

Part 1: Investigating Rowing Speed

In recent years, Canadian rowing teams have performed well in international competitions. As with any sport, many hours of training and practice are required for success. To win a race, the members of a rowing team, or crew, work to increase their rowing speed.

Since river conditions vary from place to place, the rowing speed in still water is the speed for comparison and improvement. In this investigation, you will calculate the rowing speed in still water given the following information.



- During a training session, a crew takes 40 min to row 5 km upstream and then return.
- The river flows at a speed of 4 km/h.

Think, Do, Discuss

1. How does the flow of the river affect the boat when it is being rowed upstream? downstream?
2. Suppose the crew's rowing speed in still water is x km/h. Write an expression for the upstream rowing speed.
3. If you know the distance and the speed, then how can you calculate time? Write an expression for the time the crew takes to row upstream.
4. Now write an expression for the downstream rowing speed. Then write an expression for the time taken by the crew to row downstream.
5. How long did the round trip take? Write an equation that connects your answers to steps 3 and 4. Make sure that the values in the equation use the same units. To determine the rowing speed on still water, you will need to solve this equation.

Part 2: Adding and Subtracting Rational Expressions

Adding and subtracting rational expressions is similar to adding and subtracting rational numbers.

Think, Do, Discuss

1. Make two columns on a piece of paper. At the top of the left column, write “Rational Numbers” and, at the top of the right column, write “Rational Expressions.” In the left column, add $\frac{2}{7} + \frac{3}{7}$. In the right column, use the same method to add $\frac{2}{x} + \frac{3}{x}$. Try to match the steps as you work. What is the same about the two sums? What must you state about x in the right column?
2. In the left column, add $\frac{3}{8} + \frac{1}{8}$. Reduce the answer to lowest terms. In the right column, use the same method to add $\frac{a}{2b} + \frac{3a}{2b}$. Reduce this answer to lowest terms. Remember to state any restriction(s).
3. In the left column, subtract $\frac{5}{6} - \frac{4}{3}$. Remember to use the lowest common denominator (LCD). In the right column, try to subtract $\frac{7}{4y} - \frac{5}{2y}$, using the same steps. What is the LCD in this case? What is the restriction?
4. Subtract $\frac{3}{4} - \frac{2}{3}$ in the left column. Show the steps for changing each fraction so that the denominators are common. Follow the same steps to subtract $\frac{3}{x} - \frac{2}{y}$. The result will not look simpler, but this form may be useful for some purposes.
5. To find the LCD for the sum $\frac{4}{15} + \frac{7}{10}$, factor each of the denominators. Express 15 and 10 as products of prime factors. Write the LCD as a product of prime factors. Find the sum in the left column. Use the same steps to add $\frac{4}{st} + \frac{7}{rt}$ in the right column.

You have now practised all the methods for adding and subtracting rational expressions. Even if a rational expression may have a binomial or a trinomial in the numerator or denominator, apply the same steps: factor, find the LCD, and reduce to lowest terms.

6. Simplify $\frac{a}{ab + b^2} - \frac{b}{a^2 + ab}$. Use the method in step 5. Begin by factoring the denominators. What is the LCD? By what factor do you multiply $\frac{a}{b(a + b)}$ so that it will have the common denominator? Do the same for $\frac{b}{a(a + b)}$. Now that the expressions have the same denominators, find the difference. Notice that the resulting numerator can be factored. Factor the numerator and simplify the expression. Did you remember to state the restrictions?

7. In Part 1, you should have obtained the equation $\frac{5}{x-4} + \frac{5}{x+4} = \frac{2}{3}$. To add the two rational expressions on the left side of this equation, follow the method in step 4. Treat each binomial as a single variable. What is the common denominator? By what factor do you multiply $\frac{5}{x-4}$ so that the denominator will be the common denominator? Do the same for $\frac{5}{x+4}$. Add the numerators and simplify.
8. You should now have the equation $\frac{10x}{x^2-16} = \frac{2}{3}$. Multiply both sides by (x^2-16) . Then multiply both sides by 3. Rearrange and solve the resulting quadratic equation. What was the crew's rowing speed in still water?

