

Chapter

3

Introducing Functions

The relationship between two or more quantities is one of the most fundamental concepts in mathematics. In earlier grades, you have studied relations in many different contexts. In this chapter, you will learn that some of these relations are called functions and you will examine functions in more detail.

You will find functions used in digital network design, speech recognition software, encryption, music and sound synthesis, marketing, economic forecasting, investment planning, medicine, environmental sciences, and logistics. Calculators, spreadsheet software, databases, and almost all other software applications use predefined functions.

In this chapter, you will

- learn the meaning of “function”
- use function notation for evaluating functions, representing inverse functions, and representing transformations
- investigate the properties of $f(x) = \sqrt{x}$ and $f(x) = x^2$
- investigate and explain the relationship between a function and its inverse
- solve inequalities
- describe the relationship between the graph of a function and its image under transformations
- describe the domain and range of functions under transformations

Visit the Nelson Mathematics student Web site at www.math.nelson.com. You'll find links to interactive examples, online tutorials, and more.

Connections



This is the Enigma machine, which the Germans used to send coded messages during World War II. However, the Germans did not know that the British cracked the code, and were able to read the secret messages. Historians say that this shortened the war by about two years.



Alfred J. Menezes

The Chapter Problem

Cryptography

If you have bought something over the Internet, then you have probably had to enter a credit card number when placing your order. The Internet is a vast web of interconnected servers. Your order may have passed through *many* different computers on its way to the company from which you made the purchase. It may be possible for someone to intercept the order and “read” the credit card number. Billions of transactions are made every day on the Web. It is necessary to protect the privacy of the details of transactions from people who would misuse this information.

Cryptography is the study of coding messages. A message is coded so that only the people to whom the message is sent can “read” the message. On the Internet, “public-key encryption” is one method for coding messages such as a credit card number. Public-key encryption has two parts, a public key for coding a message and a private key for decoding it. One key is used to code a message and then that key itself is coded. The coded key is the public key that is sent.

Professor Alfred J. Menezes is a pioneer in the development of elliptic curve coding systems. He teaches at the University of Waterloo, and does consulting work with Certicom. His work is critical in the development of secure wireless communications, in which Canadian companies are becoming world leaders.

Suppose you wanted to send a coded message to a friend. To code the message, assign each letter a number between 1 and 26. Then multiply each number by 17 and divide by 26. Add 1 to the remainder.

For example, the message “HI” can be coded as “GX.”

H is 8, $8 \rightarrow 8(17) = 136 \rightarrow 6$ (since $136 = 5(26) + 6$) $\rightarrow 7$, which is G.

I is 9, $9 \rightarrow 9(17) = 153 \rightarrow 23 \rightarrow 24$, which is X.

To “decode” this message, convert each number back into a letter. Danielle uses this code to send the message X WVKH NRCG to a friend. How could the code be made more complicated? Is it possible to break any code?

Design your own code to send the message “Codes are cool.” Do research and find other examples of codes used in different contexts.

For help with this problem, see pages 240, 260, 277, and 291.

Challenge 1

In some codes, letters are switched with other letters. One way to break this kind of code is to examine the frequencies of the letters used in everyday language and then to compare these frequencies with the frequencies of the letters used in the coded message. The location of the letters in a word can also help you to decide which letters are vowels.

Decode the following:

LQPITCVWNCVKQPU! AQW JCXG HKIWTGF VJKU QPG QWV!
RTGVVA LQON, G?

Challenge 2: Error-Correcting Codes

Suppose you heard someone say, “The makle eaves are playing at the Corel Centre this weekend” You would probably understand that what the person really said was, “The Maple Leafs are playing at the Corel Centre this weekend.” Your brain has a built-in error-correcting decoder! However, there is a limit to the number of errors that the decoder can correct.

Suppose you heard someone else say, “The mable eaves are staying at the morel sinner this weekend.” You might not be able to decipher the correct message. A good code produces a unique output for each input.

The bars, or optical code, printed on an envelope for mailing in Canada is a code containing the essential address information, so that optical readers can quickly direct the letter to the right location. The bars contain an error-correcting code that can “make up” for an error in the input, such as part of the bar being torn away or smudged.



Research error-correcting codes. Find a way to code “HI,” so that at least one error could be corrected.

To complete these challenges, you may need to discuss a research plan with your teacher or other students. You may also need to research beyond the resources in your classroom.

Web Challenge

For a related Internet activity, go to www.math.nelson.com.

