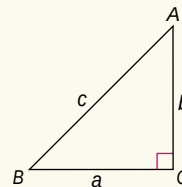


Trigonometric Functions

The Trigonometry of Right Triangles

By the Pythagorean relationship, $a^2 + b^2 = c^2$ for any right angle triangle, where c is the length of the hypotenuse, and a and b are the lengths of the other two sides.



In any right angle triangle, there are three primary trigonometric ratios associated with the measure of an angle to the ratio of two sides. For example, for $\triangle ABC$, in figure 1,

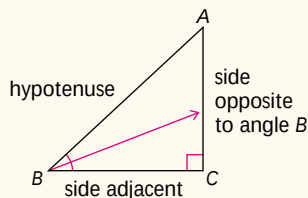


figure 1

For $\angle B$
$\sin B = \frac{\text{opposite}}{\text{hypotenuse}}$
$\cos B = \frac{\text{adjacent}}{\text{hypotenuse}}$
$\tan B = \frac{\text{opposite}}{\text{adjacent}}$

Similarly, in figure 2,

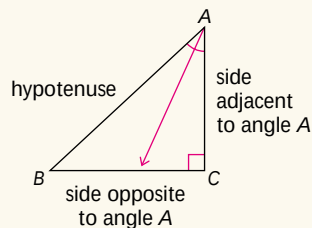


figure 2

For $\angle A$
$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$
$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$
$\tan A = \frac{\text{opposite}}{\text{adjacent}}$

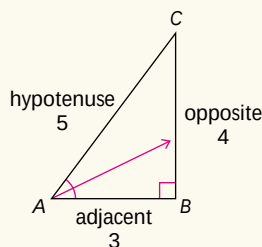
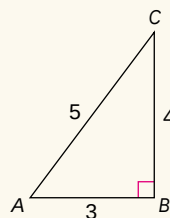
Note how the opposite and adjacent sides change in figures 1 and 2 with angles A and B .

Example 1

State the primary trigonometric ratios of $\angle A$.

Solution

Sketch the triangle, then label the opposite side, the adjacent side, and the hypotenuse.



$$\begin{aligned}\sin A &= \frac{\text{opposite}}{\text{hypotenuse}} \\ &= \frac{4}{5}\end{aligned}$$

$$\begin{aligned}\cos A &= \frac{\text{adjacent}}{\text{hypotenuse}} \\ &= \frac{3}{5}\end{aligned}$$

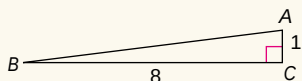
$$\begin{aligned}\tan A &= \frac{\text{opposite}}{\text{adjacent}} \\ &= \frac{4}{3}\end{aligned}$$

Example 2

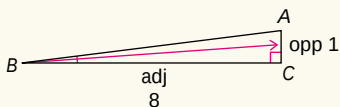
A ramp must have a rise of one unit for every eight units of run. What is the angle of inclination for the ramp?

Solution

The slope of the ramp is $\frac{\text{rise}}{\text{run}} = \frac{1}{8}$. Draw a labelled sketch.



Calculate the measure of $\angle B$ to determine the angle of inclination.



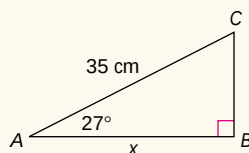
The trigonometric ratio that associates $\angle B$ to the opposite and adjacent sides is the tangent. Therefore,

$$\begin{aligned}\tan B &= \frac{\text{opposite}}{\text{adjacent}} \\ &= \frac{1}{8} \\ B &= \tan^{-1}\left(\frac{1}{8}\right) \\ &\doteq 7^\circ\end{aligned}$$

The angle of inclination is about 7° .

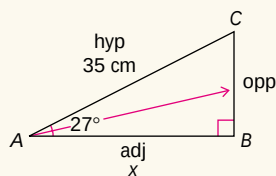
Example 3

Determine x to the nearest centimetre.



Solution

Label the sketch. The cosine ratio associates $\angle A$ to the adjacent side and the hypotenuse.



Then,

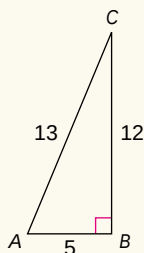
$$\begin{aligned}\cos A &= \frac{\text{adjacent}}{\text{hypotenuse}} \\ &= \frac{x}{35} \\ x &= 35 \cos 27^\circ \\ &\doteq 31\end{aligned}$$

Then, x is about 31 cm.

