

Cumulative Review Test 4

Course Review: Chapters 1 to 7

1. For each sequence, find
 - i. the general term
 - ii. t_{25}
 - iii. S_{20}
 - (a) $-4, 5, 14, \dots$
 - (b) $3, 3.3, 3.63, 3.993, \dots$
2. Determine the number of terms in each sequence.
 - (a) $3, 12, 48, \dots, 201\ 326\ 592$
 - (b) $18, 22, 26, \dots, 162$
3. George has retired from teaching and will receive his first pension cheque next month. His pension will pay him \$55 000 annually, plus an additional percentage to cover inflation. If this additional percentage averages 2.5%/a, determine the total amount of money he will receive over the next fifteen years.
4. Determine
 - (a) the amount \$8500 will grow to if invested at 8%/a compounded quarterly for 10 years
 - (b) the principal that must be invested now at 6%/a compounded annually to be worth \$10 000 in 5 years
 - (c) the total accumulated amount of \$500 invested every 6 months, starting 6 months from now, at 7%/a compounded semiannually for 8 years.
 - (d) the principal that must be invested now to provide monthly payments of \$100, beginning next month, over a 10-year period, if interest is paid at 4.8%/a compounded monthly
5. Judy bought a stereo system for \$300 down and 24 monthly payments of \$120, beginning next month.
 - (a) If the store charges 18%/a compounded monthly, what was the cash price of the system?
 - (b) What was her cost for financing the stereo system?
6. Clarrisa has just graduated from medical school. She must repay her student loans that total \$23 000. She decides that she can afford to make monthly payments of \$420. The bank is charging 5.4%/a compounded monthly in interest charges. Determine how long it will take her to repay her student loan.
7. The Morrison family is about to buy a new home for \$245 000. They intend to make a 25% down payment to qualify for a conventional mortgage and finance the remainder over 25 years. They negotiate a fixed five-year term mortgage rate of 7.9%/a compounded semiannually.
 - (a) Determine the monthly payment required to repay the loan.
 - (b) Determine the outstanding balance on the mortgage at the end of each of the first 5 years.
 - (c) At the end of the five-year term, determine how much principal has been repaid and how much interest has been paid.
 - (d) Assume interest rates do not change. Find the total interest the Morrisons can expect to pay over 25 years and the total cost of their home.

8. Simplify.

$$(a) \frac{16^{\frac{3}{4}} - 27^{-\frac{2}{3}}}{4^{-\frac{1}{2}}} \quad (b) \left(\frac{27x^2y^{-5}}{64x^{-1}y^4} \right)^{\frac{2}{3}}$$

$$(c) \left(\frac{4}{x} \right)^{n-m} (2x^2)^m$$

9. Solve each equation. Round answers to two decimal places where necessary.

$$(a) 2x^2 - 4x + 5 = 0$$

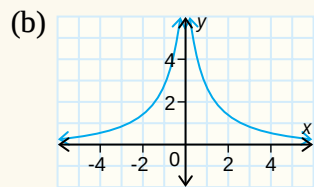
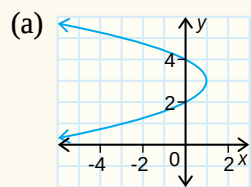
$$(b) \frac{90\,000}{x} = \frac{90\,000}{x+6} + 2500$$

$$(c) 4^{x-2} = \frac{1}{64}$$

$$(d) 2^{x^2+20} = \left(\frac{1}{8} \right)^{3x}$$

10. For each relation,

- identify the domain and range
- tell whether it is a function or not. Justify your answer.



$$(c) y = \sqrt{x-5}$$

$$(d) y = 2(x-3)^2 + 4$$

11. For $g(x) = -3x^2 + 6x + 2$

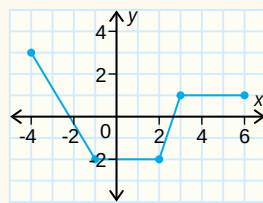
(a) graph g and g^{-1} on the same set of axes

(b) determine the equation for g^{-1}

(c) state restrictions on the domain or range of g so that its inverse is a function

(d) assume the domain of g is $\{x \mid 0 \leq x \leq 5, x \in \mathbb{R}\}$. Would the inverse be a function? Justify your answer.

