

Chapter 2 Review

Series and Financial Applications

Check Your Understanding

1. What is the difference between a sequence and a series?
2. How can you decide whether a sum of terms represents an arithmetic series or a geometric series? Use examples in your explanation.
3. Use Gauss's method to prove that the sum of all natural numbers from 1 to 60 is 1830.
4. Describe the steps and calculations for amortizing a \$2000 loan. The monthly payment is \$320. The interest rate is 6%/a, compounded monthly.
5. At the end of each year, Charlotte deposits \$1000 in a savings account for five years. The interest rate is 4%/a, compounded annually. Explain and show how the amount of the sum of the deposits can be found by using the equation for the sum of a geometric series.
6. Each of the following represents an annuity. Write a problem that would describe each annuity.
 - (a) $400 + 400(1.06) + 400(1.06)^2 + \dots + 400(1.06)^{22} + 400(1.06)^{23}$
 - (b) $100(1.02)^{-48} + 100(1.02)^{-47} + 100(1.02)^{-46} + \dots + 100(1.02)^{-2} + 100(1.02)^{-1}$
7. Find the sum of each series in question 6 using the formula for the sum of a geometric series.
8. Draw a time line for each of the following annuities.
 - (a) the accumulated value of \$200 deposited at the end of every 3 months for 4 years at 10%/a, compounded quarterly
 - (b) the present value of \$350 deposited at the end of every month for 1.5 years at 9%/a, compounded monthly
 - (c) the accumulated value of \$500 deposited at the end of every 6 months for 2 years at 12%/a, compounded quarterly
9. Represent each annuity in question 8 as a geometric sum. Find the accumulated value of each sum.
10. The amount of an annuity with monthly payments will be worth more at 12%/a, compounded weekly, rather than at 12.5%/a, compounded annually. Explain and show why this statement is true.
11. A family asks for your advice for minimizing the total cost of purchasing, and then repaying the mortgage of, a new home. Name at least four things that you would advise them to do.

Review and Practice

2.1 Arithmetic Series

1. Explain what each variable in the general term of an arithmetic sequence represents.
2. What is a series? Write the formula for finding the sum of the first n terms of an arithmetic series.
3. Find the sum of each of the following arithmetic series.
 - (a) $32 + 43 + 54 + \dots$ to 17 terms
 - (b) $19 + 14 + 9 + \dots$ to 23 terms
 - (c) $-68 - 61 - 54 - \dots + 58$
 - (d) $\frac{1}{3} + \frac{1}{2} + \frac{2}{3} + \frac{5}{6} + \dots + \frac{9}{2}$
4. In an arithmetic series of 28 terms, the 7th term is 41, and the 18th term is 96. Find the sum of the series.
5. In an arithmetic series, the 11th term is -12 and the sum of the first 15 terms is -315 . Find the first 4 terms of the series.
6. Erika has \$347 in her safety deposit box. At the end of the week, she deposits \$5. Each subsequent week, she deposits \$3 more than she deposited the previous week. How much money will be in the deposit box at the end of 20 weeks?

2.2–2.3 Geometric Series

7. Explain what each term in the general term of a geometric sequence represents.
8. Use the general term in question 7 to write the geometric series of the first n terms. What is the formula for adding the first n terms of a geometric series?
9. Add each geometric series.
 - (a) $4 + 20 + 100 + \dots$ to 9 terms
 - (b) $19 - 38 + 76 - \dots$ to 13 terms
 - (c) $192\,000 - 96\,000 + 48\,000 - \dots + 1500$
 - (d) $300 + 300(1.1) + 300(1.1)^2 + \dots + 300(1.1)^{11}$
10. Mrs. Lee has just received her first annual pension cheque of \$22 800. The payment increases each year by 2.5% to account for inflation.
 - (a) What will be the amount of the tenth pension cheque?
 - (b) How much can she expect to receive in total after ten years?
11. A sweepstakes gives away \$1 000 000 in prizes. The first ticket drawn wins \$10, the second ticket drawn wins \$30, the third ticket drawn wins \$90, and so on.
 - (a) How many tickets can the sweepstakes afford to draw?
 - (b) How much money is left after all the prizes are awarded?