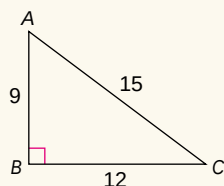
Practice

1. A rectangular lot is 15 m by 22 m. How long is the diagonal to the nearest metre?
2. State the primary trigonometric ratios for $\angle A$.

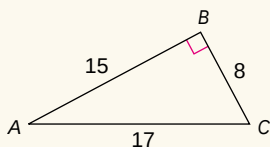
(a)



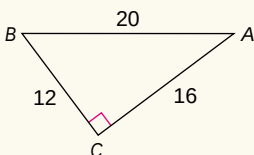
(b)



(c)



(d)



3. Solve for x to one decimal place.

(a) $\sin 39^\circ = \frac{x}{7}$ (b) $\cos 65^\circ = \frac{x}{16}$

(c) $\tan 15^\circ = \frac{x}{22}$ (d) $\tan 49^\circ = \frac{31}{x}$

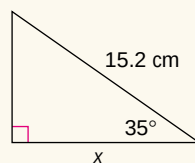
4. Solve for $\angle A$ to the nearest degree.

(a) $\sin A = \frac{5}{8}$ (b) $\cos A = \frac{13}{22}$

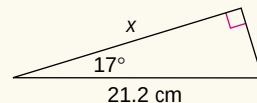
(c) $\tan B = \frac{19}{22}$ (d) $\cos B = \frac{3}{7}$

5. Determine x to one decimal place.

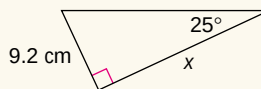
(a)



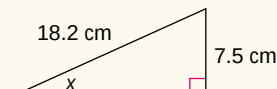
(b)



(c)



(d)



6. In $\triangle ABC$, $\angle B = 90^\circ$ and $AC = 13$ cm. Determine

(a) BC if $\angle C = 17^\circ$

(b) AB if $\angle C = 26^\circ$

(c) $\angle A$ if $BC = 6$ cm

(d) $\angle C$ if $BC = 9$ cm

7. A tree casts a shadow 9.3 m long when the angle of the sun is 43° . How tall is the tree?

8. Janine stands 30.0 m from the base of a communications tower. The angle of elevation from her eyes to the top of the tower is 70° . How high is the tower if her eyes are 1.8 m above the ground?

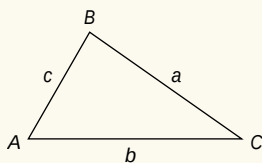
9. A surveillance camera is mounted on the top of an 80 m tall building. The angle of elevation from the camera to the top of another building is 42° . The angle of depression from the camera to the same building is 32° . How tall is the other building?

Trigonometry of Acute Triangles: the Sine Law and the Cosine Law

An acute triangle contains three angles less than 90° .

Sine Law

The sine law states that for $\triangle ABC$,

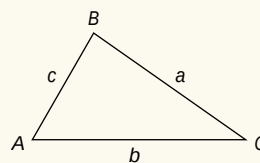


$$\bullet \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \quad \text{or} \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

To use the sine law, two angles and one side (AAS) or two sides and an opposite angle (SSA) must be given.

Cosine Law

The cosine law states that for $\triangle ABC$,



$$\begin{aligned} \bullet c^2 &= a^2 + b^2 - 2ab \cos C \quad \text{or} \\ \bullet a^2 &= b^2 + c^2 - 2bc \cos A \quad \text{or} \\ \bullet b^2 &= a^2 + c^2 - 2ac \cos B \end{aligned}$$

To use the cosine law, two sides and the contained angle (SAS) or three sides (SSS) must be given.

Example 1

Determine the length of XZ to one decimal place.

Solution

In this triangle, $\angle X = 84^\circ$, $\angle Z = 27^\circ$, and $z = 4$ cm.

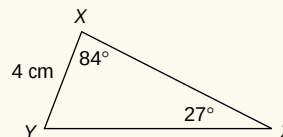
$$\begin{aligned} \angle Y &= 180^\circ - (84^\circ + 27^\circ) \\ &= 180^\circ - 111^\circ \\ &= 69^\circ \end{aligned}$$

This is not a right triangle, so the primary trigonometric ratios do not apply.

