



Part 1: Future Value of Annuities

Mr. Watts wishes to help pay for his 12-year-old granddaughter's future university expenses. Mr. Watts decides to deposit \$3000 at the end of each year, for seven years, in a savings account that pays interest at 7.5%/a, compounded annually. What will be the total value of this investment at the end of each year?

This type of investment, where fixed amounts are deposited or paid at regular intervals over a set period of time, is called an **annuity**. The **payment interval** is the time between successive payment dates. The annuity is called an **ordinary annuity** because the payments are made at the end of the payment intervals.



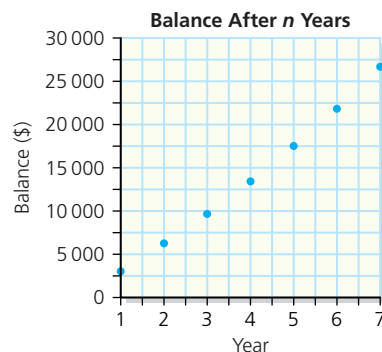
Queen's University

One way to represent an annuity is with a spreadsheet. Here are some of the values and formulas that Mr. Watts has entered in a spreadsheet to represent his annuity.

	A	B	C	D	E
1	Period	Balance	Interest at 7.5%/a	Payment	New Balance
2	1	0	=B2*0.075	\$3,000	=B2+C2+D2
3	=A2+1	E2	=B3*0.075	=D\$2	=B3+C3+D3

He obtained the following table and graph.

	A	B	C	D	E
1	Period	Balance	Interest at 7.5%/a	Payment	New Balance
2	1	\$0.00	\$0.00	\$3,000.00	\$3,000.00
3	2	\$3,000.00	\$225.00	\$3,000.00	\$6,225.00
4	3	\$6,225.00	\$466.88	\$3,000.00	\$9,691.88
5	4	\$9,691.88	\$726.89	\$3,000.00	\$13,418.77
6	5	\$13,418.77	\$1,006.41	\$3,000.00	\$17,425.17
7	6	\$17,425.17	\$1,306.89	\$3,000.00	\$21,732.06
8	7	\$21,732.06	\$1,629.90	\$3,000.00	\$26,361.97



Think, Do, Discuss

1. Why is the interest \$0 for the first period?
2. Explain how the new balance is calculated each year, or period.
3. Describe how the balance at the beginning of each period is determined.
4. In cell D3, the expression `=D$2` appears. Examine this column in the spreadsheet. What do you notice? Mr. Watts used the **Fill Down** command to complete the columns. What happens to the cell references in the spreadsheet? What effect do the \$ signs in `=D$2` have on the values in the Payment column?
5. Reproduce Mr. Watts' spreadsheet and the graph.
6. What is the balance in the account at the end of seven years? This balance is called the **amount**, or accumulated value, of an annuity. Describe two ways for finding the "accumulated" interest. Use the SUM function or the Σ toolbar button to determine the total interest earned.
7. Change the payment in cell D2 to determine the payment that will yield
 - (a) \$30 000 after seven years
 - (b) \$15 000 after seven years
8. Change the interest rate to (a) 5%/a and (b) 10%/a. What is the balance in the account after seven years for each change in interest rate? Determine the total interest earned for each of these rates. Compare the interest rates to the total interest earned. What do you notice?
9. The annual payment is \$3000. What must be the minimum annual interest rate, to one decimal place, so that the balance in the account after seven years is \$30 000?
10. Describe how you would modify the spreadsheet to find the annual balance for seven years if Mr. Watts made each payment at the beginning of the year. This type of annuity is called an **annuity due**. Modify your spreadsheet and compare it with your original spreadsheet.
11. What factors would affect the total future value of an annuity? List as many factors as possible.

Part 2: The Amortization Table

One day, you may want to buy an expensive item, such as a home, but you may not have the cash to do so. Then you can take out a loan or mortgage from a financial institution, for example, a credit union. You usually agree to repay, or amortize, the value of the loan, plus interest, by making regular equal payments for the **term**, or **amortization period**, of the loan. A portion of each **blended payment** is **interest**. The other portion is applied to reduce the **principal**, that is, the amount borrowed. In this way, a portion of each payment reduces the amount borrowed.

