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- A sequence is an ordered set of numbers. A term is a single element of a sequence. For example, $\{1, 3, 5, 7, \dots\}$ is a sequence, and 5 is the third term.
- The general term is an expression or formula for all the terms in a sequence. It is a function of a natural number, and using numbers in this formula generates the terms of this sequence.
- (a) Example, $\{1, 2, 3, \dots, 100\}$ (b) Example, $\{1, 4, 9, 16, \dots\}$
- (a) 5, 9, 13, 17, 21 (b) 0, 4, 12, 24, 40
(c) 1, 3, 9, 27, 81 (d) -1, -4, -7, -10, -13
(e) 3, 15, 35, 63, 99 (f) 10, 5, $\frac{10}{3}, \frac{5}{2}, 2$
- (a) $t_n = 5n - 3$ (b) $t_n = -6n + 36$ (c) $t_n = 3^{n+1}$
(d) $t_n = \frac{5}{2^n}$ (e) $t_n = n^2 + 1$ (f) $t_n = \frac{n+1}{n+2}$
- (a) \$18 000, \$13 500, \$10 125, \$7593.75
(b) $t_n = \$24\,000\left(\frac{3}{4}\right)^n$ (c) \$3203.61
- A recursive formula defines each term as a function of one or more previous terms, whereas a general term defines each term as a function of a natural number.
- A recursive formula defines each term as a function of previous terms, and initial conditions.
- (a) 7, -3, -13, -23, -33 (b) -3, -9, -39, -189, -939
(c) -4, -8, -16, -32, -64
(d) 3, 34, 4622, 8.545×10^7 , 2.9208×10^{16}
- (a) $t_1 = 2, t_n = t_{n-1} + 3$ (b) $t_1 = 18, t_n = t_{n-1} - 7$
(c) $t_1 = 3, t_n = (t_{n-1})^2$
(d) $t_1 = 2, t_2 = 4, t_n = t_{n-1} + t_{n-2}$
(e) $t_1 = 1, t_n = \frac{2}{3}t_{n-1}$ (f) $t_1 = 4, t_n = (1 + 2^{n-3})t_{n-1}$
- (a) 6, 9, 12, 15, 18 (b) -4, -12, -36, -108, -324
(c) 8, 4, 2, 1, 0.5 (d) 2, 8, 2, 8, 2
- (a) 333.3 mg (b) About 72 h
- If the first differences between the terms of a sequence are constant, then the sequence is arithmetic.
- (a) $t_n = a + (n-1)d$
(b) t_n - term, a - first term, n - term index, d - common difference
- (a) 4, 7, 10, 13, 16 (b) 10, 8, 6, 4, 2
- (a) $d = 6, t_n = 6n, t_{15} = 90$
(b) $d = 15, t_n = 15n - 55, t_{15} = 170$
(c) $d = -5, t_n = -5n + 41, t_{15} = -34$
(d) $d = \frac{1}{4}, t_n = \frac{1}{4}n, t_{15} = 3\frac{3}{4}$
(e) $d = 0.7, t_n = 0.7n + 0.5, t_{15} = 11$
(f) $d = 32, t_n = 32n - 28, t_{15} = 452$
- (a) dots in straight line through (0, 5) and (5, 0)
(b) dots in straight line through (0, -4) and $\left(\frac{4}{7}, 0\right)$
(c) dots in straight line through (0, 7) and $\left(\frac{7}{3}, 0\right)$
(d) dots in straight line through (0, 2) and (-4, 0)
- (a) \$162.50
(b) \$5000, \$5162.50, \$5325, \$5487.50, \$5650
(c) $t_n = \$162.50n + \$4837.50$
(e) 7 a
- (a) $a = 12, d = 3, t_n = 3n + 9$
(b) $a = -11, d = -6, t_n = -6n - 5$
(c) $a = 1, d = 9, t_n = 9n - 8$
(d) $a = 7, d = -15, t_n = -15n + 22$
- If the ratio between any two successive terms is constant, then the sequence is geometric.
- Answers will vary.