Two angles and one side are known, so the sine law can be used.

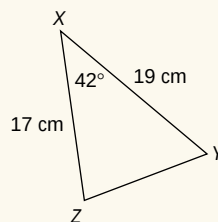
$$\begin{aligned} \frac{\sin Y}{y} &= \frac{\sin Z}{z} \\ \frac{\sin 69^\circ}{y} &= \frac{\sin 27^\circ}{4} \\ y &= \frac{4 \sin 69^\circ}{\sin 27^\circ} \\ &\doteq 8.2 \end{aligned}$$

Then XZ is about 8.2 cm.



Example 2

Determine the length of ZY to one decimal place.



Solution

In this triangle, $\angle X = 42^\circ$, $y = 17$ cm, and $z = 19$ cm.

There is not enough information to use the primary trigonometric ratios or the sine law. However, two sides and the contained angle are known, so the cosine law can be used.

$$x^2 = y^2 + z^2 - 2yz \cos X$$

$$x^2 = 17^2 + 19^2 - 2(17)(19) \cos 42^\circ$$

$$x = \sqrt{17^2 + 19^2 - 2(17)(19) \cos 42^\circ}$$

$$x \doteq 13$$

Therefore, ZY is about 13 cm.

Practice

1. Solve to one decimal place.

(a) $\frac{\sin 35^\circ}{c} = \frac{\sin 42^\circ}{12}$

(b) $\frac{15}{\sin 43^\circ} = \frac{13}{\sin B}$

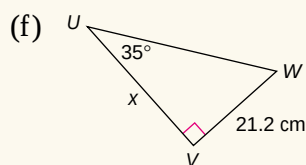
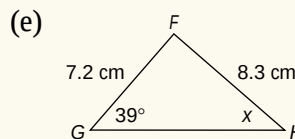
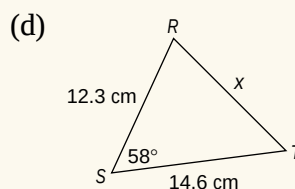
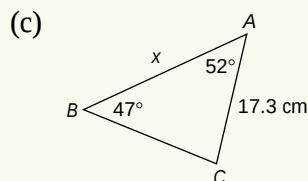
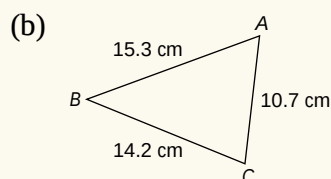
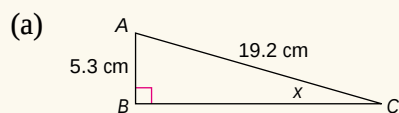
(c) $19^2 = 15^2 + 13^2 - 2(15)(13) \cos A$

(d) $c^2 = 12^2 + 17^2 - 2(12)(17) \cos 47^\circ$

(e) $\frac{\sin A}{12.3} = \frac{\sin 58^\circ}{14.2}$

(f) $\frac{\sin 14^\circ}{3.1} = \frac{\sin 27^\circ}{b}$

2. Determine x to one decimal place.



3. Solve each triangle for all missing sides and angles.
- (a) $\triangle CAT$, with $c = 5.2$ cm,
 $a = 6.8$ cm, and $\angle T = 59^\circ$
- (b) $\triangle ABC$, with $a = 4.3$ cm,
 $b = 5.2$ cm, and $c = 7.5$ cm
- (c) $\triangle DEF$, with $DE = 14.3$ cm,
 $EF = 17.2$ cm, and $\angle D = 39^\circ$
4. A swamp separates points L and R . To determine the distance between them, Ciana stands at L and looks towards R . She turns about 45° and walks 52 paces from L to point P . From P , she looks at R and estimates that $\angle LPR$ is about 60° . How many paces is it from L to R ?
5. An observation helicopter using a laser device determines the helicopter is 1800 m from a boat in distress. The helicopter is 1200 m from a rescue boat. The angle formed between the helicopter and the two boats is 35° . How far apart are the boats?
6. Neil designs a cottage that is 15 m wide. The roof rafters are the same length and meet at an angle of 80° . The rafters hang over the supporting wall by 0.5 m. How long are the rafters?
-