

Chapter 3 Review

Introducing Functions

Check Your Understanding

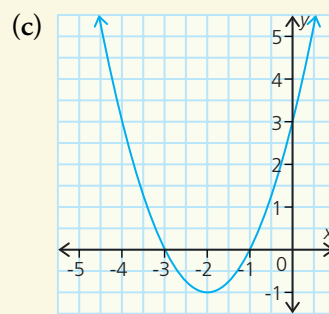
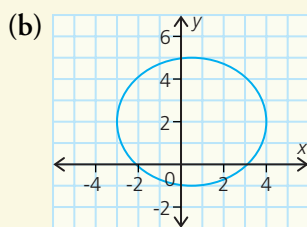
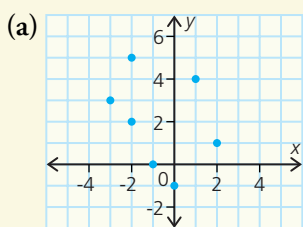
1. What is a function? Give an example of a relation that is a function and another example of a relation that is not a function.
2. A function can be described or defined in a number of ways. List the different ways and explain how you would use each to determine if a relation is a function.
3. How can you use the graph of a relation to decide if the relation is a function?
4. How do you find the domain and range of a relation?
5. How do you write the domain and the range in set notation?
6. How are the rules for solving a linear inequality algebraically different from the rules for solving a linear equation?
7. How do you write the solution to a linear inequality?
8. Describe a non-algebraic method for solving a linear inequality.
9. What is the inverse of a relation?
10. How do you graph the inverse of a relation using the graph of the relation?
11. How do you determine the equation of the inverse from the equation of a relation?
12. Why might $f^{-1}(x)$ be different from the equation of the inverse?
13. Point (a, b) is on the graph of f . Express each of the coordinates of the point using function notation.
14. Why might it be difficult to graph the inverse of a relation on a graphing calculator in function mode (**Func**)? What can you do instead?
15. Compare the domain and range of a relation with the domain and range of the inverse relation.
16. Why might the inverse of a function not be a function? How can you check whether the inverse of a function is a function?
17. Why is the inverse of $y = x^2$ not $y = \sqrt{x}$? Explain.
18. The equation of a function is expressed in terms of a base function and involves multiplication. What type of transformation could be applied to the graph of the base function to get the graph of the new function?

19. The equation of a function is expressed in terms of a base function and involves addition. What type of transformation could be applied to the graph of the base function to get the graph of the new function?
20. How does expressing a function in terms of a base function help you to determine its domain and range?

Review and Practice

3.1–3.2 Functions: Concept and Notation

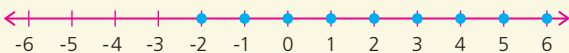
1. A relation is $\{(1, 9), (2, 7), (3, 5), (4, 3)\}$.
 - (a) State the domain and range.
 - (b) Draw an arrow diagram for the relation.
 - (c) Graph the relation.
 - (d) Determine an equation for the relation.
 - (e) Express this relation in function notation.
 - (f) Explain why this relation is a function.
 - (g) Create another ordered pair so that the relation is no longer a function.
2. For each of the following graphs, identify the domain and range, and decide if the relation is a function. Justify your answer.



3. A relation f is $f(x) = 2x^2 - 6x$.
 - (a) Evaluate $f(-2)$.
 - (b) Graph f .
 - (c) Is f a function? Explain.
 - (d) $(5, 20) \in f$. Rewrite using function notation.
 - (e) Solve $f(x) = 0$.
4. For $g(s) = \sqrt{9 - s}$,
 - (a) graph g
 - (b) state the domain and range of g

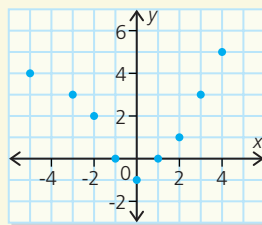
5. A student has a part-time job at an electronics store. She earns \$85 for working 12 h each week, plus 5% of her sales over \$1000 each week.
- Why is it important that this relation is a function?
 - What is a reasonable domain and range for this function? Justify your answer.
 - Let this function be g . Find $g(5000)$.
 - Graph g .

