

Practise, Apply, Solve 3.6, page 273

- (5, 0), (3, -1), (2, -2), (2, -3), (3, -4), (5, -5)
 - Example: $\{(2, -3), (3, -4), (5, -5)\}$
- connected points: (1, 6), (-1, 4), (-1, 1), (5, -2), (5, -5)
 - $D = \{x \mid -5 \leq x \leq 6, x \in \mathbf{R}\}$, $R = \{y \mid -1 \leq y \leq 5, y \in \mathbf{R}\}$
 - $D = \{x \mid -1 \leq x \leq 5, x \in \mathbf{R}\}$, $R = \{y \mid -5 \leq y \leq 6, y \in \mathbf{R}\}$
 - when domain of g is $-2 \leq x \leq 1$, g^{-1} is a function and has same domain as g^{-1}
- no restrictions; parabola opening to left with vertex (1, -2) through (0, -1) and (0, -3); $D = x \geq -2$ or $x \leq -2$
 - no restrictions; parabola opening to right with vertex (0, 3) through (2, 4) and (2, 2); $R = x \geq 3$ or $x \leq 3$
 - $R = y \geq 3$ or $y \leq 3$; parabola opening upward with vertex (3, 0) through (2, 2) and (4, 2); no restrictions
 - no restrictions; left half of downward opening parabola with vertex (2, 1) and passing through (0, -3); no restrictions
 - $R = y \geq 4$ or $y \leq 4$; parabola opening downward with vertex (4, 1) through (2, -3) and (6, -3); no restrictions
 - no restrictions; parabola opening to left with vertex (4, 3) through (-4, 1) and (-4, 5); $D = x \geq 3$ or $x \leq 3$
- $f(x) = -(x - 4)^2 + 1$; parabola opening to left with vertex (1, 4) through (0, 5) and (0, 3); when domain of relation is $x \geq 4$ or $x \leq 4$, inverse is a function;
 f^{-1} is $y = \pm\sqrt{-x+1} + 4$
 - $f(x) = (x - 1)^2 - 3$; parabola opening to right with vertex (-3, 1) through (-2, 2) and (-2, 0); when domain of relation is $x \geq 1$ or $x \leq 1$, inverse is a function;
 f^{-1} is $y = \pm\sqrt{x+3} + 1$
 - $f(x) = (x + 3)^2 - 2$, $x \leq -3$; bottom half of parabola opening to right with vertex (-2, -3) passing through (2, -5); no restrictions; f^{-1} is $y = -\sqrt{x+2} - 3$
 - $f(x) = 3 + \sqrt{-x+2}$; right half of parabola opening downward with vertex (3, 2) and passing through (4, 1); no restrictions; f^{-1} is $y = -(x - 3)^2 + 2$, $x \geq 3$
- f : parabola opening downward with vertex (-1, 3) through (-3, -5) and (1, -5)
 - f^{-1} : parabola opening to left with vertex (3, -1) through (-5, 1) and (-5, -3)
 - $D = x \geq -1$ or $x \leq -1$
- g ; bottom half of parabola opening to right with vertex (0, 0) through (1, -1)
 - g^{-1} : left half of parabola opening upward with vertex (0, 0) through (-1, 1)
 - $D = \{x \mid x \leq 0, x \in \mathbf{R}\}$, $R = \{y \mid y \geq 0, y \in \mathbf{R}\}$
 - $g^{-1}(x) = x^2$, $x \leq 0$
- g : top half of parabola opening to right with vertex (0, 3) through (4, 5)
 - g^{-1} : parabola opening up, vertex (3, 0) through (0, 9) and (6, 9)
 - $g^{-1}(x) = (x - 3)^2$, $x \geq 3$
 - $T_{\min} = 3$ for correct graph
- f : top half of parabola opening right, vertex (2, 0) and through (6, 2)
 - f^{-1} : parabola opening up, vertex (0, 2) through (2, 6) and (-2, 6)
 - $f^{-1}(x) = x^2 + 2$, $x \geq 0$
 - $T_{\min} = 0$ for correct graph
- right half of parabola opening downward with vertex (1, 7) through (2, 5), f^{-1} : top half of parabola opening to left with vertex (7, 1) through (-1, 3)
- 1
 - $f^{-1}(x) = 1 + \sqrt{\frac{7-x}{2}}$
 - 2
 - $1 + \sqrt{-a}$
 - x
- (3, 14)
 - $D = x \geq 3$
 - $f^{-1}(x) = 3 + \sqrt{-x+14}$
- $D = \{x \mid -2 < x < 3, x \in \mathbf{R}\}$, $R = \{y \mid -3 \leq y < 24, y \in \mathbf{R}\}$
 - f^{-1} is $y = 1 \pm \sqrt{\frac{x+3}{3}}$, $-2 < y < 3$
- bottom half of parabola opening right, vertex (0, 5) through (25, 0)
 - $D = \{x \mid x \geq 0, x \in \mathbf{R}\}$, $R = \{y \mid y \leq 5, y \in \mathbf{R}\}$
 - $f^{-1}(x) = (x - 5)^2$, $x \leq 5$
- $y = \sqrt{x}$ is one branch of the inverse of $y = x^2$. The inverse of $y = \sqrt{x}$ is the right-hand branch of $y = x^2$ (i.e. $y = x^2$, $x \geq 0$.)
- first differences: 12.375, 10.125, 7.875, 5.625, 3.375, 1.125, -1.125, -3.375, -5.625, -7.875, -10.125; second differences: all -2.25
 - $y = -4x^2 + 27x$, parabola opening down, vertex (3, 40.5)
 - parabola opening left, vertex (40.5, 3) through (0, 0) and (0, 6)
 - not a function: fails vertical line test
- $y = -\sqrt{x+2}$ defines the lower branch of a parabola that opens to the right. The inverse of this function is a single branch of the parabola opening up. The graph of $y = x^2 - 2$ is a parabola (both branches) with vertex at (0, -2), so it cannot be the inverse of $y = -\sqrt{x+2}$.
 - $Y = \sqrt{x+2}$ is the upper branch of a parabola opening to the right with vertex at (-2, 0). Its inverse is a function already, without any restriction.
- $f(x) = x(x - 2)$, $0 \leq x \leq 9$, $x \in \mathbf{W}$
 - f^{-1} is $y = 1 \pm \sqrt{x+1}$, $0 \leq y \leq 9$, $y \in \mathbf{W}$
 - no: fails vertical line test
 - $R = \{0, 3, 8, 15\}$
 - 3, 4, or 5
 - For f^{-1} to be a function, range of $f^{-1} = 1 \leq y \leq 9$, $y \in \mathbf{W}$;
Then each input value corresponds to a unique output value.
- Reflect the graph in $y = x$. If no two points of the function lie on the same horizontal line, then its inverse is a function; if not, then restrict its domain so that no two points are on a horizontal line.
- $P(x) = (x - 3.21)(14\,700 - 3040x)$
 - P^{-1} is $y = 4.02 \pm \sqrt{\frac{-x+2008}{3040}}$
 - \$4.21/kg or \$3.83/kg
 - \$4.02/kg
 - \$2290
 - Example: (supply cost in \$/kg, profit): (3.21, \$2008), (2.50, \$4146), (4.00, \$531), (4.80, \$0.94)
 - Example: increased supply could mean lower prices, resulting in greater sales; decreased supply could mean higher prices, resulting in fewer sales.
- $R = \{y \mid y > \frac{1}{3} \text{ or } y < -0.5, y \in \mathbf{R}\}$