

1.6 Arithmetic Sequences

Specific Expectations

The following expectations are common to the Functions and Relations course and the Functions course.

- FA1.03 identify sequences as arithmetic or geometric, or neither
- FA1.04 determine the value of any term in an arithmetic or a geometric sequence, using the formula for the n th term of the sequence
- FA2.04 demonstrate an understanding of the relationships between simple interest, arithmetic sequences, and linear growth

Evidence of Learning

Students should be able to

- identify the general term, specific terms, the initial term, and the common difference for an arithmetic sequence
- test a sequence to determine whether it is arithmetic or not by finding the differences between successive terms

Text Page 41

Time Required Functions and Relations or Functions: 75 min

Materials and Equipment

- No special equipment is required.

Overview

- Introduce arithmetic sequences and common differences.
- Present simple interest as an example of an arithmetic sequence and examine the linear relation.

Questions about recursive formulas are only for Functions and Relations students.

Arithmetic Sequences

In this lesson, students will investigate the arithmetic sequence where the differences between successive terms is constant. Students will continue to create general terms for sequences and will develop the expression for the general term of an arithmetic sequence using a , n , and d .

Management Options

Groups – Divide students into random groups of three students each and allow five to ten minutes for the groups to work through the *Think, Do, Discuss* questions of Part 1. Take a few moments for class discussion to ensure that all groups have recognized that the relationship is linear, because of the constant first differences, and that all groups have determined a general term. Then allow the groups time to respond to the *Think, Do, Discuss* questions of Part 2.

Teacher-Led Discussion – Discuss the *Think, Do, Discuss* questions in class using an acetate that lists the questions and inserting answers as students respond. Refer to the variables a , n , and d in the general term and emphasize that only some sequences—those with a constant or common difference between successive terms—are called arithmetic. Play a game of “I have ... Who has?” to let students practice using the terminology and finding expressions or values for specific terms. Or, students can provide selected information and challenge other students to supply the missing information. For example, student A gives four terms of a sequence and student B identifies the initial term and the common difference.

Part 1: The Days of a Month

Think, Do, Discuss, text page 41

1. Write the sequence of dates for the last week of October. Start with Sunday's date.

For the given calendar, the sequence is 27, 28, 29, 30, and 31.

2. How many terms are in this sequence? Describe the pattern in this sequence.

There are five terms. Each term is one more than the previous term.

3. Extend the pattern and write the next five terms of this sequence. (Assume that the dates may be more than 31.)

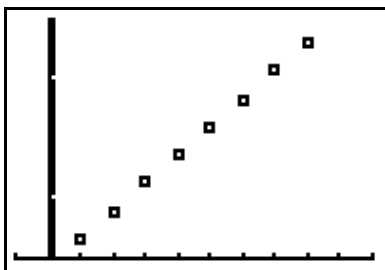
32, 33, 34, 35, 36

4. Determine the **first differences** between pairs of consecutive terms of this sequence. What do you notice? What type of relationship exists between n and the value of the term?

The first difference is always 1. The relationship between n and the value of the term is linear.

5. Graph this sequence. What is the rate of change between each pair of consecutive terms in this sequence?

The rate of change is 1.



6. Determine the general term for this sequence.

$$t_n = 27 + (n - 1)$$

7. Write the sequence of dates for all the Tuesdays of the month.

1, 8, 15, 22, 29

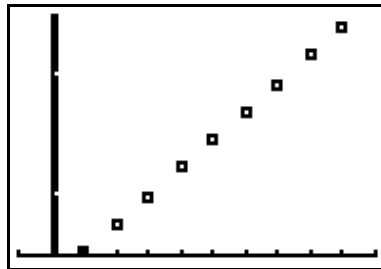
8. Repeat steps 2 to 6 for the sequence in step 7.

There are five terms. Each term is 7 more than the previous term.

The next five terms are 36, 43, 50, 57, 64.

The first differences are the same, that is, 7. The relationship between n and the value of the term is linear.

The rate of change is 7.