3.3 Solving Inequalities

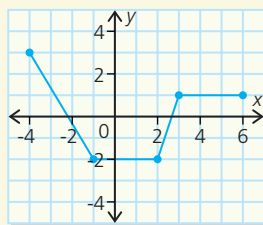
6. Graph each of the following on a number line.
- $\{x \mid -2 \leq x < 4, x \in \mathbf{R}\}$
 - $\{x \mid x < 4, x \in \mathbf{N}\}$
 - $\{x \mid x^2 > 1, x \in \mathbf{R}\}$
7. Describe each in set notation.
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8. The function f is $f(x) = \frac{4 - 2x}{5}$.
- The range of f is $\{y \mid y > 2, y \in \mathbf{R}\}$. Find the domain.
 - The domain of f is $\{x \mid -8 \leq x < 2, x \in \mathbf{R}\}$. Find the range.
9. For $g(x) = -(x + 1)^2 + 4$,
- graph g
 - find the domain of g if $g(x) > 0$

3.4 The Inverse Function

10. The graph of $y = f(x)$ is shown.
- State the domain and range of f .
 - Express f^{-1} as a set of ordered pairs.
 - Graph f and f^{-1} on the same axes.
 - Find $f(1)$.
 - Find $f^{-1}(1)$.
 - Find $f^{-1}(0)$.
 - Find $f(-4)$.



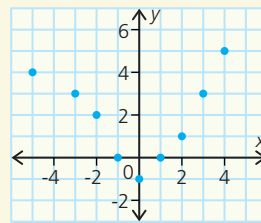
11. The graph of $y = g(x)$ is shown. Graph g^{-1} .



12. Find the equation for f^{-1} if $f(x) = \frac{x+3}{2}$.
13. A printing company charges \$100 to set up the artwork and \$11.50 for every T-shirt printed. g is the relation between the total charge for the printing in dollars, excluding taxes, and the number of shirts printed.
- Express g in function notation.
 - Determine $g^{-1}(x)$.
 - Describe g^{-1} as a rule.
 - State a reasonable domain and range for g^{-1} .
 - Write a question that would require the evaluation of $g^{-1}(1000)$.

3.5–3.6 Investigating Properties of Inverse Functions

14. Describe the relation between $y = \sqrt{x}$ and $y = x^2$.
15. The graph of $y = f(x)$ is shown. Write a subset of f that has the same domain as f^{-1} so that f^{-1} is a function.



16. A function g is $g(x) = 4(x - 3)^2 + 1$.
- Graph g and g^{-1} using technology.
 - At what points do the graphs of g and g^{-1} intersect?
 - Determine an equation for g^{-1} .
 - Solve for y in the equation that defines g^{-1} .
 - State restrictions on the domain or range of g so that its inverse is a function.
 - Suppose the domain of g is $\{x \mid 2 \leq x \leq 5, x \in \mathbf{R}\}$. Would the inverse be a function? Justify your answer.

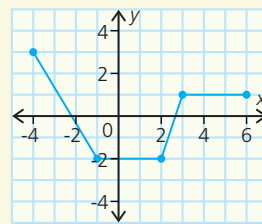
17. For $f(x) = \sqrt{3 - 2x}$,
- (a) graph f and f^{-1} using technology
 - (b) determine an equation for f^{-1}
 - (c) solve for y in the equation that defines f^{-1}

3.7 Transformations and Function Notation

18. Sketch the graph of $f(x) = \sqrt{x}$. On the same axes, sketch the graph of each function.
- (a) $y = 2f(x)$
 - (b) $y = f(2x)$
 - (c) $y = f(x + 2)$
 - (d) $y = f(x) + 2$
 - (e) $y = -f(x)$
 - (f) $y = f(-x)$
19. $f = \{(0, 5), (1, 4), (2, 7), (3, 14)\}$ and $g(x) = 3f(-x + 1) + 7$.
- (a) Evaluate $g(-1)$.
 - (b) Evaluate $g(1)$.
 - (c) Evaluate $g(3)$.
 - (d) Write g as a set of ordered pairs.
20. A function g is $g(x) = \frac{1}{2}f[3(x + 2)] - 4$.
- (a) Draw an input/output diagram for g .
 - (b) List the transformations in the correct sequence to transform f into g .
 - (c) Suppose the domain of f is $\{x \mid 2 \leq x < 5, x \in \mathbf{R}\}$ and the range of f is $\{y \mid y \geq -2, y \in \mathbf{R}\}$. Determine the domain and range of g .
21. The graph of $y = g(x)$ is shown.

