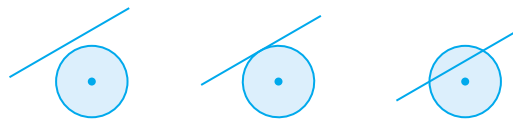


Many universities or community colleges have their own radio stations that broadcast only within a certain range or distance. You are driving to visit a campus, listening to your radio. You know that the university's radio station transmits a signal within 25 km of the station. Imagine that the transmission tower is at the origin of a graph and that you are travelling along a path that could be represented by the line $y = 2x - 10$. What are your coordinates when your radio first picks up the signal?



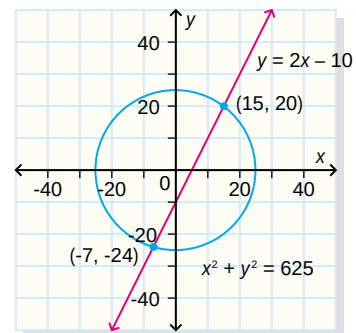
Solving Problems Graphically

There are several different ways to solve this problem. If you sketch this problem, it is easy to see that several solutions are possible, depending on how the line and the circle meet.



There can be no solution, one solution, or two solutions.

If you draw an accurate graph or use graphing technology, then you could easily solve the problem. To graph, you need to know the equations for the relations. The equation for the line is $y = 2x - 10$. The transmission of the signal can be represented by a circle with the equation $x^2 + y^2 = 625$. Here is a graph of both the circle and the line.



From the graph, you can see two solutions, points $(-7, -24)$ and $(15, 20)$.

Check the solutions by substituting them in each equation.

Solving Problems Algebraically

This problem can also be solved algebraically. Begin with the two equations, $x^2 + y^2 = 625$ and $y = 2x - 10$. Substitute $y = 2x - 10$ for y in the equation of the circle.

$x^2 + y^2 = 625$	Substitute.
$x^2 + (2x - 10)^2 = 625$	Expand.
$x^2 + 4x^2 - 40x + 100 = 625$	Simplify.
$5x^2 - 40x - 525 = 0$	Factor.
$5(x^2 - 8x - 105) = 0$	Factor again.
$5(x - 15)(x + 7) = 0$	Solve.
$x = 15 \quad \text{or} \quad x = -7$	

Substitute $x = 15$ in $y = 2x - 10$.	Substitute $x = -7$ in $y = 2x - 10$.
$y = 2(15) - 10$	$y = 2(-7) - 10$
$y = 30 - 10$	$y = -14 - 10$
$y = 20$	$y = -24$

You should also verify the solutions by substituting them in the equation of the circle. The points at which you will first hear the radio signal are (15, 20) and (-7, -24).

Consolidate Your Understanding

1. Why does drawing a diagram or a sketch of the graph help you solve an intersection problem?
2. Why is it important to check your answers for both equations in the system of equations?
3. List the algebraic methods that you know for solving systems of equations.

Focus 7.11U

Key Ideas

- Systems of equations can be solved using a graph or algebra.
- To solve a system having both a quadratic equation and a linear equation, first sketch or graph the system before solving.
- Often a graphical solution will be difficult because the intersecting points may not be integer values and may be hard to determine from the graph.

Example 1

Graph the system of equations, estimate the number of solutions, and then solve the system.

$$y = x - 2$$

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

Solution

The first equation is a line and the second equation is an ellipse. Graphing will help you to estimate the number of solutions.