Focus 4.11

Key Ideas

- When rational expressions are added or subtracted, they must have a common denominator.
- If the denominators are the same, then simply add or subtract the numerators.

$$\frac{f(x)}{g(x)} + \frac{h(x)}{g(x)} = \frac{f(x) + h(x)}{g(x)} \qquad \frac{f(x)}{g(x)} - \frac{h(x)}{g(x)} = \frac{f(x) - h(x)}{g(x)}$$
- To find a common denominator, factor the denominators. Then create the LCD from the factors.
- The LCD is not always the product of the two denominators.
- Once you have added or subtracted, it may be possible to factor the numerator and simplify the expression further.
- Always reduce the sum or difference to lowest terms.
- State the restrictions on the variables.

Example 1

Simplify.

(a) $\frac{2}{ab^2} + \frac{4}{a^2b}$ (b) $4 - \frac{3}{xy} + \frac{x}{y^2}$ (c) $\frac{3}{x^2-4} - \frac{5}{x+2}$ (d) $\frac{1}{x^2-6x+9} - \frac{1}{x^3-9x}$

Solution

(a) $\frac{2}{ab^2} + \frac{4}{a^2b}$
 $= \frac{2}{ab^2} \times \frac{a}{a} + \frac{4}{a^2b} \times \frac{b}{b}$

The LCD is a^2b^2 .

$$\begin{aligned} &= \frac{2a}{a^2b^2} + \frac{4b}{a^2b^2} \\ &= \frac{2a + 4b}{a^2b^2} \\ &= \frac{2(a + 2b)}{a^2b^2}, a \neq 0, b \neq 0 \end{aligned}$$

(b) $4 - \frac{3}{xy} + \frac{x}{y^2}$
 $= 4 \times \frac{xy^2}{xy^2} - \frac{3}{xy} \times \frac{y}{y} + \frac{x}{y^2} \times \frac{x}{x}$

The LCD is xy^2 .

$$\begin{aligned} &= \frac{4xy^2}{xy^2} - \frac{3y}{xy^2} + \frac{x^2}{xy^2} \\ &= \frac{4xy^2 - 3y + x^2}{xy^2}, x \neq 0, y \neq 0 \end{aligned}$$

$$(c) \frac{3}{x^2 - 4} - \frac{5}{x + 2}$$

$$= \frac{3}{(x - 2)(x + 2)} - \frac{5}{x + 2}$$

The LCD is $(x - 2)(x + 2)$.

$$= \frac{3}{(x - 2)(x + 2)} - \frac{5}{(x + 2)} \times \frac{(x - 2)}{(x - 2)}$$

$$= \frac{3}{(x - 2)(x + 2)} - \frac{5(x - 2)}{(x + 2)(x - 2)}$$

$$= \frac{3 - 5(x - 2)}{(x - 2)(x + 2)}$$

$$= \frac{3 - 5x + 10}{(x - 2)(x + 2)}$$

$$= \frac{13 - 5x}{(x - 2)(x + 2)}$$

$$= \frac{13 - 5x}{x^2 - 4}, x \neq \pm 2$$

$$(d) \frac{1}{x^2 - 6x + 9} - \frac{1}{x^3 - 9x}$$

$$= \frac{1}{(x - 3)^2} - \frac{1}{x(x^2 - 9)}$$

$$= \frac{1}{(x - 3)^2} - \frac{1}{x(x - 3)(x + 3)}$$

The LCD is $x(x - 3)^2(x + 3)$.

$$= \frac{1}{(x - 3)^2} \times \frac{x(x + 3)}{x(x + 3)} - \frac{1}{x(x - 3)(x + 3)} \times \frac{(x - 3)}{(x - 3)}$$

$$= \frac{x(x + 3)}{x(x - 3)^2(x + 3)} - \frac{(x - 3)}{x(x - 3)^2(x + 3)}$$

$$= \frac{x^2 + 3x - x + 3}{x(x - 3)^2(x + 3)}$$

$$= \frac{x^2 + 2x + 3}{x(x - 3)^2(x + 3)}, x \neq 0, \pm 3$$

Example 2

Simplify.

$$(a) \frac{x + 3}{x - 4} - \frac{x - 1}{x + 2}$$

$$(b) \frac{x + 1}{x^2 + 2x - 3} - \frac{x + 2}{x^2 + 4x - 5}$$

Solution

$$(a) \frac{x + 3}{x - 4} - \frac{x - 1}{x + 2}$$

$$= \frac{x + 3}{x - 4} \times \frac{x + 2}{x + 2} - \frac{x - 1}{x + 2} \times \frac{x - 4}{x - 4}$$

$$= \frac{(x^2 + 5x + 6)}{(x - 4)(x + 2)} - \frac{(x^2 - 5x + 4)}{(x - 4)(x + 2)}$$

$$= \frac{x^2 + 5x + 6 - x^2 + 5x - 4}{(x - 4)(x + 2)}$$

$$= \frac{10x + 2}{(x - 4)(x + 2)}$$

$$= \frac{2(5x + 1)}{(x - 4)(x + 2)}, x \neq -2, 4$$

The LCD is $(x - 4)(x + 2)$.