$$f(x) = -g\left(\frac{1}{2}x - 3\right) + 1$$

- (a) Draw an input/output diagram for f .
- (b) Determine the domain and range of f .
- (c) Evaluate $f(4)$.
- (d) Graph f .



Chapter 3 Summary

In this chapter, you saw what a function is and how to evaluate a function. You also saw how to find the inverse of functions and to describe the image of a function under various transformations.

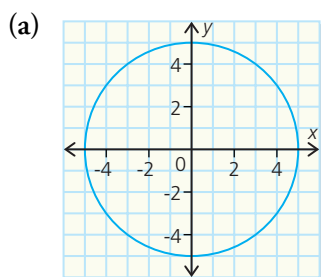
Chapter 3 Review Test

Introducing Functions

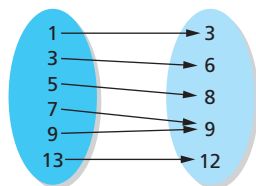
1. Knowledge and Understanding

For each case,

- determine $f(5)$
- state the domain and range
- state whether it is a function or not, and justify your answer



(b) $y = f(x)$



(c) $f = \{(1, 2), (1, 5), (4, 7), (5, -2), (8, 10)\}$

(d) $f(x) = 3x - 7$

(e) $f(x) = \sqrt{x - 3}$

(f) $f(x) = (x + 3)^2 - 5$

- Solve each inequality.
- Graph each solution set for $x \in \mathbb{R}$.

(a) $-3(3x - 2) + 4x \geq -2x + 5(2x - 4)$

(b) $\frac{2x}{3} + \frac{3(x-5)}{2} \leq x$

- Determine the equation of the inverse, f^{-1} , for each function.

(a) $f(x) = 5x + 6$

(b) $f(x) = \sqrt{x + 4}$

(c) $f(x) = x^2 - 5$

- Communication:** Will the inverse of a given function also be a function? Clearly explain your reasoning and use an example to support your argument.

- Explain why the vertical line test is a good test of whether or not the graph of a relation is a function.

- A relation is defined by $g(x) = -x^2 + 6x - 4$.

(a) Graph $g(x)$.

(b) State the domain and range of $g(x)$.

(c) Determine the equation of g^{-1} .

(d) Graph g^{-1} using the same set of axes used in (a).

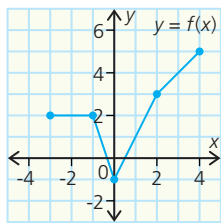
(e) State the domain and range of g^{-1} .

(f) Which relation, $g(x)$ or g^{-1} , is a function? Explain.

- Application:** A baseball club pays a peanut vendor \$50 per game for selling bags of peanuts for \$2.50 each. The club also pays the vendor a commission of \$0.05 per bag.

(a) Determine a function that describes the income the vendor makes for each baseball game. Define the variables in your function.

- (b) Determine a function that describes the revenue the vendor generates each game for the baseball club. Define the variables in your function.
- (c) Determine the number of bags of peanuts the vendor must sell before the baseball club makes a profit from his efforts.
8. (a) For the graph of $y = f(x)$, state the transformations that must be applied to $f(x)$ to obtain the graph of $y = -2f(-2x - 2) + 3$.
- (b) Graph $y = -2f(-2x - 2) + 3$.



9. **Thinking, Inquiry, Problem Solving**

Make a conjecture with respect to the points of intersection between a function and its inverse. Verify your conjecture using

- (a) a linear function and its inverse
- (b) a quadratic function and its inverse
- (c) a root function and its inverse