12. The graph of $f(x)$ is shown, and $g(x) = -2f(2x+6) + 3$. Sketch the graph of $g(x)$.



13. Solve each inequality and graph the solution set, $x \in \mathbb{R}$.

$$(a) 4(x-3) + x \leq 6 - 2(x+2)$$

$$(b) \frac{x}{3} - \frac{2x-3}{4} > \frac{5}{6}$$

14. (a) What transformations must you apply to the graph of $y = x^2$ to obtain the graph $y = \frac{2}{3}x^2 - \frac{3}{4}x + 1$? Show all your work and explain each step.

(b) Graph $y = x^2$ and $y = \frac{2}{3}x^2 - \frac{3}{4}x + 1$ on the same set of axes.

15. Find the value(s) of k so that the graph of each function has

- two x -intercepts
- one x -intercept
- no x -intercepts

$$(a) f(x) = kx^2 - 9x + 6$$

$$(b) f(x) = 3x^2 + kx + 12$$

$$(c) f(x) = 9x^2 + 3x + k$$

- 16.** In each case, draw the graph of $y = f(x)$ and use it to graph $y = \frac{1}{f(x)}$ on the same axes.

(a) $f(x) = x^2 + 2$
 (b) $f(x) = (x + 4)^2 - 2$
 (c) $f(x) = \sqrt{x + 5}$

- 17.** For each function

- i. state the zeros, asymptotes, and domain
 ii. graph the function and estimate the range

(a) $f(x) = \frac{x}{x + 1}$
 (b) $g(x) = \frac{1}{x^2 - x - 6}$
 (c) $h(x) = \frac{x^2 - 4}{x}$

- 18.** Simplify.

(a) $2x(3x - 1)(3x + 1) - (4x + 2)^3$
 (b) $(6x - 2y)(x^2 + 4xy - 3y^2)$

- 19.** Simplify. State any restrictions.

(a) $\frac{5x^2 - 5}{x^2 - 4x - 5}$
 (b) $\frac{4x^4y}{3x^2y^4} \times \frac{-6x^3y^2}{10x^4}$
 (c) $\frac{2m^2 - m - 15}{m + 2} \times \frac{m^2 - m - 6}{m^2 - m + 9}$
 (d) $\frac{x^2 - y^2}{2x^2 - 8x} \times \frac{(x - y)^2}{2xy}$
 (e) $\frac{1}{x} - \frac{3}{y} + \frac{3x - y}{xy}$
 (f) $\frac{x + 2}{x^2 + 5x + 6} - \frac{x - 3}{x^2 - 3x - 10}$

- 20.** Point $P(-6, -8)$ lies on the terminal arm of an angle, θ , in standard position. Determine the exact value of $\frac{\sin \theta + \cos \theta}{\tan \theta}$.

- 21.** Express each angle in radians.

(a) 35° (b) 320°
 (c) -180° (d) 225°

- 22.** Express each angle in degrees.

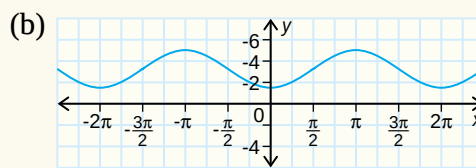
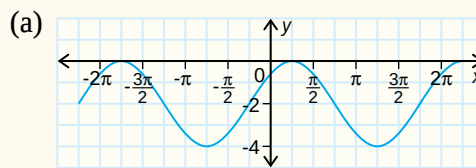
(a) $\frac{7\pi}{8}$ (b) $-\frac{\pi}{12}$
 (c) $\frac{8\pi}{3}$ (d) $\frac{5\pi}{16}$

- 23.** Verify that $\sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{6} = 1$.

- 24.** Sketch the graph of each function.

(a) $f(x) = 3 \sin x - 2, 0 \leq x \leq 2\pi$
 (b) $f(x) = -2 \cos(x - 30^\circ) + 2, 0^\circ \leq x \leq 360^\circ$

- 25.** Determine an equation that models each function.