$$(b) \frac{x + 1}{x^2 + 2x - 3} - \frac{x + 2}{x^2 + 4x - 5}$$

$$= \frac{x + 1}{(x + 3)(x - 1)} - \frac{x + 2}{(x + 5)(x - 1)}$$

$$= \frac{x + 1}{(x + 3)(x - 1)} \times \frac{x + 5}{x + 5} - \frac{x + 2}{(x + 5)(x - 1)} \times \frac{x + 3}{x + 3}$$

$$= \frac{x^2 + 6x + 5}{(x + 3)(x - 1)(x + 5)} - \frac{x^2 + 5x + 6}{(x + 5)(x - 1)(x + 3)}$$

$$= \frac{x^2 + 6x + 5 - x^2 - 5x - 6}{(x + 3)(x - 1)(x + 5)}$$

$$= \frac{x - 1}{(x + 3)(x - 1)(x + 5)}$$

$$= \frac{1}{(x + 3)(x + 5)}, x \neq -5, -3, 1$$

The LCD is $(x + 3)(x - 1)(x + 5)$.

Example 3

Ero and Jamal set off at the same time on a 30-km walk for charity. Ero, who has trained all year for this event, walks 1.4 km/h faster than Jamal, but sees a friend on the route and stops to talk for 20 min. Even with this delay, Ero finishes the walk 2 h ahead of Jamal. How fast was each person walking, and how long did it take for each person to finish the walk?

Solution

Since speed and time are related, choose one variable to represent either speed or time. Let x km/h represent Jamal's walking speed. Then express the other unknown quantities in terms of x . Organize the information in a table.

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} \quad \text{so} \quad \text{Time} = \frac{\text{Distance}}{\text{Speed}}.$$

	Distance (km)	Speed (km/h)	Time (h)
Ero	30	$x + 1.4$	$\frac{30}{x + 1.4}$
Jamal	30	x	$\frac{30}{x}$

Ero took 2 h 20 min less time than Jamal to walk 30 km. The difference between the times in the Time column must be $2\frac{1}{3}$ h. Write this statement as an equation.

$$\frac{30}{x} - \frac{30}{x + 1.4} = \frac{7}{3} \quad \text{Note that the restrictions of } x \neq 0, -1.4 \text{ do not apply here, since } x > 0.$$

$$\begin{aligned} \frac{30}{x} \times \frac{x + 1.4}{x + 1.4} - \frac{30}{x + 1.4} \times \frac{x}{x} &= \frac{7}{3} \\ \frac{30(x + 1.4) - 30x}{x(x + 1.4)} &= \frac{7}{3} \\ \frac{30x + 42 - 30x}{x(x + 1.4)} &= \frac{7}{3} \\ \frac{42}{x(x + 1.4)} &= \frac{7}{3} \\ x(x + 1.4) \times \frac{42}{x(x + 1.4)} &= \frac{7}{3} \times x(x + 1.4) \end{aligned}$$

$$42 = \frac{7}{3}x(x + 1.4)$$

$$3 \times 42 = \frac{7}{3}x(x + 1.4) \times 3$$

$$126 = 7x(x + 1.4) \quad \text{Divide by 7.}$$

$$18 = x(x + 1.4)$$

$$18 = x^2 + 1.4x$$

$$0 = x^2 + 1.4x - 18$$

$$x = \frac{-1.4 \pm \sqrt{1.4^2 - 4(1)(-18)}}{2(1)}$$

Use the quadratic formula.

$$x = \frac{-1.4 \pm 8.6}{2}$$

Ignore the negative root, since $x > 0$.

$$x = 3.6$$

Jamal walked at 3.6 km/h and Ero walked at $3.6 + 1.4 = 5$ km/h.

Jamal's total walking time was $\frac{30}{3.6} = 8.\bar{3}$ or 8 h 20 min. Ero's total walking time was $\frac{30}{5} = 6$ h, but he took 6 h 20 min to complete the route because he stopped for 20 min during the walk.

