

Using Technology to Analyze Canadian Mortgages

2.12

By regulation, the annual interest rates of the majority of Canadian mortgages are compounded, *at most*, semiannually. However, the payment interval is much more frequent. Monthly, weekly, or bi-weekly payment intervals are the most popular.

Mortgage amortizations in Canada can be represented by general annuities. The periodic payment is usually determined using a repayment or amortization period of 20 years or 25 years. But the interest rate is not guaranteed for either length of time. The rate is usually guaranteed, or fixed, for a specific time, such as six months, one year, two years, three years, or five years. At the end of this time, the term and interest rate may be **renegotiated**.

Some mortgages are completely **open**. The word “open” means that the interest rate is not fixed but changes as the bank’s prime lending rate changes. Many mortgages allow you to make extra payments during the term or to increase the periodic payment by as much as 20% per year. Many financial institutions offer loan and mortgage calculators on their Web sites. Visit the Nelson Thomson Learning Web site at www.math.nelson.com for links to some of these sites.

Example 1

Carmen took out a mortgage of \$75 800 to purchase a farmhouse outside of Kingston. She will repay the mortgage with equal monthly blended payments. The interest rate is 8.4%/a, compounded semiannually. The term is 20 years.

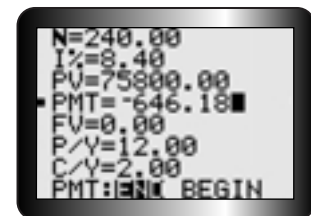


- (a) Determine the monthly payment.
- (b) How much interest will Carmen pay over the life of the mortgage?

Solution: Using the TVM Solver

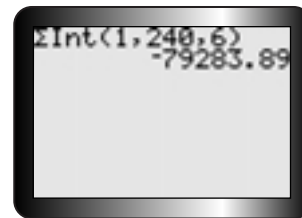
- (a) Solve for the payment, R , using the **TVM Solver**. Enter the values, except for the value for PMT, shown in the screen. Scroll to the line containing **PMT** and press **[ALPHA]** **[ENTER]**. The value of the payment is negative, because the payment is an outflow of Carmen’s cash.

The monthly payment is \$646.18.



- (b) The total interest that Carmen will pay is the sum of the interest payments from the first payment to the 240th payment. Use ΣInt from the Finance CALC menu.

Carmen will pay \$79 283.89 in interest over the life of the mortgage.



Solution: Using a Spreadsheet

- (a) First find the interest rate, i , compounded monthly, that is equivalent to 8.4%/a, compounded semiannually.

$$8.4\% \div 2 = 4.2\%$$

$$(1 + i)^{12} = (1.042)^2 \quad \text{Take the 12th root of each side.}$$

$$1 + i = \sqrt[12]{(1.042)^2}$$

$$1 + i = (1.042)^{\frac{1}{6}}$$

$$i = (1.042)^{\frac{1}{6}} - 1$$

$$i \doteq 0.006\,880\,553\,5$$

Create a spreadsheet to produce the amortization table. As you did in section 2.5, change the payment value until you find the value that results in a balance of \$0 after 240 payments.

	A	B	C	D	E
1	Payment Number	Payment	Interest Paid	Principal Paid	Outstanding Balance
2	0	—	—	—	\$75,800.00
3	=A2+1	\$646.18	=E2*0.0068805535	=B3-C3	=E2-D3

- (b) As in section 2.5, use the SUM function or the Σ toolbar button to determine the total interest paid.

Solution: Using Algebra

- (a) Use $i = 0.006\,880\,553\,5$, the value calculated above, because the interest is compounded semiannually, but the payments are monthly.

