

Performance Tasks for Part 3

Trigonometric Functions

THE FOLLOWING ACTIVITIES SHOULD EACH TAKE LESS THAN A PERIOD TO COMPLETE.

1. Sunrise, Sunset

Using the Internet or an almanac, find the sunrise times for each day of the year in a particular city. Plot the sunrise times for every tenth day as a function of the day of the year.

- Write an equation that approximately models this data.
- Test how well your model fits the data and use the model to predict the sunrise for some of the days that are not in the plot.
- Write a report that details how you arrived at the model and discusses how well the model fits the data.

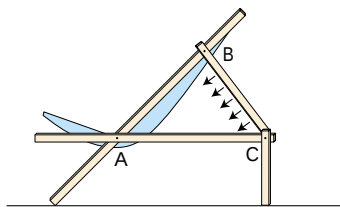
2. Amusement Rides

A popular ride at an amusement park is called the “Ring of Terror.” It is like a Ferris wheel but is inside a haunted house. Riders board a platform at the centre of the “ring” and the ring moves counterclockwise. When a rider is moving above the platform, he or she meets flying creatures and when the seat descends to a level below the platform, creatures emerge from a murky, slimy pit. The radius of the ring is 6 m.

- Graph the height of a rider with respect to the platform through three revolutions of the ride.
- Create an equation to model the motion of a person in the ride.
- How would your graph and equation change if the rider got on the ride in the pit, at the bottom of the ring? Explain.

3. Lawn Chairs

The manufacturer of a reclining lawn chair would like to have the chair be positioned at the following angles: 95° , 120° , 135° , 160° , and 175° . In the figure, AC is 65 cm and AB is 45 cm. Find the positions for the notches on BC that will produce the required angles. Give a complete solution.



4. Solving Systems of Trigonometric Equations

Solve the following system of equations, both graphically and algebraically, for values where $-2\pi \leq x \leq 2\pi$.

$$y = \cos^2 x$$

$$y = \frac{1}{2} + \sin^2 x$$

5. How High Is the Tower?

A skier sees the top of a radio tower, due south of him, at an angle of elevation of 32° . He then skis on a bearing of 130° for 560 m and finds himself due east of the tower. Calculate

- (a) the height of the radio tower
- (b) the distance from the tower to the skier
- (c) the angle of elevation of the top of the tower from the new position

6. Proving Identities

Use graphing technology to determine which statements are trigonometric identities. For those that are, prove the identity. For those that are not, decide for which values the statements are true.

(a) $\tan^2 x \cos^2 x = 1 - \cos^2 x$

(b) $\cos^2 x = \sin^2 x - 1$

(c) $\frac{1}{\cos x} + \frac{1}{\sin x} = 1$

(d) $\frac{\tan x \cos x}{\sin x} = 1$

THE FOLLOWING ACTIVITIES COULD EACH TAKE MORE THAN A PERIOD TO COMPLETE.

7. The Tide Is High.

This data was collected on the east coast of Canada on August 15 and relates the height, h , of a tide to the time of day, t .

t	4 A.M.	5 A.M.	6 A.M.	7 A.M.	8 A.M.	9 A.M.	10 A.M.	11 A.M.
h (m)	0.7	1.2	2.4	4.1	5.8	7.1	7.6	7.2

t	12 P.M.	1 P.M.	2 P.M.	3 P.M.	4 P.M.	5 P.M.	6 P.M.	7 P.M.	8 P.M.
h (m)	6.1	4.5	2.9	1.6	0.8	1.1	2.1	3.7	5.4

- (a) Graph this data using time as the x -axis and height as the y -axis. You may need to use a 24-h clock for the time. In other words, 2 P.M. should be represented as 14.
- (b) How high is high tide and how low is low tide?
- (c) Determine the amplitude and period for this periodic function.
- (d) Determine an equation that approximates the data. Justify all components of your equation.
- (e) Determine the height of the tide at 2:00 A.M. on August 15, 11:00 P.M. on August 15, and 3:00 A.M. on August 16.

8. A Trigonometric Trip

Create a trigonometric trip. Use your surrounding area to create a problem of finding an inaccessible height or distance. Your problem should be three-dimensional and include a right triangle and the use of the cosine and sine law. Submit the problem and full solution. Perhaps the whole class could go on a trigonometric walk to take measurements and solve one another's problems.

9. Math Music

A pure tone produces a sine wave when shown on an oscilloscope. When an instrument is played, the tone is not pure. For instance, when a guitar string or piano string vibrates it does not produce a simple sine wave. It does produce other, less distinguishable harmonious waves of higher pitch called *harmonic* waves. For instance, $y = \sin(2x)$ is called the second harmonic, $y = \sin(3x)$ is called the third harmonic, and $y = \sin(4x)$ is called the fourth harmonic.

(a) Use a table to graph the second harmonic for values between $-2\pi \leq \theta \leq 2\pi$.

(b) When a note is played on a musical instrument, the fundamental pitch usually has the greatest amplitude. The second, third, and fourth harmonics can be heard but their amplitudes are usually lower than the fundamental harmonic.

Graph $y = \frac{1}{2} \sin(2x)$ to see how the amplitude is reduced.

(c) The wave of a musical instrument is actually an additive waveform produced by combining the fundamental sine wave form and different combinations of harmonics.

Graph $y = \sin x + \frac{1}{2} \sin(2x)$ by hand. Do this by graphing both $y = \sin x$ and $y = \frac{1}{2} \sin(2x)$ on the same set of axes and then adding the y -values for corresponding x -values. Use graphing technology to determine if your graph is accurate.

(d) Use graphing technology to graph each equation. Sketch the graph on paper:

i. $y = \sin x + \frac{1}{2} \sin(2x) + \frac{1}{4} \sin(4x)$

ii. $y = \sin x + \frac{1}{3} \sin(3x) + \frac{1}{5} \sin(5x) + \frac{1}{7} \sin(7x)$

Research Extension (Extra Time)

(e) By examining waves produced on an oscilloscope by a synthesizer (or other technology set-ups), match the above graphs to the instruments imitated by the synthesizer. Create an equation of your own with harmonics and see if you can determine which instrument it most closely resembles.