Practise, Apply, Solve 4.11

A

1. State the LCD for each pair of expressions.

(a) $\frac{3}{x}, \frac{5}{y}$

(b) $\frac{-2}{ab}, \frac{1}{b}$

(c) $\frac{4}{x-1}, \frac{2}{x}$

(d) $\frac{b}{b+2}, \frac{3b}{b+1}$

(e) $\frac{5}{x^2}, \frac{7}{xy}$

(f) $\frac{m-1}{2m+3}, \frac{m}{m-3}$

(g) $\frac{11x}{yz^2}, \frac{8x}{y^2z}$

(h) $\frac{3s}{(s-1)(s+1)}, \frac{s+2}{s-1}$

(i) $\frac{4}{x^2-4}, \frac{3}{x+2}$

2. Find each sum.

(a) $\frac{2}{a} + \frac{7}{b}$

(b) $\frac{3}{x} + \frac{2}{x^2}$

(c) $\frac{4}{xy} + \frac{5}{yz}$

(d) $\frac{7}{x^2y} + \frac{4}{xy^2}$

(e) $\frac{6x}{y} + \frac{3}{y^2}$

(f) $\frac{2x}{x+1} + \frac{5}{x}$

(g) $\frac{3}{5y} + \frac{2}{y+2}$

(h) $\frac{2}{x+3} + \frac{3}{x+1}$

(i) $\frac{x}{x-1} + \frac{2x}{x+3}$

3. Find each difference.

(a) $\frac{3}{4x} - \frac{1}{3x}$

(b) $\frac{3}{y^2} - \frac{2}{y}$

(c) $\frac{2}{3x} - \frac{4}{x}$

(d) $\frac{10}{x^2y} - \frac{6}{xy^2}$

(e) $\frac{9}{4ab^2} - \frac{5}{6a^2b}$

(f) $\frac{6}{y+1} - \frac{3}{y-1}$

(g) $\frac{3x}{x+5} - \frac{2x}{x-3}$

(h) $\frac{t+3}{t-1} - \frac{2t}{t+1}$

(i) $\frac{4}{x(x+2)} - \frac{3}{x}$

4. Simplify.

(a) $\frac{-2}{x} + \frac{5}{x^2} - \frac{8}{3x}$

(b) $\frac{6}{pq} - \frac{4}{q} - \frac{8}{p^2q}$

(c) $\frac{2x}{yz} - \frac{3x}{y} - \frac{9x}{y^2z}$

(d) $\frac{2a}{(a+1)(a+3)} + \frac{5}{(a+3)}$

(e) $\frac{1}{(x+2)^2} - \frac{3}{(x+2)}$

(f) $\frac{1}{2(x+4)} - \frac{3x}{(x+4)^2} + \frac{1}{2}$

5. Simplify.

$$\begin{array}{lll} \text{(a)} \frac{x}{3x-6} + \frac{3}{2x-4} & \text{(b)} \frac{3x-1}{x^2+3x} + \frac{2}{5x} & \text{(c)} \frac{6}{y} + \frac{3}{y^2-2y} \\ \text{(d)} \frac{4p}{p^2+3p} - \frac{2}{p+3} & \text{(e)} \frac{2x+1}{x^2-x-2} + \frac{3}{x-2} & \text{(f)} \frac{4a}{a^2-9} - \frac{2}{a+3} \\ \text{(g)} \frac{1}{y+4} - \frac{y-3}{y^2+3y-4} & \text{(h)} \frac{3m+1}{2m-3} + \frac{m}{m^2-9} \end{array}$$

B

6. Simplify and state each result in lowest terms.

$$\begin{array}{ll} \text{(a)} \frac{x}{x^2-1} - \frac{1}{x^2-1} & \text{(b)} \frac{x-3}{x^2-7x+10} - \frac{x+2}{x^2-25} \\ \text{(c)} \frac{3a}{6a^2-a-2} + \frac{2a}{10a^2-a-3} & \text{(d)} \frac{1}{h} \left(\frac{1}{3x+h} - \frac{1}{3x} \right) \\ \text{(e)} \frac{k+1}{2k^2-7k+6} - \frac{k-3}{2k^2-k-3} & \text{(f)} \frac{1}{a-3} + \frac{1}{a^2-9} - \frac{1}{a^2-2a-3} \end{array}$$

7. **Communication:** Explain why it is better to find the LCD instead of multiplying the denominators when adding or subtracting rational expressions. Use an example to illustrate your points.