9. Describe how to locate, in the calendar, a sequence where the rate of change between each pair of consecutive terms is 8.

Draw a diagonal line from upper left to lower right passing through the next day in each subsequent week, for example, Monday in week 1, Tuesday in week 2, ..., and so on.

10. Describe how to locate, in the calendar, a sequence where the rate of change between each pair of consecutive terms is 6.

Draw a diagonal line from upper right to lower left passing through the previous day in each subsequent week, for example, Wednesday in week 1, Tuesday in week 2, ..., and so on.

Part 2: Simple Interest

Think, Do, Discuss, text page 42

1. Calculate the interest earned at the end of the first year.

$$0.05(\$10\ 000) = \$500$$

2. Determine the balance in the account on Juan's first birthday.

$$\$10\ 000 + \$500 = \$10\ 500$$

3. Calculate the interest earned at the end of the second year.

The interest earned is constant (simple interest), so interest earned in year 2 is \$500.

4. Determine the balance in the account on Juan's second birthday.

The account balance after two years is \$11 000.

5. Write the sequence that represents the balance in the account on each birthday from the 1st to the 18th.

\$10 500, \$11 000, \$11 500, \$12 000, \$12 500, \$13 000, ... , \$19 000

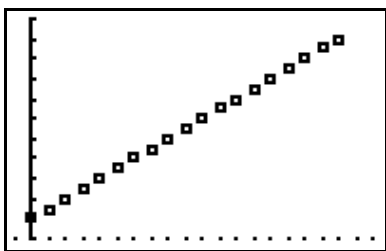
6. What is the balance in the account on Juan's 18th birthday? How much of the balance is interest?

\$19 000, \$9000 of which is interest

7. Explain the pattern in this sequence. What is the rate of change between each pair of consecutive year-end balances?

The pattern is an arithmetic sequence with initial term \$10 500 and the common difference is \$500. The yearly rate of change is \$500.

8. Graph this sequence.



(vertical axis in increments of \$1000 starting at \$9000)

9. Is the growth of the balance in the account linear or nonlinear? Explain.

The growth of the balance is linear, because first differences are a constant \$500. The graph suggests a line.

10. Let n represent the number of years. Determine (a) the general term and (b) the recursive formula for this sequence.

The general term is

$$t_n = 10\,500 + (n - 1)(500)$$

The recursive formula is

$$t_n = t_{n-1} + 500, t_1 = 10\,500$$

11. Suppose Juan leaves the money in the account until he retires at 65. What would be the balance in the account on his 65th birthday?

$$t_n = 10\,500 + (n - 1)(500)$$

$$t_n = 10\,500 + (64)(500)$$

$$t_n = 42\,500$$

The balance in the account is \$42 500.

12. Mary deposits \$5000 in an account that earns simple interest at a rate of 8%/a. If she makes no other deposits, determine the sequence that represents the balance at the end of each year for ten years. Determine the general term of this sequence.

The initial term is \$5400.

The common difference, or yearly interest, is $0.8(\$5000) = \$400$.

The sequence is 5400, 5800, 6200, 6600, 7000, 7400, 7800, 8200, 8600, 9000.

EXTENSION SUGGESTION

Allow students to create and calculate "what if?" scenarios. For example,

If I work over the holidays and invest ■ at ■ % simple interest, how much will I have in ■? Or If I want to have ■ in ■ years, how much must I invest now at ■ % simple interest?

Additional Example: Functions and Relations

When Sue was ten years old, she inherited a collection of 34 china figurines from her grandmother. Each year thereafter on her birthday, her aunts and her mom gave her three new figurines to add to the collection.

- (a) Use a sequence to show the total number of figurines in her collection from when she is 10 to when she is 18.
- (b) Find a general term and a recursive formula to describe the sequence.

Solution

- (a) The sequence is 34, 37, 40, 43, ... , 58.
- (b) The general term is $t_n = 34 + 3(n - 1)$.
The recursive formula is
 $t_n = t_{n-1} + 3, t_1 = 34$.

