

Chapter 7U Review

Investigating Loci and Conics

Check Your Understanding

1. Create a table that displays all of the standard forms of the equations of the conics. Include information about the foci, vertices, asymptotes, and so on.
2. The equation of any conic has the general form $ax^2 + by^2 + 2gx + 2fy + C = 0$. Create a table to show the restrictions on these values for each of the four conics. For example, for a circle, $a = b$.
3. Explain “asymptote” in your own words. Draw several graphs that have asymptotes.
4. Describe the procedure for finding the intersection of a line and a conic.
 - (a) What things do you need to consider?
 - (b) What different situations could arise?
 - (c) Draw diagrams to show the different cases.
5. In your own words, explain the purpose of a locus definition.
6. Draw diagrams to show the reflective properties of the conics.

Review and Practice

7.1U Introducing Locus Definitions

1. Name several ways that you can define a mathematical “object.”
2. State the locus definition of a circle.
3. Describe the shape defined by the locus of points that are equidistant from two fixed points.
4. Describe the shape defined by the locus of points that are equidistant from a fixed point and a fixed line.
5. Find the locus of the points that are equidistant from $(-6, 2)$ and $(4, 1)$.
6. Find the locus of points that are 2 units from the circle $x^2 + y^2 = 25$. Express this locus using geometry and as an equation, or equations.

7.2U–7.3U Revisiting the Circle

7. State the centre and radius of each circle.
 - (a) $4x^2 + 4y^2 = 1$
 - (b) $x^2 + y^2 + 16x - 6y + 72 = 0$
 - (c) $2x^2 + 2y^2 + 6x - 10y - 1 = 0$
8. Find the equation of the circle with centre $(0, 0)$ and that passes through $(8, -6)$.
9. Find the equation of the circle with centre $(2, 1)$ and that passes through $(5, 2)$.
10. Find the equation of the circle that is tangent to the x -axis and has centre $(-3, 4)$.
11. Graph $y = \sqrt{16 - x^2}$ and describe the graph.

7.4U The Ellipse

12. State the locus definition of an ellipse.
13. Give the standard form for an ellipse with
 - (a) its major axis along the x -axis
 - (b) its major axis along the y -axis
14. For each ellipse in question 13, state the length of both the major and minor axes.
15. State the relationship between a , b , and c .
16. State the x - and y -intercepts of $\frac{x^2}{25} + \frac{y^2}{64} = 1$.
17. Graph $x^2 + 4y^2 = 100$, labelling the vertices, the foci, and the major and minor axes.
18. Find the equation of the ellipse with centre $(0, 0)$, foci $(5, 0)$ and $(-5, 0)$, and y -intercepts ± 6 .
19. The ceiling in a hallway 10 m wide is in the shape of a semiellipse. The semiellipse is 9 m high in the centre. The walls of the hallway are 6 m high. Find the height of the ceiling 2 m from either wall.

7.5U–7.6U The Parabola

20. Draw a diagram showing a parabola, with its focus, its directrix, and its vertex.
21. State the locus definition of a parabola.

- 22.** State the equation of the parabola
- (a) with a vertical axis of symmetry and the vertex at the origin
 - (b) with a vertical axis of symmetry and the vertex (h, k)
 - (c) with a horizontal axis of symmetry and the vertex at the origin
 - (d) with a horizontal axis of symmetry and the vertex (h, k)
- 23.** A point on a parabola is 8 units from the focus. How far is this point from the directrix? Use a diagram to show your answer.
- 24.** Find the coordinates of the focus and the vertex and write the equation of the directrix for each parabola.
- (a) $y^2 = 12x$
 - (b) $(x - 3)^2 = 8(y + 2)$
- 25.** Find the equation of the parabola.
- (a) with vertex $(0, 0)$ and focus $(0, 2)$
 - (b) with vertex $(1, 2)$ and directrix $y = -2$

7.7U–7.9U The Hyperbola

- 26.** State the locus definition of a hyperbola. Include a diagram.
- 27.** State the standard form of a hyperbola with the transverse axis along
- (a) the x -axis
 - (b) the y -axis
- 28.** Draw a diagram of each hyperbola, clearly marking the transverse and the conjugate axes, the foci, the vertices, and the asymptotes.
- (a) centred at the origin and having x -intercepts
 - (b) centred at the origin and having y -intercepts
- 29.** Find the equation of the locus of the points such that the difference between the distance to $A(-14, 6)$ and the distance to $B(6, 6)$ is 6 units.
- 30.** Graph $16x^2 - 9y^2 = 144$, labelling the transverse and the conjugate axes, the foci, the vertices, and the asymptotes.
- 31.** Find the equation of the hyperbola with centre $(3, -5)$, vertex $(7, -5)$, and one focus at $(8, -5)$. Include a diagram with your solution.

7.10U The General Form of Conics

- 32.** Describe the steps for graphing a conic in general form.
- 33.** Describe how to determine the type of conic when the conic is in general form.

