four rows, the number of seats are 14, 18, 22, 26, ... How many seats does the last row have?
12. **Thinking, Inquiry, Problem Solving:** Bernie invests \$5000 in a savings account that pays 6.75%*a*, compounded semiannually. Three years later, he deposits another \$2000 in the account. What will be the balance in the account at the end of five years?