- (a) $t_n = ar^{n-1}$
(b) t_n - term, a - first term, r - common ratio, n - term index
- (a) $r = 7, t_n = 2(7)^{n-1}, t_{12} = 3\,954\,653\,486$
(b) $r = 4, t_n = -3(4)^{n-1}, t_{12} = -12\,582\,912$
(c) $r = \frac{1}{2}, t_n = 160\left(\frac{1}{2}\right)^{n-1}, t_{12} = 0.078\,125$
(d) $r = -3, t_n = 7(-3)^{n-1}, t_{12} = -1\,240\,029$
(e) $r = 0.2, t_n = 0.4(0.2)^{n-1}, t_{12} = 8.19 \times 10^{-9}$
(f) $r = \frac{1}{3}, t_n = \frac{1}{3^{n-9}}, t_{12} = \frac{1}{27}$
- (a) \$149 500, \$127 075, \$108 013.75, \$91 811.69, \$78 039.93, \$66 333.94
(b) $t_n = \$149\,500(0.85)^{n-1}$ (c) \$47 926.27
- In simple interest, only the principal amount earns interest, whereas in compound interest, in addition to the principal amount, the interest earns interest as well.
- (a) $A = P(1 + i)^n$
(b) A - the amount of the investment, P - the principal amount invested, i - interest rate, n - number of compounding periods
- Solve for P in $A = P(1 + i)^n$
- (a) $A = \$7518.15, I = \$2518.15$
(b) $A = \$12\,717.67, I = \$2217.67$
- (a) $P = \$13\,367.01, I = \$8132.99$
(b) $P = \$78\,119.84, I = \$21\,880.16$
- \$16 783.48
- \$15 745.09
- (a) $x^{\frac{1}{n}} = \sqrt[n]{x}$ (b) $x^{\frac{m}{n}} = \sqrt[n]{x^m}$
(c) $x^{-\frac{m}{n}} = \frac{1}{(\sqrt[n]{x})^m} = \frac{1}{\sqrt[n]{x^m}}$
- (a) 3 (b) -3 (c) -2 (d) 2 (e) 2
(f) $\frac{1}{16}$ (g) $\frac{1}{3}$ (h) $\frac{1}{16}$ (i) $\frac{8}{27}$ (j) $\frac{1}{16}$
- (a) 5.10 (b) -2.71 (c) -3.06 (d) 3.27 (e) 2.89
(f) 0.91 (g) 0.01 (h) 0.01 (i) 1.57 (j) 0.27
- (a) \$5207.48 (b) \$6470.94
- (a) \$1466.03 (b) \$77 702.94
- (a) $t_n = \$1200(1.05)^n$ (b) \$1954.67
(c) \$1344.69
- multiplication: $a^m \times a^n = a^{m+n}$; division: $a^m \div a^n = a^{m-n}$;
power of a power: $(a^m)^n = a^{mn}$; power of a product: $(xy)^m = x^m y^m$;
power of a quotient: $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$; zero as an exponent: $a^0 = 1$;
negative exponents: $a^{-m} = \frac{1}{a^m}$; rational exponents: $x^{\frac{m}{n}} = \sqrt[n]{x^m}$
 $= (\sqrt[n]{x})^m$
- (a) 13 (b) -7 (c) $-\frac{19}{216}$
- (a) $-\frac{2}{3}$ (b) $\frac{1}{19\,683}$ (c) $\frac{1}{243}$ (d) $-\frac{4}{45}$
- (a) $\frac{4b^2}{9a^4}$ (b) $\frac{16x^4z^8}{y^6}$ (c) $\frac{625x^7}{y^3}$
- (a) a^{p+2} (b) $(2^3 - 2^m)x^{(6-6m)}$ (c) 1 (d) x^{n-4m}
- Example: $21 = 3^x$
- when both sides of an exponential equation are expressible as a power of common base
- (1) Consider an exponential equation. (2) Graph $y = L.S.$ and $y = R.S.$ (3) Carefully determine the intersection point.
- (a) 2 (b) 2 (c) -1 (d) $\frac{7}{4}$
- (a) 2.43 (b) 1.39 (c) 5.03 (d) 8.31
- (a) -3, -4 (b) -1 (c) $\frac{3}{2}, 1$ (d) $\frac{\pm\sqrt{17}-3}{2}$
- (a) 8 (b) 10
- 13.87 a