An **amortization table**, or schedule, shows

- the regular blended payment
- the portion of each payment that is interest
- the portion of each payment that reduces the principal
- the outstanding balance after each payment.

Here is the amortization table for the repayment of a \$4000 loan. The monthly payment is \$520. The interest rate is 12%/a, compounded monthly. The loan would be paid after eight months if the eighth payment was adjusted to \$542.88 (\$520 + \$22.88). The loan would also be paid after nine months with a final payment of \$23.11 (\$22.88 + \$0.23).

	A	B	C	D	E
1	Payment Number	Payment	Interest Paid	Principal Paid	Outstanding Balance
2	0				\$4,000.00
3	1	\$520.00	\$40.00	\$480.00	\$3,520.00
4	2	\$520.00	\$35.20	\$484.80	\$3,035.20
5	3	\$520.00	\$30.35	\$489.65	\$2,545.55
6	4	\$520.00	\$25.46	\$494.54	\$2,051.01
7	5	\$520.00	\$20.51	\$499.49	\$1,551.52
8	6	\$520.00	\$15.52	\$504.48	\$1,047.03
9	7	\$520.00	\$10.47	\$509.53	\$537.50
10	8	\$520.00	\$5.38	\$514.62	\$22.88
11	9	\$520.00	\$0.23	\$519.77	\$(496.89)

Lise takes out a \$9500 loan to purchase a used car. The interest rate is 9%/a, compounded monthly. She agrees to repay the loan in one year by making the same payment of \$830.79 at the end of every month.

Here is part of a spreadsheet that shows the amortization table of this loan.

	A	B	C	D	E
1	Payment Number	Payment	Interest Paid	Principal Paid	Outstanding Balance
2	0	_____	_____	_____	9500
3		830.79			

Think, Do, Discuss

1. Enter the information in the spreadsheet in a new spreadsheet. What expression would you enter in cell A3 to increase the payment number?
2. The interest rate is $9\%/a$, compounded monthly. What is the interest rate of the interest conversion period? Does the payment interval correspond to the interest conversion period? Explain.
3. Calculate the interest paid for the first month *on the principal*. Write an expression for this calculation and enter it in cell C3. How is the interest calculated for subsequent periods?
4. Subtract the interest for the first month from the payment. What remains? Write an expression to represent the remaining value and enter it in cell D3.
5. What is the outstanding balance at the end of the first month? Describe how to calculate the outstanding balance for a given period. Write an expression for the outstanding balance and enter it in cell E3.
6. Enter the values and expressions for the third row in your spreadsheet. Are any of these values constant? Explain. Use the **Fill Down** command to complete the amortization table.
7. What is the outstanding balance after the 12th payment? How do you know that the loan has been completely paid? How can you adjust the last payment so that the outstanding balance will be \$0? Make the adjustment.
8. Use the features of the spreadsheet to obtain a scatter plot that shows the outstanding balance at the end of each month.
9. Use the **SUM** function or the Σ toolbar button to find the total payments, the total interest paid, and the total principal paid. Do these values make sense? Explain.
10. Estimate how long it would take Lise to repay the loan if she reduced each payment by almost one-half to \$415.40. Modify the spreadsheet to determine how long it would take her to repay the loan.
11. Suppose each payment is still \$830.79, but the interest rate changes to $12\%/a$, compounded monthly. How much more would Lise have to pay in interest? What would be her last payment?
12. Lise may have chosen to purchase a new car of the same make. To buy the new car, she would have had to borrow \$28 000. How long would it take her to repay this loan under the original terms? How much interest altogether would she pay?
13. Suppose the monthly payment is \$415.40. How long would it take for Lise to repay this \$28 000 loan? How much total interest would Lise pay?
14. What would you tell someone who is purchasing a car about minimizing the total interest paid on a car loan?