2.4–2.5 Using Spreadsheets to Analyze and Represent Financial Situations: Future Value and Amortization Tables

12. Describe an annuity, and the difference between an ordinary annuity and an annuity due.
13. Describe the amortization period, or term, of a loan or mortgage. What is the purpose of an amortization table? Describe the components of an amortization table.
14. Ewen would like to volunteer as an aid worker in a third-world country after graduating from high school. He deposits \$110 at the end of every month in a savings plan that pays 6%/a, compounded monthly.
 - (a) What is the monthly rate of interest?
 - (b) Explain why Ewen's savings plan is an ordinary simple annuity.
 - (c) Use a spreadsheet to find the value of his savings after two years.
 - (d) How much interest is earned by his savings?
 - (e) After the first deposit, Ewen decides to increase each deposit by \$4 every month. Determine the new balance after two years.
15. Madonna buys a stereo by financing \$2400 at 18%/a, compounded monthly. Each month, she pays \$130 until the debt is paid in full.
 - (a) Without using a spreadsheet, construct an amortization table for the first eight months. How much does she still owe after eight months?
 - (b) Use a spreadsheet to create an amortization of the loan.
 - (c) How long will it take her to repay the loan? What will be the final payment?
 - (d) Determine the total cost of the stereo purchase.
 - (e) What monthly payment would let her repay the loan in 18 months?
16. Using a spreadsheet, determine the annual interest rate, compounded monthly, of the finance plan for the computer system shown in the ad.
17. Amanda has just started grade 10. She wants to save \$7500 over the next three years to help pay for her first year of college. Use a spreadsheet to find, to the nearest cent, how much she would have to deposit at the end of every three months in an account. The account pays 7%/a, compounded quarterly.

Computer System

Includes free digital camera, CD burner, 20" screen, and much more.

Take home price \$2,890.00

OR

\$450 down and \$158.02 per month for 18 months

2.6–2.7 Using Series to Analyze Financial Situations:

Future Value

- 18.** The future value of an ordinary simple annuity can be written as the geometric series $R + R(1 + i) + R(1 + i)^2 + \dots + R(1 + i)^{n-2} + R(1 + i)^{n-1}$. Describe what each variable represents.
- 19.** Give two equations for finding the accumulated value of any ordinary simple annuity.
- 20.** Since the birth of his daughter, Kay, Mr. Kemp has deposited \$450 at the end of every three months in an education savings plan. The interest rate is 8%/a, compounded quarterly. What will be the fund's value when Kay turns 17?
- (a) Draw a time line to represent the annuity.
 - (b) Write the series that represents the annuity.
 - (c) Find the amount in the savings plan when Kay turns 17 and the total interest earned.
- 21.** Singh Lee wants to travel to China in 20 months. The trip will cost \$3200. How much should she deposit at the end of each month in an account that pays 9%/a, compounded monthly, to raise \$3200?
- (a) Draw a time line to represent the annuity.
 - (b) Write the series that represents the annuity.
 - (c) Find how much Singh Lee must deposit at the end of each month.
- 22.** The Pilons are buying a home in the future. They want to accumulate enough money for a \$30 000 down payment. They plan on depositing \$475 at the end of each month in an account that pays 6%/a, compounded monthly.
- (a) Draw a time line to represent the annuity.
 - (b) Write the series that represents the annuity.
 - (c) Determine how long it will take them to save for the down payment.
 - (d) Determine the monthly deposit needed to buy a home at the end of the next four years.

