

1.3U Sequences and Recursive Formulas

The expectations for the Functions course do not include recursive formulas, so this section is only for Functions and Relations students. However, this section could be a source of enrichment for selected Functions students.

Specific Expectations

Unless stated otherwise, the following expectations are common to the Functions and Relations course and the Functions course.

- FA1.01(U) write terms of a sequence, given the formula for the n th term or given a recursion formula
- FA1.02(U/C) determine a formula for the n th term of a given sequence (e.g., the n th term of the sequence $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \dots$ is $\frac{n}{n+1}$)
- OC3.01 explain mathematical processes, methods of solution, and concepts clearly to others
- OC3.02 present problems and their solutions to a group, and answer questions about the problems and the solutions
- OC3.03 communicate solutions to problems and to findings of investigations clearly and concisely, orally and in writing, using an effective integration of essay and mathematical forms
- OC3.04 demonstrate the correct use of mathematical language, symbols, visuals (e.g., diagrams, graphs), and conventions

Evidence of Learning

Students should be able to

- generate terms of a sequence given the initial term and the recursive general term
- create a recursive expression to describe a sequence
- solve problems using recursive descriptions of relations, for example, problems about investment and population growth

Text Page 27

Time Required for Functions and Relations: 75 min

Materials and Equipment

- spreadsheet software or graphing calculator
- bulletin board displays of familiar Fibonacci sequences
- current information on depreciated values of popular automobiles

Overview

- This section introduces the recursive sequence in which each term is described by referring to a previous term or terms. Many students will intuitively recognize this recursive pattern. Encourage students to identify terms using subscripts and to describe the same sequence in different ways, where possible.

Management Options

Partners or Individuals – Let students work through the *Think, Do, Discuss* questions while you circulate to answer questions.

Teacher-Led Discussion – Use an overhead to present the *Think, Do, Discuss* questions about depreciation. Highlight the answers in a different colour. Allow students time to respond to each question before showing the answer.

Part 1: Depreciation

Think, Do, Discuss, text page 27

1. *Determine the dollar amount of depreciation for the first year.*

$$0.10(\$26\ 000) = \$2600$$

2. Determine the value of the car at the end of the first year.

$$\$26\,000 - \$2600 = \$23\,400$$

3. What percent of the original purchase price does the value in step 2 represent?

The value of the car at the end of the first year is 90% of the original price.

4. Determine the value of the car at the end of the second year.

Recursive expression: The value after two years is

$$r_2 = 0.90(r_1)$$

$$r_2 = 0.90(\$23\,400)$$

$$= \$21\,060$$

General term: The resale value after two years is

$$r_n = (0.90)^n (\$26\,000)$$

$$r_2 = (0.90)^2 (\$26\,000) = \$21\,060$$

5. Write the first five terms of the sequence that represents the value of the car at the end of each year for the first five years.

$$r_1 = \$23\,400$$

$$r_2 = \$21\,060$$

$$r_3 = \$18\,954$$

$$r_4 = \$17\,058.60$$

$$r_5 = \$15\,352.74$$

So the sequence is 23 400, 21 060, 18 954, 17 058.60, 15 352.74.

6. Let t_n represent any term of this sequence. How could you represent the term before t_n ?

$$t_n = 0.9t_{n-1}$$

7. For this sequence, write a formula that uses the previous term to calculate any term, except the first term.

$$t_n = 0.9t_{n-1}$$

8. At the end of which year will the car's value be about one-half of its original purchase price?

Students can generate the sequence using a calculator until the value of the car is about \$13 000 and track the number of terms. Or, they can use a general term and solve the resulting equation.

$$(0.90)^n (26\,000) = 13\,000$$

$$(0.90)^n = 0.5$$

$$n \log(0.9) = \log(0.5)$$

$$n = \frac{\log 0.5}{\log 0.9}$$

$$= 6.6$$

The resale value will be half the original value a few months before the end of the seventh year.

Part 2: The Fibonacci Sequence

Think, Do, Discuss, page 28

1. Starting with the first generation, write the sequence of the number of honeybees in each generation.

$$1, 1, 2, 3, 5, 8$$

2. Identify the pattern in this sequence.

Each term is the sum of the two preceding terms.

3. Use this pattern to determine the next five terms of the sequence.

$$13, 21, 34, 55, 89$$

4. Let t_n represent any term in this sequence. How could you represent the term before t_n ? How could you represent the term before the term that is before t_n ?

$$t_{n-1}, t_{n-2}$$

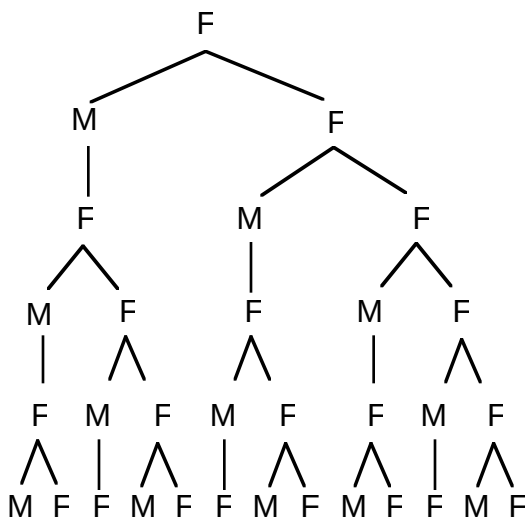
5. Given that $t_1 = 1$, $t_2 = 1$, and n is a natural number, determine a formula for the general term of this sequence.

$$t_n = t_{n-2} + t_{n-1}$$

6. This sequence is known as the **Fibonacci sequence**, and it occurs in many different situations in nature. Could you easily calculate the 50th term of this sequence? Explain.