Key Ideas

- An **annuity** is a series of payments, or deposits, at regular intervals.
- The **payment interval** is the time between successive payment dates.
- The **term** of an annuity is the time from the beginning of the first payment interval to the end of the last payment interval.
- The **periodic payment** of an annuity is the amount deposited or paid for each payment interval.
- In an **ordinary annuity**, the payment is made at the end of each payment interval. In an **annuity due**, the payment is made at the beginning of each payment interval. The annuity is called a **simple annuity** when the payment interval corresponds to the interest conversion period. For example, if the interest conversion period of the loan is a month, then the payment interval is a month. If the interest conversion period of the loan is three months (quarterly), then the payment interval is three months (quarterly).
- The annuity is called a **general annuity** when the payment interval does not correspond to the interest conversion period. This annuity will be examined in a later section.
- The **amount** of an annuity is the final value at the end of the term of the annuity. The amount includes all of the periodic payments and the compound interest.
- A spreadsheet allows you to change the parameters of an annuity problem and analyze the effects of the changes.
- Many people will take out a **loan** or **mortgage** to borrow a large sum of money to buy an expensive item, such as a car or a home. The components of a loan or mortgage are the **principal**—the amount borrowed—the interest rate, the interest conversion period, and the **periodic payment**.
- The periodic payment may be described as a **blended payment**, because a portion of the payment is interest and a portion of the payment reduces the principal. The **amortization period** of a loan or mortgage is the time taken for the debt to be paid in full if all the blended payments are made punctually.
- An **amortization table** shows the amount of the regular blended payment, the interest portion of each payment, the principal portion of each payment, and the outstanding balance after each payment. A spreadsheet is useful for creating amortization tables. A spreadsheet is also useful for analyzing the effects of changing the parameters of a loan problem.

Example 1

Samantha has a part-time job and is saving money for a trip to Europe. She deposits \$600 at the end of every three months in a savings plan that pays 8%/a, compounded quarterly. Determine the quarterly balances for the first 30 months.

Solution: Using a Spreadsheet

Both the \$600 deposit and interest are paid quarterly. The quarterly interest rate is 2%. Adjust the spreadsheet that you created for the Think, Do, Discuss of Part 1. In cell C2, enter the expression $=B2*0.02$ and, in cell D2, enter 600.

	A	B	C	D	E
1	Period	Previous Balance	Interest at 8%/a	Payment Made	New Balance
2	1	0	$=B2*0.02$	600	$=B2+C2+D2$
3	$=A2+1$	$=E2$	$=B3*0.02$	$=D2$	$=B3+C3+D3$

Use the **Fill Down** command to complete the table for ten payments that take place over 30 months.

	A	B	C	D	E
1	Period	Balance	Interest at 8%/a	Payment Made	New Balance
2	1	\$0.00	\$0.00	\$600.00	\$600.00
3	2	\$600.00	\$12.00	\$600.00	\$1,212.00
4	3	\$1,212.00	\$24.24	\$600.00	\$1,836.24
5	4	\$1,836.24	\$36.72	\$600.00	\$2,472.96
6	5	\$2,472.96	\$49.46	\$600.00	\$3,122.42
7	6	\$3,122.42	\$62.45	\$600.00	\$3,784.87
8	7	\$3,784.87	\$75.70	\$600.00	\$4,460.57
9	8	\$4,460.57	\$89.21	\$600.00	\$5,149.78
10	9	\$5,149.78	\$103.00	\$600.00	\$5,852.78
11	10	\$5,852.78	\$117.06	\$600.00	\$6,569.83

Example 2

Rodrigo plans to buy a new car in three years. At the end of three years, he would like to pay \$12 000 in cash. How much must he put in an account at the end of every month to achieve his goal? The account pays interest at 7.5%/a, compounded monthly.

Solution

The term of Rodrigo's annuity is 3 years, or 36 periods. The interest rate is 7.5%/a, or 0.625%/month. Adjust the spreadsheet that you created for the Think, Do, Discuss of Part 1. Estimate a reasonable monthly payment, for example, \$300.

	A	B	C	D	E
1	Period	Balance	Interest at 7.5%/a	Payment Made	New Balance
2	1	0	$=B2*0.00625$	600	$=B2+C2+D2$
3	$=A2+1$	$=E2$	$=B3*0.00625$	$=D$2$	$=B3+C3+D3$

Complete the table.