Now find the weekly deposit, R , using the formula for present value, PV , of an ordinary simple annuity.

$$PV = 75\,800, n = 20 \times 12 \text{ or } 240, \text{ and } i = 0.006\,880\,553\,5$$

$$PV = R \times \frac{1 - (1 + i)^{-n}}{i} \quad \text{Substitute the values for } PV, n, \text{ and } i.$$

$$75\,800 = R \times \frac{1 - (1.006\,880\,553\,5)^{-240}}{0.006\,880\,553\,5} \quad \text{Simplify and solve for } R.$$

$$R = 646.18$$

Carmen's monthly payment is \$646.18.

- (b) Find the total interest that Carmen will pay by multiplying the monthly payment by the number of payments and subtracting the principal.

$$\begin{aligned}\text{total interest paid} &= \$646.18 \times 240 - \$75\,800 \\ &= \$79\,283.20\end{aligned}$$

To find a more accurate answer for the interest paid, use the unrounded value of R , 646.182 868 9, in your calculation.

$$\text{total interest paid} = \$646.182\,871 \times 240 - \$75\,800, \text{ or about } \$79\,283.89.$$

Notice that the difference is \$0.69.

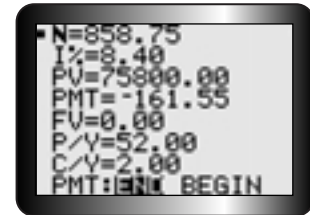
Example 2

Analyze the effect of changing the payment from \$646.18 monthly to \$161.55 weekly on the term of Carmen's loan. Discuss your findings.

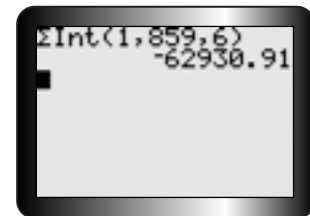
Solution: Using the TVM Solver

$$\$646.18 \div 4 = \$161.55$$

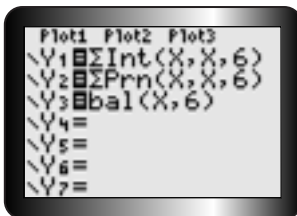
Adjust the payment, **PMT**, to -161.55 and the payments per year, **P/Y**, to 52. Solve for **N**. Carmen will pay the mortgage in full in just under 859 weeks by making weekly payments of \$161.55 each. For a monthly payment of \$646.18, the term is $52 \times 20 = 1040$ weeks. Carmen can reduce the amortization period by 181 weeks, or almost 3.5 years, by reducing the payment and paying more frequently.



Find the total interest paid using **ΣInt**. The total interest paid is about \$62 930.91, which is about \$16 353 less than \$79 283.89.



Enter **ΣInt**, **ΣPrn**, and **bal** into Y1, Y2, and Y3, respectively. Set **Tblstart** to 854. Look at the amortization table. As you can see in the table, the loan is paid in full after 859 weeks. The final payment can be adjusted to $\$161.55 - \$40.54 = \$121.01$.



X	Y ₂	Y ₃
854.00	-160.1	763.74
855.00	-160.3	603.37
856.00	-160.6	442.77
857.00	-160.8	281.92
858.00	-161.1	120.82
859.00	-161.4	-40.54
860.00	-161.6	-202.153356

The significant savings in interest, \$16 353, is achieved by changing the monthly payment of \$646.18 to a weekly payment of \$161.55. This change suggests that the frequency of payments has a substantial effect on the amortization.

Take a closer look at the payment amount. Dividing \$646.18 by 4 to get \$161.55 is based on the assumption that there are four monthly payments. You may assume

further that there are $4 \times 12 = 48$ payments in a year. However, the payment of \$161.55 is weekly, which means that there are 52 payments for the year. In this way, four additional payments are gained for the year, and these payments further reduce the principal owed. You will address this topic in more detail in the Practise, Apply, Solve of this chapter.

