

The conics share several similar features. Most conics have a focus or foci. You can describe each conic by a locus definition. And you can express all conics as second-degree equations.

The equation of each conic has a general form. But all conics can have the general form $ax^2 + by^2 + 2gx + 2fy + C = 0$. This section will consolidate what you know about conics.

Part 1: Recognizing Conics in General Form

By this point, you have become very familiar with different forms of the conic sections.

Think, Do, Discuss

1. What type of conic is $9x^2 + 4y^2 = 36$? State the values of a , b , g , f , and C for the general form of this equation.
2. Change $\frac{(x-1)^2}{4} - \frac{y^2}{25} = 1$ to the general form and state the values of a , b , g , f , and C . Name the conic.
3. Predict the conic represented by $x^2 - 4x - 8y - 28 = 0$. Then rearrange the equation in standard form and determine if your prediction is correct.
4. If $ax^2 + by^2 + 2gx + 2fy + C = 0$ and $a = b = 0$ or $ab = 0$, then what is the conic?
5. If $ax^2 + by^2 + 2gx + 2fy + C = 0$ and $a = b$, then what is the conic?
6. Consider the case of $ab > 0$. What is the conic? What is the conic if $ab < 0$?
7. Name each conic.
 $4y^2 - 8y + 9x^2 - 54x + 49 = 0$
 $x^2 - 4x - y^2 - 5 - 4y = 0$
8. If $f = g = 0$, then describe the conic.



Part 2: Organizing What You Know

You can write any conic in the general form $ax^2 + by^2 + 2gx + 2fy + C = 0$.

To identify the conic section represented by a given equation, write the equation in standard form. However, you can also identify the conic section by comparing it with the general form. The following table will help you organize information.

Identifying Conics by the General Form $ax^2 + by^2 + 2gx + 2fy + C = 0$

Conic Section	Standard Form	Relationship of Values in the General Form
Circle	$(x - h)^2 + (y - k)^2 = r^2$	$a = b$
Ellipse	$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$ $\frac{(x - h)^2}{b^2} + \frac{(y - k)^2}{a^2} = 1$	$ab > 0$ $a \neq 0, b \neq 0$
Parabola	$(x - h)^2 = 4p(y - k)$ $(y - k)^2 = 4p(x - h)$	$ab = 0$, either $a = 0$ or $b = 0$
Hyperbola	$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$ $\frac{(x - h)^2}{b^2} - \frac{(y - k)^2}{a^2} = -1$	$ab < 0$ $a \neq 0, b \neq 0$

Example 1

Identify the conic.

(a) $x^2 + y^2 - 4x - 12 = 0$

(b) $2x^2 + y^2 + 4x + 2y = 20$

Solution

For each equation, find the values of a and b for the general form $ax^2 + by^2 + 2gx + 2fy + C = 0$.

(a) In this equation, $a = b = 1$. Since $a = b$, then the conic is a circle.

(b) In this equation, $ab > 0$, and the conic is an ellipse.

Example 2

Name the conic and write it in standard form.

$$4x^2 + y^2 - 8x - 12 = 0$$

Solution

Since $ab > 0$, the conic is an ellipse. To write the equation in standard form, complete the square.

$$\begin{aligned}
4x^2 + y^2 - 8x - 12 &= 0 \\
(4x^2 - 8x) + y^2 &= 12 \\
4(x^2 - 2x) + y^2 &= 12 \\
4(x^2 - 2x + ?) + y^2 &= 12 \\
4(x^2 - 2x + 1) + y^2 &= 12 + 4 \\
4(x - 1)^2 + y^2 &= 16 \\
\frac{(x - 1)^2}{4} + \frac{y^2}{16} &= 1
\end{aligned}$$

The equation of the conic in standard form is $\frac{(x - 1)^2}{4} + \frac{y^2}{16} = 1$.

Consolidate Your Understanding

1. Explain how you can determine what type of conic an equation in general form represents.
2. Explain how you can tell whether a conic is centred at the origin given the general form of the equation.

Focus 7.10U

Key Ideas

- A conic that is centred about the origin has the form $ax^2 + by^2 + C = 0$.
- A conic that is not centred about the origin has the **general form**
 $ax^2 + by^2 + 2gx + 2fy + C = 0$.
- The following conditions determine the type of conic.
 - ♦ If $a = b$, then the conic is a circle.
 - ♦ If $ab > 0$, then the conic is an ellipse.
 - ♦ If $ab < 0$, then the conic is a hyperbola.
 - ♦ If $ab = 0$, then the conic is a parabola.