	A	B	C	D	E
1	Period	Balance	Interest at 7.5%/a	Payment Made	New Balance
2	1	\$0.00	\$0.00	\$300.00	\$300.00
3	2	\$300.00	\$1.88	\$300.00	\$601.88
4	3	\$601.88	\$3.76	\$300.00	\$905.64
5	4	\$905.64	\$5.66	\$300.00	\$1,211.30
6	5	\$1,211.30	\$7.57	\$300.00	\$1,518.87
7	6	\$1,518.87	\$9.49	\$300.00	\$1,828.36
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
34	33	\$10,590.85	\$66.19	\$300.00	\$10,957.05
35	34	\$10,957.05	\$68.48	\$300.00	\$11,325.53
36	35	\$11,325.53	\$70.78	\$300.00	\$11,696.31
37	36	\$11,696.31	\$73.10	\$300.00	\$12,069.41
38	37	\$12,069.41	\$75.43	\$300.00	\$12,444.85

The monthly payment should be lower, because the balance after 36 periods is more than \$12 000. Try other values for the payment so that the final balance after 36 months is as close as possible to \$12 000. The closest balance, after 36 months, is shown in this portion of another spreadsheet.

Rodrigo should deposit \$298.28 each month.

	A	B	C	D	E
34	33	\$10,530.13	\$65.81	\$298.28	\$10,894.23
35	34	\$10,894.23	\$68.09	\$298.28	\$11,260.59
36	35	\$11,260.59	\$70.38	\$298.28	\$11,629.25
37	36	\$11,629.25	\$72.68	\$298.28	\$12,000.22
38	37	\$12,000.22	\$75.00	\$298.28	\$12,373.50

Example 3

An ad in an electronics store window reads, “Big-screen television: Was \$3270: now \$2890, or \$500 down and \$122.81 for 24 months on approved credit.” What is the annual interest rate, compounded monthly, of the store’s payment plan?

Solution

The problem represents an annuity with an initial value of \$2390. Find the interest rate that would reduce the outstanding balance to \$0 after 24 payments. Estimate a reasonable monthly interest rate, for example, 1.5%/month, or 18%/a, and create a spreadsheet. Notice that, in cell E2, the payment in cell D2 is subtracted from the balance, since the payment reduces the debt. Use the **Fill Down** command, as necessary, to complete the spreadsheet.

	A	B	C	D	E
1	Period	Balance	Interest at 18%/a	Payment Made	New Balance
2	1	2390	=B2*0.015	122.81	=B2+C2-D2
3	=A2+1	=E2	=B3*0.015	=D\$2	=B3+C3-D3

	A	B	C	D	E
22	21	\$379.17	\$5.69	\$122.81	\$262.04
23	22	\$262.04	\$3.93	\$122.81	\$143.16
24	23	\$143.16	\$2.15	\$122.81	\$22.05
25	24	\$22.50	\$0.34	\$122.81	-\$99.97
26	25	\$99.97	-\$1.50	\$122.81	-\$224.28
27	26	-\$224.28	-\$3.36	\$122.81	-\$350.45

The loan is paid in full before the 24th payment, which means that the interest rate is too low. Try other interest rates. Notice that 1.75%/month is the best estimate, as shown.

	A	B	C	D	E
22	21	\$470.53	\$8.23	\$122.81	\$355.95
23	22	\$355.95	\$6.23	\$122.81	\$239.37
24	23	\$239.37	\$4.19	\$122.81	\$120.75
25	24	\$120.75	\$2.11	\$122.81	\$0.05
26	25	\$0.05	\$0.00	\$122.81	-\$122.76
27	26	-\$122.76	-\$2.15	\$122.81	-\$247.72

The store's interest charge is 1.75%/month, or 21%/a, compounded monthly.

Example 4

Mina took out a mortgage of \$75 800 to purchase a house in Massey. She agrees to repay the mortgage over 20 years with equal monthly blended payments of \$653.02 each. The interest rate is 8.4%/a, compounded monthly. Her final payment must be increased by \$0.25 to pay off the mortgage.