- 34.** For each of the following,
- identify the conic
 - write the equation in standard form
 - graph the conic
- (a) $4x^2 + 9y^2 + 8x - 36y + 4 = 0$ (b) $3x^2 + 3y^2 - 6x - 9 = 0$
(c) $3x^2 - 6y^2 - 2y + 4 = 0$ (d) $x^2 + 2y^2 - 12y + 2 = 0$
(e) $4x^2 - y^2 + 56x + 2y + 199 = 0$

7.11U When Lines Meet Conics

- 35.** Draw diagrams to show the different ways in which each of the four conics can intersect with a line.
- 36.** Discuss why graphing a system of equations is not always the best solution.
- 37.** Use both algebra and a graph to determine the intersection of $x - 2y = -8$ and $x^2 + 4y^2 = 32$.
- 38.** The centre of a circle is on the y -axis and is on the line $x - 5y - 15 = 0$. If the circle passes through $(4, 2)$, then find its equation.
- 39.** Solve $x^2 + 4y^2 = 40$ and $x + 2y = 8$.
- 40.** Find the intersection of $x^2 + y^2 = 5$ and $x + y - 1 = 0$.

Chapter 7U Summary

In this chapter, you studied locus definitions and conic sections.

You may define a geometric figure, a region, or an equation of a relation using a locus definition that sets specific conditions.

You also studied these conic sections: the circle, the ellipse, the parabola, and the hyperbola. You saw these conics in different ways: as locus definitions, standard and general forms of the equations, and graphs. You compared similar and different features of the conics. You also compared conics that were first centred at the origin and then translated.

The reflective properties of conic sections are very useful in automobile headlights, navigation systems, satellite dishes, and architectural structures. You explored these applications of conics.

You investigated the intersection of conics with lines and explored both graphical and geometric solutions.

Chapter 7U Review Test

Investigating Loci and Conics

- Identify the conic and state the coordinates of the vertices and foci.
 - $\frac{x^2}{16} + \frac{y^2}{9} = 1$
 - $x^2 - y^2 = 1$
 - $2x^2 + 2y^2 = 18$
 - $y^2 = 8(x - 3)$
 - $\frac{(x+2)^2}{25} + \frac{(y+1)^2}{9} = 1$
- Knowledge and Understanding:** Find the equation of each conic.
 - a circle with radius 3 units and centre $(0, -2)$
 - a hyperbola with vertices at $(-2, 0)$ and $(2, 0)$ and a conjugate axis 6 units long
 - a parabola with vertex $(0, 0)$ and directrix $x = -6$
- State the focus and directrix of $x^2 = -5y$.
- Communication:** Describe how you can determine whether the transverse axis of a hyperbola is horizontal or vertical.
- For the conic $25x^2 - 9y^2 = -225$,
 - state the vertices
 - find the asymptotes
 - graph the relation
- For the conic $\frac{x^2}{25} + \frac{y^2}{64} = 1$,
 - name the x - and y -intercepts
 - state the coordinates of the foci
 - state the length of the major axis
 - name the vertices
 - graph the conic
- Graph each conic and then state the vertices and find the foci.
 - $25x^2 + 9y^2 = 900$
 - $4x^2 - y^2 = 100$
 - $\frac{x^2}{4} + \frac{y^2}{12} = 1$
 - $y^2 - 4y + 4x = 0$
 - $\frac{x^2}{25} - \frac{y^2}{144} = 1$
- Find the equation for the ellipse, centred at $(0, 0)$, with foci on the x -axis. The length of the major axis is four times the length of the minor axis, and the point $(-3, 2)$ is on the ellipse.
- Application:** A conical act came to town with the circus. The Bolic Twins performed the great feat of walking across an untight cable that happened to be so untight that it fell into a parabolic curve. (Assume the cable will stay in the parabolic shape, even as they walk across it.) The cable was attached to two 30 m high poles placed 20 m apart. The lowest point of the cable was 5 m above the ground. After the Bolic twins had taken two steps on the cable, they became quite nervous and reached out for the pole. They could just reach it. If the arms of each twin are 1 m long, then how high from the ground are they?

10. Determine the type of conic.

(a) $x^2 + y^2 - 6x + 8y - 9 = 0$

(b) $4x^2 - 8y^2 - 16y - 40 = 0$

(c) $x^2 + 9y^2 + 2x = 0$

(d) $y^2 + 4x - 12y + 4 = 0$

(e) $2x^2 - 3y^2 - 12x - 12y - 6 = 0$

11. Obtain the equation of the parabola with the vertex at the origin and the focus on the x -axis, that opens to the right, and with a focus on the line $4x - 7y - 12 = 0$.

12. Find the equation of the diameter of the circle $x^2 + y^2 - 4x + 6y - 1 = 0$ that passes through point $(0, -2)$.

13. Thinking, Inquiry, Problem Solving

The base of a triangle lies on the x -axis with one vertex at the origin. The length of the base is 12 units. The product of the tangents to the base angles is $\frac{9}{16}$. Let $P(x, y)$ be the vertex of the triangle that is not on the x -axis. Find the equation for the locus of this vertex of the triangle.

14. Find the intersection of $5x - y - 20 = 0$ and $y^2 = 50x$. Use both a graphical solution and an algebraic solution.