Practise, Apply, Solve 7.10U

A

1. Identify the conic and list the coordinates of the centre (or vertex in the case of a parabola).

(a) $(y - 2)^2 = 9(x + 1)$

(b) $\frac{(x + 1)^2}{16} - \frac{y^2}{25} = -1$

(c) $\frac{(x - 3)^2}{9} + \frac{(y - 8)^2}{12} = 1$

(d) $\frac{x^2}{16} + \frac{(y + 6)^2}{8} = 1$

(e) $(x + 1)^2 = -12(y - 1)$

(f) $\frac{(y - 2)^2}{36} - \frac{(x + 2)^2}{9} = 1$

2. Knowledge and Understanding: Identify the conic.

- (a) $9x^2 - 4y^2 - 36x - 40y = 100$ (b) $x^2 + 25y^2 + 2x = 124$
(c) $4x^2 - 16x - y + 17 = 0$ (d) $4x^2 + 9y^2 + 9x - 3y + 4 = 0$
(e) $3x^2 + 3y^2 - 2x + 1 = 0$ (f) $3x^2 - 6y^2 - 2y + 4 = 0$
(g) $x^2 + 2y^2 - 2y + 1 = 0$ (h) $y^2 - 2y + 9x + 19 = 0$

3. For each conic,

- i. identify the conic
ii. list the coordinates of the centre
iii. tell whether the foci are on a horizontal line or on a vertical line

- (a) $x^2 - y^2 + 8x + 4y + 9 = 0$
(b) $12x^2 + 4y^2 - 24x = 44$
(c) $4x^2 - y^2 + 24x - 8y + 12 = 0$

4. For $3x^2 - 6x - y + 1 = 0$,

- (a) determine the type of conic
(b) write the equation in standard form
(c) graph the equation

5. For $y^2 - 3x^2 - 4y + 6x + 2 = 0$,

- (a) determine the type of conic
(b) write the equation in standard form
(c) graph the equation

6. For $2x^2 + 2y^2 - 8x + 12y + 24 = 0$,

- (a) determine the type of conic
(b) write the equation in standard form
(c) graph the equation

7. State the type of conic and determine the general form of the equation for each conic.

- (a) $\frac{x^2}{9} + \frac{(y-3)^2}{4} = 1$ (b) $(x+3)^2 + (y-1)^2 = 9$
(c) $\frac{(x+4)^2}{16} - \frac{(y-3)^2}{4} = -1$ (d) $(x-2)^2 = -6(y+1)$
(e) $\frac{(x-1)^2}{4} - \frac{(y+2)^2}{5} = 1$ (f) $(y+3)^2 = 2(x-1)$

B

8. Write each equation in standard form and sketch its graph.

- (a) $4x^2 + 9y^2 - 16x + 54y + 61 = 0$
(b) $9x^2 - 4y^2 - 54x - 36y + 81 = 0$
(c) $2x^2 + 3y^2 - 12x + 12y + 24 = 0$
(d) $y^2 - 2x^2 - 2y + 6x - 12 = 0$
(e) $y^2 - 8x - 8y = 0$

9. **Communication:** A classmate claims that $2x^2 - 4x + 2y^2 + 5y - 16 = 0$ is an ellipse. Is she correct? Explain how you know.
10. A conic is given by the equation $y^2 - 5x^2 + 20x = 50$.
- What are the coordinates of its centre?
 - Find the coordinates of the foci and the vertices.
 - Find the lengths of the transverse and the conjugate axes.
 - Find the equations of the asymptotes.
 - Graph the conic.
11. Determine the equation in general form for a parabola with vertex (4, 2) and focus (1, 2).
12. Determine the equation in general form for a circle with centre (2, 3) and radius 5 units.
13. Identify each conic and then graph it.
- $y^2 + 6y - 4x + 25 = 0$
 - $9x^2 - 16y^2 - 36x + 96y + 36 = 0$
 - $4x^2 - 8x + y^2 - 12 = 0$
 - $x^2 + 4x - y + 8 = 0$
 - $25x^2 + 4y^2 + 100x - 16y + 16 = 0$
14. **Application:** Find the equation in general form of the conic defined by the locus of points that are equidistant from the line $x = 5$ and point (4, 1).
15. What translation maps $x^2 + y^2 + 6x + 6y + 2 = 0$ onto a circle whose centre is at the origin?
16. Determine the equation in general form for a hyperbola that is centred about the origin with one vertex at (5, 0) and one focus at (6, 0).
17. **Thinking, Inquiry, Problem Solving:** The end points of the diameter of a circle are $A(-2, 4)$ and $B(6, 2)$. Find the equation of the circle in general form.
18. **Check Your Understanding:** Explain, in your own words, the meaning of the restrictions for a and b . For instance, why does the restriction $ab < 0$ tell you that the conic is a hyperbola?

C

19. Find an equation of the hyperbola whose foci are the vertices of the ellipse $7x^2 + 11y^2 = 77$ and whose vertices are the foci of this ellipse.
20. What is the equation, in general form, of each conic?
- The foci are $(-2, 0)$ and $(4, 0)$, and the sum of the focal radii is 10.
 - The foci are $(3, 0)$ and $(-5, 0)$, and the difference of the focal radii is 6.
 - The foci are $(-6, 3)$ and $(2, 3)$, and the sum of the focal radii is 10.
 - The foci are $(5, 4)$ and $(-7, 4)$, and the difference of the focal radii is 10.