- How much interest will she pay over the life of the mortgage?
- Determine the outstanding balance for each of the first ten months.
- How much more interest would her purchase cost if her mortgage rate had been 9%/a?

Solution

- You could use the SUM function of the spreadsheet to find the sum of the interest column in an amortization table. Alternatively, find the interest by multiplying the payment by the total number of payments and subtracting the principal. Remember that \$0.25 is added to the last payment.

$$\begin{aligned}
 \text{total interest} &= \$653.02 \times 20 \times 12 - \$75\,800 + \$0.25 \\
 &= \$80\,925.05
 \end{aligned}$$

- (b) Create an amortization spreadsheet.

	A	B	C	D	E
1	Payment Number	Payment	Interest Paid	Principal Paid	Outstanding Balance
2	0	_____	_____	_____	\$75,800.00
3	=A2+1	653.02	=E2*0.007	=B3-C3	=E2-D3

	A	B	C	D	E
1	Payment Number	Payment	Interest Paid	Principal Paid	Outstanding Balance
2	0				\$75,800.00
3	1	\$653.02	\$530.60	\$122.42	\$75,677.58
4	2	\$653.02	\$529.74	\$123.28	\$75,554.30
5	3	\$653.02	\$528.88	\$124.14	\$75,430.16
6	4	\$653.02	\$528.01	\$125.01	\$75,305.15
7	5	\$653.02	\$527.14	\$125.88	\$75,179.27
8	6	\$653.02	\$526.25	\$126.77	\$75,052.51
9	7	\$653.02	\$525.37	\$127.65	\$74,924.85
10	8	\$653.02	\$524.47	\$128.55	\$74,796.31
11	9	\$653.02	\$523.57	\$129.45	\$74,666.86
12	10	\$653.02	\$522.67	\$130.35	\$74,536.51

- (c) Change the expression in cell C3 to =E2*.0075 and scroll down the spreadsheet until the outstanding balance is close to \$0.

	A	B	C	D	E
272	270	\$653.02	\$22.24	\$630.78	\$2,334.30
273	271	\$653.02	\$17.51	\$635.51	\$1,698.78
274	272	\$653.02	\$12.74	\$640.28	\$1,058.50
275	273	\$653.02	\$7.94	\$645.08	\$413.42
276	274	\$653.02	\$3.10	\$649.92	-\$236.50
277	275	\$653.02	-\$1.77	\$654.79	-\$891.29

If the interest rate is 9%/a, Mina would need to make 273 payments of \$653.02 each and one final payment of \$416.52 (\$413.42 + \$3.10) to pay the mortgage.

The total interest paid at 9%/a is

$$273 \times \$653.02 + \$416.52 - \$75\,800 = \$102\,890.98$$

The total interest paid at 8.4%/a is \$80 925.05

A 0.6% increase in the mortgage rate to 9% would cost Mina an additional \$21 965.93 in interest. This is a substantial difference. This example suggests that people should research mortgage rates to find the lowest possible interest rate.

Practise, Apply, Solve 2.5

A

Refer to Example 1 on page 134 to answer questions 1 and 2.

1. (a) What is Samantha's periodic payment?
(b) Is her savings plan an example of an ordinary annuity or an annuity due? Explain.
(c) Why is her savings plan a simple annuity?
(d) Determine the interest earned and the new balance after the 11th payment.
(e) How would you modify the spreadsheet if Samantha already had \$1500 in her savings plan prior to making deposits?
2. (a) Without using a spreadsheet, determine Samantha's quarterly balances for the first two years if she deposited \$600 at the beginning of each month for three months.
(b) What type of annuity is described in (a)?
(c) How much more interest would she have earned over the two years?
(d) Describe how you would modify the spreadsheet for the change in payment.

Refer to Example 2 on page 134 to answer questions 3 and 4.