Additional Example: Functions

Patti's grandmother created a savings account for her by depositing \$1000 when she was born. The account earns simple interest at 3%/a.

- (a) List the balances in the account on Patti's birthday each year until she is 6.
- (b) Provide a general term for the annual balance.

Solution

- (a) The sequence is 1030, 1060, 1090, 1120, 1150, 1180.
- (b) $t_n = 1000(0.03)^n + 1000$

Practise, Apply, Solve 1.6

Questions with a Functions and Relations Focus:

1–4, 6, 8–11, 14–19, 22, 23

Questions with a Functions Focus:

1–3, 5–13, 15, 17–20

6. Knowledge and Understanding, p. 48

Management Suggestion

Collect and assess the responses to this question using a specific marking scheme.

Achievement Chart Connection

Students are required to demonstrate understanding of the characteristics of an arithmetic sequence by finding specific terms and a general term.

Level 3 Performance

Students must identify the common difference and use it to find the next three terms. When the initial term and the common difference are known, the general term can be created. Any term can be found by substitution.

9. Communication, p. 49

Management Suggestion

Collect and assess the responses to the question.

Achievement Chart Connection

Students are required to find the same term using a graph and algebra. They are required to compare the methods, to choose one they prefer, and to explain their choices.

Level 3 Performance

Students must compare visual and algebraic approaches. Students should use correct mathematical form and precise vocabulary to enhance their descriptions.

ASSESSMENT SUGGESTION

Collect the students' responses to questions 6 and 9 at the end of the period and assess them using an analytic marking scheme.

15. Application, p. 50

Management Suggestion

Discuss with the class, supported by some extra visual information or other information obtained from the Internet about Halley's comet.

Achievement Chart Connection

Planetary motion presents a complex and somewhat imprecise example of a sequence. Students may recognize that a mathematical description of an observed phenomenon allows for prediction of future events.

Level 3 Performance

Although the prediction of the comet's appearance cannot be based strictly on an arithmetic sequence, the orbit suggests a sequence. This question presents an opportunity to discuss an algebraic model and to check the confidence or accuracy of

students. Students should be able to recognize a sequence and make a reasonable prediction.

18. Thinking, Inquiry, Problem Solving, p. 50

Management Suggestion

Ask students to prepare a two-page response suitable for bulletin-board display. Spend a few minutes in class discussing the possible approaches and expectations and ensure that all students have equitable access to computers or calculators.

Achievement Chart Connection

Invite students to apply the steps of a problem-solving process—generating data, searching for a pattern, creating a model, testing and adjusting the model, drawing a conclusion, and justifying and communicating a decision.

Level 3 Performance

Students will recognize the characteristics of an arithmetic sequence and identify the common difference. The initial value is known and the number of terms can easily be predicted, so students have the task of finding the sum. For some students, generating the terms and then adding may be appropriate, but encourage other students to find a general approach. This question will lead to future lessons about arithmetic series.

19. Check Your Understanding, p. 50

Management Suggestion

To find the general term of an arithmetic sequence, students must identify the initial term and the common difference. To calculate a specific term, students must substitute the term number into the general term. The constant rate of change, that is, the difference between successive terms is always the same, indicates the sequence is arithmetic.

Ask students to create four numbers where three numbers are part of an arithmetic sequence and the fourth number is not. Partners could be asked to identify and explain the problem number.

The Chapter Problem – Controlling Non-Native Plant Populations

By the end of this section, students should be able to determine whether a sequence is arithmetic by looking for a constant difference between pairs of successive terms in a group of numbers. If the sequence is arithmetic, the students may create a general term.

The answers to the Chapter Problem are on p. 1-54 of the *Teacher Resource*.

Challenge 1

Remind students that they should be able to

- check the differences between successive terms
- describe a general term if the differences are constant
- recognize sequences that are not arithmetic

Challenge 2

Remind students that they should be able to

- check the differences between successive terms
- describe a general term if the differences are constant
- recognize sequences that are not arithmetic