It appears that there may be two solutions. To solve the system algebraically, substitute $y = x - 2$ in the equation of the ellipse.

$$\frac{x^2}{4} + \frac{(x-2)^2}{9} = 1$$

$$9x^2 + 4(x-2)^2 = 36$$

$$9x^2 + 4(x^2 - 4x + 4) = 36$$

$$9x^2 + 4x^2 - 16x + 16 = 36$$

$$13x^2 - 16x - 20 = 0$$

Using the quadratic formula,

$$x = \frac{-(-16) \pm \sqrt{(-16)^2 - 4(13)(-20)}}{2(13)}$$

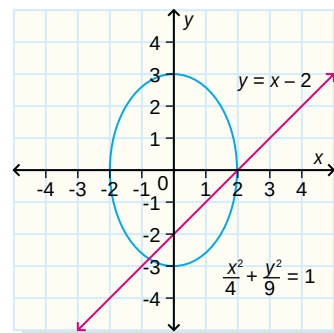
$$x = \frac{16 \pm \sqrt{256 + 1040}}{26}$$

$$x = \frac{16 \pm 36}{26}$$

$$x = 2 \text{ or } x = \frac{-10}{13}$$

If $x = 2$, then $y = 0$; if $x = \frac{-10}{13}$, then $y = \frac{-36}{13}$.

The solutions are $(2, 0)$ and $(\frac{-10}{13}, \frac{-36}{13})$.



Example 2

Find the intersection of $y = -x^2 + 5x$ and $y = 3x + 1$.

Solution

Substitute $3x + 1$ for y in the equation $y = -x^2 + 5x$.

$$-x^2 + 5x = 3x + 1 \quad \text{Rearrange to isolate the variable.}$$

$$-x^2 + 2x - 1 = 0 \quad \text{Divide by } -1.$$

$$x^2 - 2x + 1 = 0 \quad \text{Factor.}$$

$$(x-1)(x-1) = 0 \quad \text{Solve.}$$

$$x = 1$$

Substitute $x = 1$ in one of the original equations.

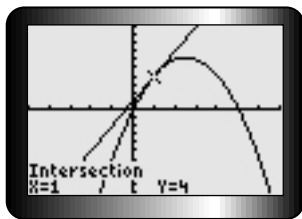
$$y = 3x + 1$$

$$y = 3(1) + 1$$

$$y = 4$$

The solution is $(1, 4)$.

Use your graphing calculator to verify the solutions.



Practise, Apply, Solve 7.11U

A

1. For each system of equations, state which of the given ordered pairs are solutions to the system.

(a) $x^2 + y^2 = 25$ and $x + y = 1$ $(-3, 4), (3, -4), (4, -3), (-4, 3)$

(b) $x^2 - y^2 = 3$ and $x + y = -1$ $(1, -2), (-1, 2), (2, -1), (-2, 1)$

(c) $x^2 + y^2 = 5$ and $2x^2 + y = 0$ $(1, 2), (-1, 2), (-1, -2), (1, -2)$

Graphing technology would be useful for questions 2 to 6.

2. Graph $2x + y - 4 = 0$ and $4x^2 - y^2 = 16$.

(a) What are the coordinates of the intersection points?

(b) Verify the coordinates by substitution.

3. Graph $x^2 = \frac{1}{2}y$ and $y = -3x + 5$.

(a) What are the coordinates of the intersection points?

(b) Verify the coordinates by substitution.

4. Graph $(x - 7)^2 + (y + 4)^2 = 5$ and $2x + y - 5 = 0$.

(a) What are the coordinates of the intersection points?

(b) Verify the coordinates by substitution.

5. Graph $x - y = 0$ and $(x - 1)^2 + 4(y - 1)^2 = 20$.

(a) What are the coordinates of the intersection points?

(b) Verify the coordinates by substitution.

6. **Knowledge and Understanding**

(a) Graph $\frac{x^2}{6} + \frac{y^2}{12} = 1$ and $2x + y = 2$.

(b) Determine the intersection points.

(c) Solve the system algebraically to verify your graph.