8. **Knowledge and Understanding:** Simplify.

$$\text{(a)} \frac{5x-7y}{12x} + \frac{2x-9y}{8y} \qquad \text{(b)} \frac{1}{2x+8} - \frac{3x}{(x+4)^2} + \frac{1}{2}$$

9. Simplify.

$$\begin{array}{l} \text{(a)} \frac{x-2}{x^2-7x+10} + \frac{x+2}{x^2-4x-5} \\ \text{(b)} \frac{x-3}{x^2+x-12} - \frac{x-2}{x^2+3x-4} \\ \text{(c)} \frac{3k}{6k^2+13k-5} + \frac{2k+1}{6k^2+7k-3} \\ \text{(d)} \frac{4-5q}{24q^2+2q-12} - \frac{5-4q}{12q^2-15q-18} \\ \text{(e)} \left(\frac{3x-2}{2x^2-5x-3} - \frac{x+2}{x^2-9} \right) \div \frac{2x-3}{2x^2+7x+3} \\ \text{(f)} \frac{a-2}{6a^2-7a-5} \div \frac{2a}{3a^2-5a} - \frac{3a+2}{2a^2+11a+5} \end{array}$$

10. Find the zeros of each function. (Hint: Rewrite $f(x)$ by putting all terms over a common denominator.)

$$\text{(a)} f(x) = 3x - 1 + \frac{1}{x+1} \qquad \text{(b)} f(x) = 6x - 17 + \frac{28}{x+2}$$

11. Solve for x .

$$\begin{array}{ll} \text{(a)} \frac{x-2}{x} + \frac{4}{5x} = -\frac{1}{5} & \text{(b)} \frac{2x+3}{x-1} - \frac{3}{x} = 2 \\ \text{(c)} \frac{1}{x} = \frac{2}{x+1} + \frac{1}{1-x} & \text{(d)} \frac{3x}{x^2-1} = \frac{x}{x+1} - 4 \\ \text{(e)} \frac{15}{x^2-1} = \frac{4}{x-1} - \frac{3}{x+1} & \text{(f)} \frac{x}{x-2} + 2 = \frac{5x}{x+2} + \frac{3x+1}{x^2-4} \end{array}$$

- 12.** On the 42-km go-cart course at Sportsworld, Arshia drives 0.4 km/h faster than Sarah, but she has engine trouble part way around the course and has to stop to get the go-cart fixed. This stop costs Arshia one-half hour, and so she arrives 15 min after Sarah at the end of the course. How fast did each girl drive and how long did each girl take to finish the course? Answer to one decimal place.
- 13.** Rowing at 8 km/h, in still water, Rima and Bhanu take 16 h to row 39 km down a river and 39 km back. Find the speed of the current to two decimal places.
- 14.** A river flows at 2 km/h, and John takes 6 h to row 16 km up the river and 16 km back. How fast did he row?
- 15. Application:** Jaime bought a case of concert T-shirts for \$450. She kept two for herself and sold the rest for \$560, making a profit of \$10 on each shirt. How many shirts were in the case?
- 16.** Stuart agrees to a house-painting job for \$900. He takes 4 days longer than expected, and he has earned \$18.75 less per day than expected. In how many days did he expect to complete the house?
- 17.** A grade 11 class, on a field trip to Montreal, had lunch in a restaurant. The bill came to \$239.25. Four students had birthdays that day, and it was agreed that these four should not have to pay for lunch. The other students had to pay \$1 more than if all the students had paid. How many students had lunch?



- 18. Check Your Understanding:** Explain the steps for adding or subtracting rational expressions. Use $\frac{x-2}{x^2-4x-32} - \frac{x-1}{x^2-2x-48}$ as an example and explain each step fully.



- 19. Thinking, Inquiry, Problem Solving:** Find A and B such that $\frac{2x-1}{(x+1)(3x+2)} = \frac{A}{x+1} + \frac{B}{3x+2}$.
- 20.** Find A , B , and C such that $\frac{(x-1)}{(x+1)(x-2)^2} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$.
- 21.** It takes Paulina x hours to travel a kilometres. If she increases her speed by b kilometres per hour, the journey will take her c hours less time. Find x .



The Chapter Problem—Fundraising

Apply what you have learned in this section to answer these questions about the Chapter Problem on page 300.

CP18. Write a proposal, for this fundraiser, that the representative could present to the school council. Include all of your graphs and analyses.