

Chapter 6

Extending Skills with Trigonometry

People have used trigonometry for thousands of years. Architects and engineers have used it to design and build structures ranging from the pyramids in Egypt to the CN Tower in Toronto. Astronomers have used trigonometry to measure the distance of the planets and stars from the Earth, and travellers have used it to navigate on land, on sea, and in the air.

In Chapter 5, you used trigonometric functions to model periodic phenomena. In earlier grades, you used trigonometry to determine sides and angles of acute and right triangles. In this chapter, you will apply what you have learned to obtuse triangles and extend your skills to deal with more complex trigonometric equations.

In this chapter, you will

- extend the sine law and cosine law to obtuse triangles
- use trigonometry to model and solve problems in two and three dimensions
- determine the exact values of special trigonometric ratios
- learn about a type of equation called an identity
- prove whether a given equation is an identity or not
- solve quadratic trigonometric equations.

Visit the Nelson Mathematics student Web site at www.math.nelson.com. You'll find links to interactive examples, online tutorials, and more.

Connections



The Chapter Problem

What Time Is It?

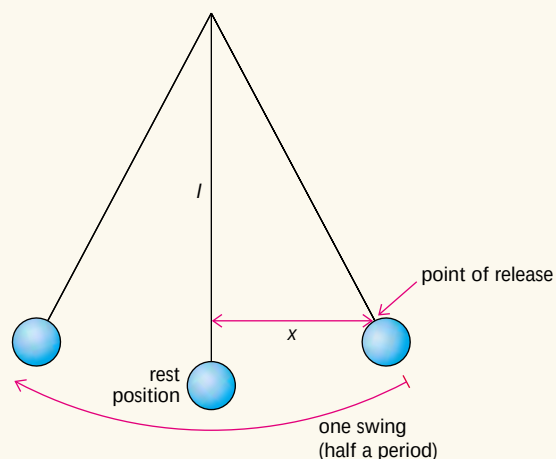
In early times, people used the repeating patterns of nature to keep track of time. For instance, the sundial uses the fact that as the Earth rotates on its axis, the position of the sun changes, causing the position of shadows to change. The ancient Greeks used their knowledge of geometry to make sundials that were very complex and accurate. However, even the best sundial has its drawbacks: it cannot be used on cloudy days or at night.

Mechanical clocks were invented sometime in the 1300s. These crude devices were inaccurate and had to be constantly reset. In the 1500s, Galileo discovered that a pendulum could be used to keep uniform time. You may have seen grandfather clocks that use a pendulum to do this. Owing to friction, a swinging pendulum left on its own will eventually stop swinging. In a clock, a device called a yoke pushes the pendulum to ensure that it swings through the same angle on each swing.

When a pendulum is set in motion, it swings back and forth through an arc. Let x be the distance that the pendulum swings from the centre point and l be the length of the pendulum. Then the motion can be modelled by the function $x = M \cos\left(t \sqrt{\frac{980}{l}}\right)$, where M is the maximum value of x and t is the time, in seconds, that has elapsed since the pendulum was set in motion.

The period of a pendulum is the time the pendulum takes to return to its point of release. In this diagram, the period is equivalent to two swings, one to the left and the return swing to the right. When the value of M is small, the period depends entirely on the length of pendulum.

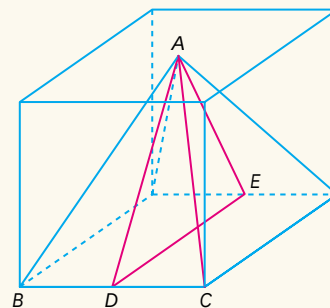
How long must the pendulum in a grandfather clock be for one swing to take exactly one second?



Challenge 1

The largest right pyramid that can be contained entirely in a cube, shares the same base and height of the cube. Determine:

- (a) all three angles in the triangular faces of the pyramid, $\angle ABC$, $\angle ACB$, and $\angle BAC$.
- (b) the angle at which each triangular face of the pyramid meets with the base, $\angle ADE$.
- (c) the angle at the top of the pyramid formed between opposite triangular faces, $\angle DAE$.



Challenge 2

In this chapter, you will learn about special types of equations called fundamental trigonometric identities. There are also other kinds of trigonometric identities. Two of these are called addition formulas. For the sine and cosine functions, where x and y are angles, the formulas are

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

- (a) Use the special triangles to show that these relationships are true when $x = \frac{\pi}{3}$ and $y = \frac{\pi}{6}$.
- (b) Use the addition formula to develop equivalent expressions for $\sin 2x$ and $\cos 2x$.
- (c) Prove that $\frac{1 - \cos 2x + \sin 2x}{1 + \cos 2x + \sin 2x} = \tan x$.

Web Challenge

For a related Internet activity, go to www.math.nelson.com.