B

7. Solve each system algebraically.
- (a) $y = x + 2$ and $x^2 + y^2 = 100$
 - (b) $x^2 - y^2 = -4$ and $y = 5$
 - (c) $y = 2 - 3x$ and $x^2 = y + 2$
 - (d) $4x^2 + y^2 = 13$ and $2x - y = 1$
 - (e) $y = 2 - x$ and $x^2 + y^2 = 25$
8. **Application:** The path of a sonic boom on the Earth's surface is described by the equation $x^2 - y^2 = -45$. The path of a road is described by the equation $5x - 2y = 19$. What are the coordinates of points on the road at which the sonic boom will be heard?
9. Verify your results to this problem both graphically and algebraically. For the ellipse $4x^2 + 9y^2 = 36$, find the equation of a line that will
- (a) intersect the graph at two points
 - (b) intersect the graph at only one point
 - (c) not intersect the ellipse
10. Verify your results to this problem both graphically and algebraically. For the conic $2x^2 - 3y^2 + 18 = 0$, find the equation of a line that will
- (a) intersect the graph at two points
 - (b) intersect the graph at only one point
 - (c) not intersect the graph
11. Find the point(s) of intersection of $y = x + 1$ and $(x - 1)^2 + (y + 2)^2 = 9$.
12. Sketch each system and solve it algebraically.
- (a) $y^2 - 8x = -8$ and $y = 2x - 10$
 - (b) $x^2 - 4x - y^2 - 5 - 4y = 0$ and $x - 2y = 1$
13. Show that the intersection points of the circle $(x - 2)^2 + (y + 3)^2 = 25$ and the line $4x + 3y + 1 = 0$ are the end points of the diameter of the given circle.
14. Determine the equation of the line that has two intersection points with the hyperbola $4x^2 - 16y^2 = 64$, but only one intersection point with the ellipse $4x^2 + 16y^2 = 64$.
15. A motion-detector light, which is used at night, was installed above the middle of the school gym's floor. The light can detect motion within a range defined by the equation $x^2 + y^2 = 100$. If someone walks through the path defined by the equation $x - 2y - 22 = 0$, then will the person be detected by the motion detector? Justify your answer.



- 16. Check Your Understanding:** Draw diagrams to represent each scenario. Include the number of solutions and describe the algebraic solution to the problem.
- (a) a circle with a line
 - (b) a parabola with a line
 - (c) an ellipse with a line
 - (d) a hyperbola with a line

C

- 17.** Use a graph and algebra to show that the intersection points of $x - 2y - 4 = 0$ and $4x^2 + 16y^2 = 64$ lie on the conic defined by $x^2 + y^2 - 8x + 10y + 16 = 0$.
- 18. Thinking, Inquiry, Problem Solving:** Prove that the centre of the circle given by $x^2 + y^2 - 10x - 10y + 25 = 0$ lies on the perpendicular bisector of the circle's chord defined by the equation $y = 11 - 3x$.
- 19.** For the two conics $x^2 + 4y^2 = 49$ and $2x^2 - 3y^2 = -12$,
- (a) graph the conics to determine the number of possible solutions
 - (b) isolate x^2 in the first expression and substitute the resulting expression in the second equation to solve
 - (c) does your answer for (b) support your answer in (a)?
- 20.** Solve this system of equations using algebra and a graph.
- $$\begin{aligned}y - 2 &= -x^2 \\ x^2 + y^2 &= 25\end{aligned}$$

Did You Know?

It would be so much easier to reach outer space if you could just take an elevator instead of a spaceship. Think of the benefits—no expensive rocket fuel and more people could visit space more frequently. According to an article in *The National Post*, this dream might not be a fantasy.

Several writers and scientists, such as Yuri Artsutanov, Jerome Pearson, and Arthur C. Clarke, have shown that a “super elevator” is technically possible. It would have to be built on the equator, so its centre of gravity could stay in geostationary orbit around the Earth. This elevator would soar 36 000 km into the sky. The speedy elevator cars would be powered by electromagnetism, just as high-speed trains in Japan are now.

So far, no one has built such an elevator, because there is no material strong enough to withstand the great stress it would face. However, in 1991, Japanese scientist Sumio Iijima discovered carbon nanotubes. These nanotubes are about one hundred times stronger than steel and are strong enough to be used in the elevator. At the moment, carbon nanotube material costs about \$500 per gram, so the cost to make a space elevator is too great. But researchers hope to reduce the cost to \$1 per gram. And then—away we go!