3. (a) What is the payment interval of Rodrigo's annuity?
(b) Why are there 36 payment intervals in this example?
(c) How much interest is earned in the 34th payment interval?
(d) In the final spreadsheet, the balance at the end of the 36th period is \$12 000.22. Explain why the payment cannot be adjusted to give a balance of exactly \$12 000.
(e) How would you modify the spreadsheet so that the interest rate is 6%/a, compounded monthly?
4. For a monthly payment of \$300, the balance will grow to \$12 069.41 in three years. For a monthly payment of \$298.28, the balance will grow to \$12 000.22 in three years.
(a) Explain the difference between the balances.
(b) Find the interest earned for each situation and determine the difference.

B

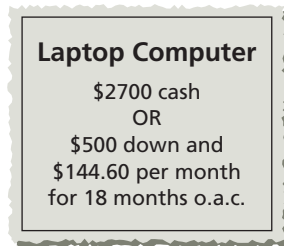
5. **Communication:** Phong wants to purchase a motorcycle. He can borrow \$6500 at 10%/a, compounded quarterly, if he agrees to repay the loan by making equal quarterly payments for four years. Estimate a reasonable quarterly payment. Explain your answer.

6. For four years, you deposit \$250 at the end of every three months in an annuity earning 10%/a, compounded quarterly.
 - (a) What is the term of the annuity?
 - (b) What is the payment interval?
 - (c) How many payments would you make altogether?
 - (d) What is the interest rate for the conversion period?
7. **Knowledge and Understanding:** Meredith borrows \$3600 to help pay for nurse's college. She will repay the loan by making equal blended payments of \$625 each at the end of every six months. The interest rate is 11%/a, compounded semiannually.
 - (a) Without using a spreadsheet, construct an amortization table for the loan repayment.
 - (b) What is the final payment if she takes all of the four years to repay the loan?
 - (c) What is the last payment if she chooses to repay the loan in seven payments?
8. Verify your work in question 7 by using a spreadsheet to create an amortization table of Meredith's loan.
9. Mr. Abourbih has \$12 000 in a savings account that pays 5%/a, compounded quarterly. Mr. Abourbih intends to deposit \$500 at the end of every three months in the account for five years.
 - (a) Use a spreadsheet to determine the balance in the account over the five years.
 - (b) How much more would the balance be at the end of five years if each deposit were \$550?
 - (c) How much more would the balance be at the end of five years if the interest rate were 6.5%/a, compounded quarterly?
 - (d) How much do you think you could realistically save every three months? How much are your current savings?
 - (e) Use the values in (d) to determine how much you could save over the same five years.
10. Prima just started grade 9 and would like to save \$12 000 over the next four years to help pay for her first year of university.
 - (a) Determine, to the nearest cent, how much she should deposit at the end of every three months in an account that pays 6%/a, compounded quarterly, to attain her goal.
 - (b) Determine how much she should deposit at the end of each month for four years if the account pays 6%/a, compounded monthly.
 - (c) Compare the payments in (a) and (b). Explain why the payment in (a) is not quite three times the payment in (b).

11. Kacee borrows \$1800 to buy a computer. She will repay the loan by making equal blended payments of \$165.25 each at the end of each month. The interest rate is 18%/a, compounded monthly.
- What is the interest rate per conversion period?
 - Without using a spreadsheet, create an amortization of her loan.
 - Verify the amortization in (b) by using a spreadsheet.
 - Graph the amortization of her loan.

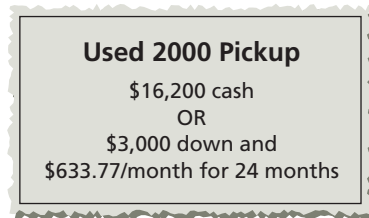


12. Determine the annual interest rate, compounded monthly, of the payment plan for the computer.



13. **Thinking, Inquiry, Problem Solving:** Two twin sisters, Bev and Cindy, just celebrated their 15th birthdays. Bev decides to start making regular annual deposits of \$1000 each at 7%/a, compounded annually, beginning today and ending on her 26th birthday. At that time, she will not make any more deposits. On her 26th birthday, Cindy decides to start making regular annual deposits of \$1000 each at the same interest rate. Determine when or if Cindy's investment will be equal to Bev's investment. Include tables and arguments in your solution.
14. Lakshmi Samson is saving to buy a house. After every six months, she deposits \$1800 in an account that pays 8%/a, compounded semiannually.
- How much money will she have saved in six years?
 - After making the first deposit of \$1800, she increases each subsequent payment by 10%. How much money will she have saved in six years?
15. **Application:** Stephanie buys a boat by paying 10% down and by financing the balance with a bank loan at 15%/a, compounded monthly. She arranges to amortize the loan by making monthly payments of \$225 each for three years. Determine what Stephanie paid for the boat.