2.8 Using Series to Analyze Financial Situations: Present Value

- 23.** Describe, in your own words, the difference between the present value of an ordinary simple annuity and the future value of an ordinary simple annuity.
- 24.** The present value of an ordinary simple annuity can be written as the geometric series $R(1+i)^{-n} + R(1+i)^{-n+1} + R(1+i)^{-n+2} + \dots + R(1+i)^{-2} + R(1+i)^{-1}$. Use the sum of a geometric series, $S_n = \frac{a(r^n - 1)}{r - 1}$, to show that this value can also be found using $PV = R \times \frac{1 - (1+i)^{-n}}{i}$.
- 25.** For each of the following,
- draw a time line to represent the present value of the annuity
 - write the series that represents the present value of the annuity
 - find the present value of the annuity
- (a) \$3000 deposited at the end of every year for 9 years at 5%/a, compounded annually
- (b) \$500 paid at the end of every 3 months for 5 years at 6%/a, compounded quarterly
- 26.** Rolly bought a Windsurfer for \$350 down and 18 monthly payments of \$75 each. The first payment is due at the beginning of next month.
- (a) The interest rate is 21%/a, compounded monthly. What is the selling price of the Windsurfer?
- (b) What is the finance cost?
- 27.** C.C. Ryder wants to buy a motorcycle. He can borrow \$9200 at 10%/a, compounded quarterly, and repay the loan with equal quarterly payments for four years.
- (a) Draw a time line to represent the annuity.
- (b) Write the series that represents the present value of the annuity.
- (c) Find the quarterly payment that Mr. Ryder must make.

2.9–2.10 Equivalent Rates and General Annuities

- 28.** What is the difference between a simple annuity and a general annuity?
- 29.** Find the nominal rate that is equivalent to 15%/a, compounded quarterly, if interest is paid
- (a) annually (b) monthly (c) weekly

- 30.** Nevia wishes to save some money to purchase a used jeep. She can save \$270 each month. She deposits her savings at the end of each month for three years in an account that pays 5%/a, compounded annually.
- Draw a time line to represent the annuity.
 - Write the series that represents the future value of the annuity.
 - Find the amount she will have after three years.
 - After three years, she uses her savings as a down payment on a jeep that costs \$14 800. She finances the remaining balance over two years at 8%/a, compounded semiannually. Determine the monthly payment.
- 31.** Yolanda wants to buy a snow machine for \$8700. The store will let Yolanda finance the purchase at 18%/a, compounded annually. Yolanda plans to repay the purchase in four years by making a payment at the end of each month.
- Draw a time line to represent the repayment of the loan.
 - Write the series that represents the present values of all the payments.
 - Determine the monthly payment. Verify your solution using technology.
- 32.** The Cooper family borrows \$17 300 to purchase a house trailer for camping. They decide to repay the loan by making bi-weekly payments for three years. The interest rate is 9%/a, compounded monthly.
- Determine the bi-weekly payment that lets them repay the loan in three years.
 - Use technology to determine how much sooner the Coopers could repay the loan if they increased the bi-weekly payment by \$50.
 - Use technology to determine the bi-weekly payment that enables them to repay the loan in three years if the interest rate is 9%/a, compounded annually.

2.11–2.12 Using Technology to Analyze Canadian Mortgages

- 33.** A couple is considering a \$98 000 mortgage at 7.6%/a, compounded semiannually.
- Calculate and compare the monthly mortgage payment for a term of
 - 25 years
 - 20 years
 - How much will be saved in interest by choosing the shorter term?
 - What would you suggest to the couple?