- 26.** During high tide, at 12:06 a.m., the water in an inlet is 18 m deep. During low tide, which occurs 12 h later, it is 11.5 m deep.

- (a) Determine an expression for the water depth in the inlet, t hours after high tide.
 (b) Graph the function over a 24-h period.
 (c) Determine the times on this day when the water is 15 m deep and 12 m deep.

- 27.** Solve. Round to the nearest tenth where necessary.

(a) $2 \sin x = \sqrt{3}$, $0 \leq x \leq 2\pi$
 (b) $5 \cos x - 4 = 0$, $0^\circ \leq x \leq 360^\circ$
 (c) $\sin 2x(2 \cos x + 1) = 0$, $0 \leq x \leq 2\pi$
 (d) $3 \sin^2 x - 11 \sin x - 4 = 0$,
 $0^\circ \leq x \leq 360^\circ$

- 28.** Prove each identity.

(a) $\frac{2}{\sin^2 \theta} = \frac{1}{1 - \cos \theta} + \frac{1}{1 + \cos \theta}$
 (b) $\tan^2 \theta + \cos^2 \theta + \sin^2 \theta = \frac{1}{\cos^2 \theta}$

- 29.** Two bike trails diverge at an angle of 102° . Two riders leave the intersection at the same time, with one rider heading northwest at 18 km/h and the other going northeast at 20 km/h. How far apart are they after 1.5 h?

- 30.** Solve $\triangle LMN$, given $\angle L = 38^\circ$, $l = 30$ cm, and $n = 45$ cm.

- 31.** A forest ranger at the top of a 95-m high observation tower sees a tent at an angle of depression of 12° . Turning in a different direction, the ranger sees a second tent at an angle of depression of 17° . The angle between these two lines of sight is 40° . How far apart are the campsites?

Functions and Relations

- 32U.** Determine the first six terms of the sequence in which $t_1 = 5$, $t_2 = 3$, and $t_n = 4t_{n-1} + 2t_{n-2}$.

- 33U.** Evaluate.

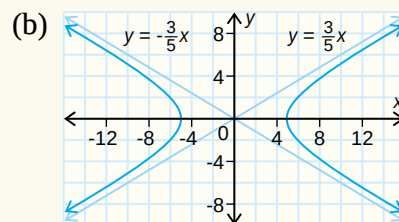
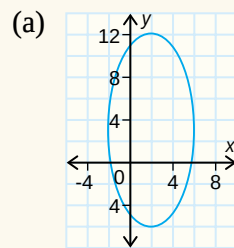
(a) $(i + 3)^2$
 (b) $3i^4$
 (c) $(4 - 5i)(3 + 2i)$
 (d) $(5 - 7i) + (6 - 4i) - (3 - 2i)$
 (e) $\frac{5 + 2i}{4 + 6i}$
 (f) $[2i + (1 - i)]^2$
 (g) $\frac{(3 + 6i)}{(2 - 3i)(3 + 2i)}$
 (h) $\frac{(5 + 8i) + (-3 + 5i)}{(6 + 7i) - (5 - 4i)}$

- 34U.** Sketch the graph of $x^2 - 6x + 4y + y^2 - 9 = 0$.

- 35U.** Graph each conic. Then state the coordinates of the vertices and the foci.

(a) $9x^2 + 16y^2 = 144$
 (b) $x^2 - 9y^2 = 9$
 (c) $y^2 - 8y + 2x = 0$
 (d) $4x^2 - y^2 = -16$

- 36U.** Determine the equation of each conic.



37U. For each equation

i. identify the type of conic that it represents

ii. graph the equation

(a) $9x^2 + 4y^2 + 18x - 16y = 11$

(b) $x^2 - 12x - 4y = 0$

(c) $4x^2 - y^2 - 16x - 6y + 3 = 0$

(d) $x^2 - 10x + y^2 - 10y = -49$

38U. Sketch the locus and determine the equation of each situation.

(a) The product of the coordinates of any point is 8.

(b) The set of points that are equidistant from $(0, 4)$ and $(2, 6)$

39U. Determine the intersection points of $y = 2x - 4$ and $x^2 - 6x + y^2 + 8y + 16 = 0$.