16. Ms. Reynen is thinking about retirement plans that would enable her to retire at age 60. She may deposit \$1000 each year in an registered retirement savings plan (RRSP), beginning at age 30. Or she may deposit \$3000 each year in an RRSP, beginning at age 50. Both plans offer the same annual compound interest rate. Analyze the effect that the interest rate would have on the value of each plan when Miss Reynen turns 60. Report your findings.
17. On his way to work, Mr. Pretz sees this ad for a pickup truck.



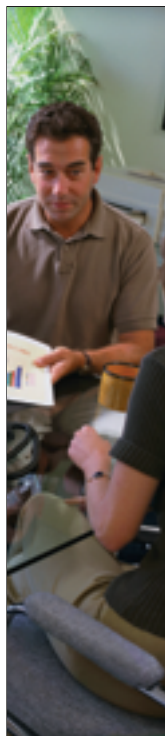
- (a) Determine the annual interest rate, compounded monthly, of the payment plan.
 - (b) Mr. Pretz is interested in buying the truck. He knows that he can get a loan from his bank to finance the cash price of the truck. The interest rate is 8%/a, compounded monthly. What monthly payment would enable him to repay the bank loan in two years?
 - (c) Why is the payment in (b) greater than the advertised payment?
 - (d) Mr. Pretz decides to make the \$3000 down payment and finance the remaining amount through a bank loan as in (b). Determine how much money he will save by financing the loan through his bank instead of through the truck dealer.
18. The Larsen family has just financed the purchase of a motor home for \$48 000. They agree to repay the loan by making equal monthly blended payments of \$595 each at 8.4%/a, compounded monthly.
- (a) Create an amortization of their loan.
 - (b) How long will it take to repay the loan?
 - (c) How much will be the final payment?
 - (d) Determine how much interest they will pay over the life of the loan.
 - (e) Use technology to graph the amortization of the loan.
 - (f) How much sooner would the loan be paid if they had made a 10% down payment?
 - (g) How much would the Larsens have saved in interest had they obtained a loan at 7.2%/a, compounded monthly?

19. Check Your Understanding

- (a) Without using a spreadsheet, show how an annuity that pays 6.6%/a, compounded monthly, will grow over eight months. A periodic payment of \$200 is paid at the end of each month.
- (b) Determine the value of the annuity if the monthly payment is made at the beginning of each month. Explain the differences between the two payment options.



- 20.** Ansley wants to borrow \$40 000. She must decide how to repay the loan. She could either make quarterly payments of \$3000 each at 12%/a, compounded quarterly, or make monthly payments of \$1000 each at 12%/a, compounded monthly.
- (a) Describe the advantages and disadvantages of each option.
 - (b) What advice would you give Ansley? Justify your advice.
- 21.** Fatuma has decided to invest \$3000 at the end of each year for the next five years in a savings account that pays 8%/a, compounded semiannually. Use a spreadsheet to determine the amount of the annuity.



The Chapter Problem—Financial Planning

In this section, you studied the use of spreadsheets to analyze financial situations. Apply what you learned to answer these questions about the Chapter Problem on page 106.

- CP5.** Bart's education fund earns 6%/a, compounded monthly. Create a spreadsheet to represent the monthly balance in the fund. Use it to determine the balance after the 20th payment.
- CP6.** How many payments will take place by the time Bart reaches his 18th birthday? Determine the accumulated value of Bart's education fund on his 18th birthday.
- CP7.** Create an amortization table to represent the repayment of Mr. Sacchetto's car loan. Print the table.
- CP8.** When will the car loan be paid? What is the amount of the final payment?
- CP9.** Can you create an amortization table to represent the repayment of Mr. Sacchetto's mortgage? Explain.