Solution: Using a Spreadsheet

First find the interest rate, i , compounded weekly, that is equivalent to 8.4%/a, compounded semiannually.

$$8.4\% \div 2 = 4.2\%$$

$$(1 + i)^{52} = (1.042)^2$$

Take the 52nd root of each side.

$$1 + i = \sqrt[52]{(1.042)^2}$$

$$1 + i = (1.042)^{\frac{1}{26}}$$

$$i = (1.042)^{\frac{1}{26}} - 1$$

$$i \doteq 0.001\,583\,635\,1$$

The equivalent weekly interest rate is 0.001 583 635 1, as a decimal.

Adjust the spreadsheet in Example 1 to reflect the changes in the interest rate and the payment. Notice that the mortgage is repaid after the 859th payment. Again this final payment can be adjusted to $\$161.55 - \$40.54 = \$121.01$. Find the total interest paid, \$62 930.91, by using the SUM function or the Σ toolbar button.

	A	B	C	D	E
854	852	\$161.55	\$1.97	\$159.58	\$1,083.63
855	853	\$161.55	\$1.72	\$159.83	\$923.80
856	854	\$161.55	\$1.46	\$160.09	\$763.71
857	855	\$161.55	\$1.21	\$160.34	\$603.37
858	856	\$161.55	\$0.96	\$160.59	\$442.78
859	857	\$161.55	\$0.70	\$160.85	\$281.93
860	858	\$161.55	\$0.45	\$161.10	\$120.82
861	859	\$161.55	\$0.19	\$161.36	-\$40.54
862			\$62 930.91		

Consolidate Your Understanding

1. Compare using the TVM Solver and using a spreadsheet for solving mortgage and general annuity problems. Does one form of technology have more advantages than the other?
2. Compare using a spreadsheet and using the TI-83 Plus calculator for producing repayment schedules and amortization tables. Does one form of technology have more advantages than the other?
3. Discuss some of the factors that could affect the time it might take for someone to amortize a mortgage.

Key Ideas

- By regulation, the interest rate on any Canadian mortgage can be compounded, at **most**, semiannually. The repayment of a mortgage in Canada can be represented by a general annuity, because the payment interval of a Canadian mortgage does not coincide with the interest conversion period.
- First find the equivalent interest rate that coincides with the payment interval, before creating an amortization table for a Canadian mortgage.
- An alternative to solving general annuities algebraically is the **TVM Solver** of the TI-83 Plus calculator.
- You may use both the TI-83 Plus calculator and a spreadsheet effectively to produce amortization schedules, or tables, of loans.
- You may use both the **TVM Solver** and a spreadsheet effectively to analyze the effects of changing the conditions of a mortgage or the effects of changing the conditions of long-term savings plans.
- Many financial institutions offer loan and mortgage calculators on their Web sites to assist clients with investment decisions.

Practise, Apply, Solve 2.12

A

- Solve for i to eight decimal places.

(a) $(1 + i)^{12} = (1.05)^2$	(b) $(1 + i)^{12} = (1.035)^2$
(c) $(1 + i)^{24} = (1.04)^2$	(d) $(1 + i)^{52} = (1.06)^2$
(e) $(1 + i)^{365} = (1.055)^2$	(f) $(1 + i)^{52} = (1.055)^2$
- Find the equivalent nominal rate, compounded monthly, for each of the following mortgage rates.

(a) 7.0%/a, compounded semiannually	(b) 8.5%/a, compounded semiannually
(c) 9.9%/a, compounded semiannually	(d) 12%/a, compounded semiannually
- Visit a local financial institution or visit the Nelson Thomson Learning Web site at www.math.nelson.com to research the different types of mortgages available. Write a report that describes the differences and similarities of these mortgages. Give some advantages and disadvantages of each type. Your report should address some of the following topics or phrases: length of term, fully open, limited open, fully closed, pre-payment options, lump-sum payment options, and variable or fixed rates.