The calculation would be tedious, requiring the calculation of 49 preceding terms to find the 50th term.

7. Create a tree diagram that represents six generations of a female honeybee. Write the sequence that represents the number of honeybees in each generation.



1, 2, 3, 5, 8, 13

8. Does this sequence have the same pattern as the sequence in step 1? Is this sequence a Fibonacci sequence?

The pattern is the same but the values of the first terms of the sequence are different. Yes, this is also a Fibonacci sequence.

9. Determine a formula for this sequence.

$$t_n = t_{n-2} + t_{n-1}, t_1 = 1, t_2 = 2$$

10. Use your formula to find the next five terms of this sequence.

21, 34, 55, 89, 144

11. For 11 generations of honeybees, who has more ancestors, a male honeybee or a female honeybee?

neither

EXTENSION SUGGESTION

Ask students to find some information about examples of Fibonacci sequences in nature or in other applications.

ASSESSMENT SUGGESTION

Ask students to explain the difference between using a recursive formula or expression and using the position number when creating a general term of a sequence.

Additional Example

The recursive formula for a sequence is

$$t_{n+1} = (t_n)^2 - 1.$$

- (a) Find the first five elements if $t_n = 2$.
 (b) What would change if the value of the initial term were decreased by 1?
 (c) Find the first five elements if $t_n = 1$.

Solution

(a) 2, 3, 8, 63, 3968

(b) Answers will vary.

(c) 1, 0, -1, 0, -1

Note the importance of knowing the value of one term when using a recursive formula.

Practise, Apply, Solve 1.3U

Questions with a Functions and Relations Focus:

1-5, 7, 9, 10, 12, 13, 15, 20, 21, 24

Questions with a Functions Focus:

N/A.

7. Knowledge and Understanding, p. 33

Management Suggestion

Challenge capable students by insisting that the work be done mentally or by using technology to write a short program.

Achievement Chart Connection

Students are required to demonstrate understanding of recursion and to perform an algorithm.

Level 3 Performance

This question requires simple interpretation of the given recursive formula and substituting specific values to calculate the required term.

12. Communication, p. 34

Management Suggestion

Turn the question into a game. Divide the class into five teams. Each team should describe four different sequences with a recursive formula for each sequence. When their team is selected (using a spinner or draw), a team member hands the judge a list of the first five terms, while another team member shows the recursive formula on the overhead or board. Other teams can try to list the sequence within a specific time and would gain a point if the list were perfectly correct. The judge can be scoring the efforts of one team while a second judge monitors another team.

Achievement Chart Connection

Students are expected to use symbols and conventions to clearly describe a sequence with a recursive formula.

Level 3 Performance

Students must use accepted notation including subscripts to identify terms in a sequence and to show clearly the calculations for determining specific terms of the sequence.

ASSESSMENT SUGGESTION

Observe students as they work among themselves. Do they cooperate or do they work individually?

15. Application, p. 34

Management Suggestion

Encourage students to record unsuccessful approaches to the problem as well as successful ones. Collect their work and make comments or suggestions, but this assessment is formative rather than summative.

Achievement Chart Connection

Students are expected to use the given recursive formula in an unfamiliar way to determine the initial value.

Level 3 Performance

Students must substitute given facts and express the first term using the recursive formula.

20. Thinking, Inquiry, Problem Solving, p. 35

Management Suggestion

Give students a few minutes to consider the problem and record their first ideas or attempts. Then move them into random or selected groups of three or four students each

and allow some more time for the groups to discuss and compare ideas. At the end of this time, students should return to their desks and respond individually to the question.

Achievement Chart Connection

For this complex situation, students should use a recursion formula to describe successive impurity levels.

Level 3 Performance

Students should recognize the pattern in the declining level of impurity in the water, find several specific terms of the sequence, describe the sequence with a general term or a recursive expression, and then use this model to find specific information.

21. Check Your Understanding: p. 35

Management Suggestion

Students are asked to recognize the advantages and disadvantages of general terms and recursive expressions for describing sequences. The request for an example provides an opportunity to notice and correct any misunderstanding. You might collect students' work and retain it as part of a portfolio.

The Chapter Problem—Controlling Non-Native Plant Populations

Now students might consider using a recursive formula to describe the number of plants or seeds. The number of plants or seeds in any year depends partially on the number remaining from the previous year as well as the production of new plants.

The Chapter Problem Answers are on pp. 1-54 to 1-56 of the *Teacher Resource*.

Challenge 1

Remind students that they should be able to describe the number of grains using a recursive formula.

Challenge 2

Remind students that they should be able to see that each stage are dependent upon the previous stage and write a recursive formula to describe the shaded area.