- 34.** Mrs. Graywolf took out a \$43 800 mortgage to purchase a cottage in Temagami. She will repay the mortgage over eight years with equal monthly blended payments. The interest rate is 8.4%/a, compounded semiannually.
- (a) Determine the monthly payment.
 - (b) Create an amortization table of the mortgage.
 - (c) Determine how much interest she will pay over the life of the mortgage.
- 35.** The Willis family wants to buy their first home for \$137 800. They have \$22 000 for a down payment. They can afford a weekly mortgage payment of \$225. The interest rate is 8.8%/a, compounded semiannually.
- (a) Determine how long it would take to repay the mortgage.
 - (b) What is the cost of the house, including the interest charge?
 - (c) How much would be saved in interest if the weekly payment were \$250?
- 36.** The Wong family purchase a new home for \$233 000. They put \$51 000 down and mortgage the remainder for a term of 25 years. They obtain a fixed five-year mortgage. The interest rate is 8.6%/a, compounded semiannually.
- (a) Determine the monthly payment needed to repay the loan.
 - (b) Find the outstanding balance at the end of each of the first five years.
 - (c) How much of the 20th and 120th payment is applied to the principal?
 - (d) At the end of the five-year mortgage, the Wongs make an \$18 000 lump-sum payment before renewing the mortgage for a fixed three-year term at 7.4%/a, compounded semiannually. Find the new monthly payment.
 - (e) How much do the Wongs still owe after eight years?

Chapter 2 Summary

In this chapter, you have seen that a series is the sum of the terms of a sequence and found the general term of an arithmetic series and a geometric sequence. You have used spreadsheets to find the future value and present value of a deposit or an annuity. You have also seen how payments are amortized over a period of time and how Canadian mortgages operate.

Chapter 2 Review Test

Series and Financial Applications

- Find the sum of each series.
 - $-11 - 3 + 5 + 13 + \dots + 125$
 - $6 - 18 + 54 - \dots + 39\,366$
- A hockey arena has a total seating capacity of 15 690. The first row of seats around the rink has 262 seats. The number of seats in each subsequent row increases by 18.
 - How many rows of seats are in the arena?
 - By what percent would the capacity increase if three more rows were added after the last row?
- Application:** A lottery plans to give out \$5 000 000 in prizes. The first ticket drawn wins \$20, the second ticket drawn wins \$50, the third ticket drawn wins \$125, and so on. Can the lottery afford to give out 14 prizes?
- Romika buys a laptop computer by borrowing \$1400 at 15.6%/a, compounded monthly. She repays the loan by paying \$110 at the end of each month.
 - Create an amortization table for the first six payments.
 - How much of the fourth payment reduces the principal of the loan?
- Knowledge and Understanding**

Calvin buys a new electric guitar and amplifier for a total cost of \$2300. He paid a deposit of \$200 and borrowed the balance at 16.2%/a compounded monthly. He agrees to repay the loan by making equal monthly payments over 2.5 years.

 - Draw a time line to represent the repayment of the loan.
 - Write the series that represents the present values of the payments.
 - Find the monthly payment that Calvin must make.
- Thinking, Inquiry, Problem Solving**

Penny decides on her 25th birthday to start planning for her future by depositing \$3200 at the end of each year in an investment that pays 9%/a, compounded annually, until her 45th birthday. She then leaves the money in the investment to collect interest until her 55th birthday.

 - Determine how much money she will have on her 55th birthday.
 - Penny's best friend, Susie, decides to wait until her 35th birthday to start planning for her future. Susie can get the same rate of return on her investment. Find the yearly payment that she would be required to make until her 55th birthday so that her investment is equivalent to Penny's investment.

7. Marisa has just won the Mega Bucks Sweepstakes. She must decide between two prize options: \$50 000 cash today or receive \$1250 at the end of every three months for 20 years. Marisa expects a return of 8%/a, compounded quarterly, if she invests the cash prize.
- (a) Draw a time line to represent the present value of the payment option.
 - (b) Write the series that represents the annuity.
 - (c) Which option should Marisa choose and why?
8. **Communication:** The Walton family want to purchase a home. They are considering a mortgage of \$116 800 at 7.6%/a, compounded semiannually.
- (a) Determine the bi-weekly payment if the term of the mortgage is 20 years.
 - (b) How much will the Waltons pay in interest over the life of the mortgage?
 - (c) What would you suggest to the Waltons for minimizing the interest they must pay?