4. In Example 2, the monthly payment of \$646.18 changes to a weekly payment of \$161.55.
 - (a) Show that these payments do not yield equivalent amounts at the end of one year.
 - (b) Find, to the nearest cent, the weekly payment that is equivalent to a monthly payment of \$646.18.
 - (c) Analyze the effect of changing the \$646.18 monthly payment to the weekly payment in (b) on the term of Carmen's loan. Discuss your findings.
 - (d) Review your work and the solution of Example 2. Which of the following has a greater effect on the repayment of a mortgage, the frequency of payment or the size of the payment?
5. Describe, in your own words, how the interest is calculated for a Canadian mortgage.

B

Use technology to solve questions 6 to 19. Use the TVM Solver on the TI-83 Plus calculator, a spreadsheet, or visit the Nelson Thomson Learning Web site at www.math.nelson.com to use one of several online mortgage calculators.

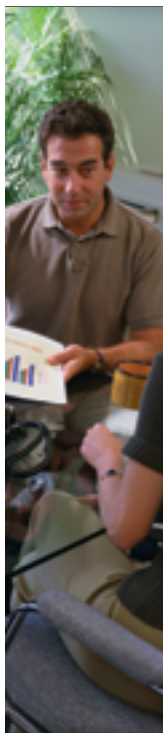
6. **Knowledge and Understanding:** Lynn may apply for a mortgage of \$80 000 at 8.8%/a, compounded semiannually.
 - (a) Calculate and compare the monthly mortgage payment for a term of
 - i. 25 years
 - ii. 20 years
 - (b) How much will Lynn save in interest by choosing the shorter term?
 - (c) What would you suggest to Lynn?
7. Mr. Bowhey may buy a cottage near Algonquin Provincial Park by taking out a \$60 000 mortgage. He expects to make a total monthly payment of \$624 and he assumes the interest rate will be fixed at 8.0%/a, compounded semiannually, until the cottage is fully paid for.
 - (a) Use the TVM Solver of the TI-83 Plus Calculator to find the term of the mortgage, in months and to two decimal places, if
 - i. the monthly payment is \$624
 - ii. the semimonthly payment is \$312 ($\$624 \div 2$)
 - iii. the weekly payment is \$144 ($\$624 \times 12 \div 52$)
 - (b) Based on your solutions, does the frequency of the payments have a significant effect on the term of the mortgage?
8. Mr. Bowhey (see question 7) decides to make weekly payments of $\$624 \div 4$, or \$156.
 - (a) Find the term of the mortgage in months. Express your answer to two decimal places.
 - (b) Determine how much interest Mr. Bowhey would save by choosing to make weekly payments of \$156 instead of \$144.

9. The Tang family is about to purchase a new home for \$203 000. They intend to make a down payment of \$43 000 and mortgage the remaining amount over 25 years. They obtain a fixed five-year term mortgage rate of 8.6%/a, compounded semiannually.
- Determine the monthly payment.
 - Find the outstanding balance on the mortgage at the end of each of the first five years.
 - Determine how much principal has been repaid and how much interest has been paid at the end of the five-year term.
 - Assume that the interest rates do not change. Determine the total interest the Tangs can expect to pay over 25 years and the total cost of their home.
10. (a) For question 9, determine the monthly payment if the Tangs were able to obtain a mortgage rate of 7.8%/a. How much would they save in total interest over 25 years?
- (b) Assume that the Tangs pay the payment in question 9 (a). How much sooner will they be able to completely pay their mortgage?
11. The Tangs decide to increase the monthly payment in question 9 (a) by 10% at the end of each of the first two years.
- Determine the new monthly payments for year 2 and year 3 to year 5.
 - Determine the new outstanding balance on the mortgage at the end of each of the first five years.
 - How much sooner will the house be paid for if the Tangs continue to make the same monthly payment that is made in year 3 to year 5?
12. Mrs. Rice purchases a cottage on Lake Nipissing for \$61 500. She puts 20% down and takes out a mortgage to finance the remaining amount. The term of the mortgage is seven years and the interest rate is 6.8%/a, compounded semiannually.
- Determine the monthly payment.
 - Create an amortization table of her mortgage and obtain a graph of its amortization.
 - How much of the 12th payment is
 - principal?
 - interest?
 - How much of the 50th payment is
 - principal?
 - interest?
 - What is the total cost of her cottage, including the total interest?
 - Suppose Mrs. Rice had managed to make a 25% down payment. Determine the total cost of her cottage, including all the interest.



- 13. Communication:** Friends of your parents plan to visit them to discuss mortgages. They plan to purchase a home soon and intend to amortize the mortgage over 25 years. They would like suggestions and strategies for minimizing the amortization period. Write a report outlining your suggestions and strategies for minimizing the total cost of paying for their home. Use what you have discovered in this section and from your own independent research, including visits to financial institutions.
- 14.** Mr. Los is planning to purchase a home. He decides to deposit \$440 at the end of each month in an account that pays 6%/a, compounded yearly. At the end of four years, he uses his savings for a down payment on a \$156 000 home. He obtains a mortgage for the balance at a rate of 8%/a, compounded semiannually. He can afford semi-monthly payments of \$525 each. Assume the interest rates remain constant. How long it will take him to repay the mortgage?
- 15.** In question 14, Mr. Los decides to start saving for the purchase of the house 1.5 years sooner. Then he will save for 5.5 years. He also decides that he can afford semi-monthly payments of \$545 each. How much sooner will he be able to repay the mortgage? How much less will be the total cost of his home?
- 16. Application:** In 1979, a new home could be purchased for \$85 000 by paying \$5000 down and financing the remaining amount. Typically a mortgage was amortized over 20 years. The initial term was three years and the interest rate was 10%/a, compounded semiannually.
- (a) Find the monthly payment and the balance owing at the end of the term.
 - (b) In the early 1980s, mortgage rates increased dramatically. When it was time to renew the term in 1982, the mortgage rate had grown to 22%/a, compounded semiannually. Determine the new payment required to maintain the mortgage.
 - (c) Determine the percent by which the original payment was increased to maintain the 20-year amortization.
 - (d) Explain why bankruptcy rates were extremely high in the early 1980s.
- 17. Thinking, Inquiry, Problem Solving:** A major Canadian bank advertises a 3% “cash-back” mortgage offer that may be used to reduce the principal on a mortgage. The offer is only valid for a five-year fixed rate mortgage where the interest rate is 7.4%/a, compounded semiannually, with monthly payments over a 25-year amortization. A competing bank does not make this offer, but it offers a five-year fixed rate mortgage at 7%/a, compounded semiannually, with monthly payments. Which option should you choose if your mortgage is for \$120 000?

18. Visit a bank or credit union or visit the Nelson Thomson Learning Web site at www.math.nelson.com to get information on the different mortgage rates available. Select and record the highest and lowest rates that you find.
- (a) Is there a relationship between the mortgage interest rates and the conditions and terms? Explain.
 - (b) Predict how long it would take to repay a \$100 000 mortgage using both rates if you repay the loan with monthly payments of \$600.
 - (c) Determine how long it would take to repay a \$100 000 mortgage using the highest and lowest rates that you found if you made monthly payments of \$600. Compare the results with your predictions.
19. **Check Your Understanding:** List three factors that determine the monthly payment on a mortgage. For each factor, explain how they affect the payment.



The Chapter Problem—Financial Planning

In this section, you studied Canadian mortgages. Apply what you learned to answer these questions about the Chapter Problem on page 106.

- CP21.** Use technology to create an amortization table for the repayment of his mortgage for the first two years.
- CP22.** Determine how much principal he still owes on the mortgage after five years.
- CP23.** Determine the total amount of interest paid and the amount of principal repaid after the 30th payment.
- CP24.** At the end of the second year of his mortgage, Mr. Sacchetto wins \$7500 at bingo. He decides to make a lump-sum payment on the mortgage at the end of the second year. How much sooner will the mortgage be repaid if he continues to make payments equal to the payment in question CP19?