

1.7 Geometric Sequences

Specific Expectations

Unless stated otherwise, the following expectations are common to the Functions and Relations course and the Functions course.

- FA1.01(U) write terms of a sequence, given the formula for the n th term or given a recursion formula
- FA1.01(U/C) write terms of a sequence, given the formula for the n th term
- FA1.02 determine a formula for the n th term of a given sequence (e.g., the n th term of the sequence $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \dots$ is $\frac{n}{n+1}$)
- FA1.03 identify sequences as arithmetic or geometric, or neither
- FA1.04 determine the value of any term in an arithmetic or a geometric sequence, using the formula for the n th term of the sequence

Evidence of Learning

Students should be able to

- use ratios or first differences to determine whether a sequence is arithmetic, geometric, or neither
- create a general term to describe a given sequence
- substitute values into a general term to find specific terms of a sequence
- recognize situations that involve sequences

Text Page 51

Time Required Functions and Relations or Functions: 75 min

Materials and Equipment

- graph paper
- graphing calculator

Overview

- Introduce sequences where successive terms have a common ratio. The terms of the sequence may increase or decrease, but each term is a result of multiplying the previous term by a factor.

Management Options

Independent Work – This lesson revisits and extends concepts about sequences developed in section 1.6. Most students should be able to answer the *Think, Do, Discuss* questions on their own. If you choose this approach, you might pair some students together. Allow students about ten minutes to

work through the questions before discussing them in class.

Teacher-Led Discussion – As a class, work through the *Think, Do, Discuss* questions. Use acetate to record the answers and a TI-83 Plus calculator and the view screen to graph the relation. Encourage students to describe the relations first in everyday language and then to use terms such as “geometric,” “common ratio,” and “general term.”

ASSESSMENT SUGGESTION

KNOWLEDGE AND UNDERSTANDING QUICK QUIZ

Ask students to respond in writing to several “quick-answer questions,” that you can mark and then return to students next class.

Give five elements of a geometric sequence.

Give four elements of an arithmetic sequence.

Find the common difference in the sequence 3, 7, 11, 15, ...

Find the common ratio in the sequence 9, 3, 1, ...

Explain how to tell whether a sequence is arithmetic or geometric.

Part 1: Doubling

Think, Do, Discuss, text page 51

1. Assume that 100 *E. coli* bacteria double in number every hour. Copy and complete the table to show how many bacteria are present after each hour for 8 h.

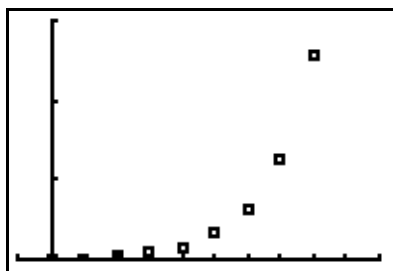
Time (h)	0	1	2	3	4
Number of Bacteria	100	200	400	800	1600
Time (h)	5	6	7	8	
Number of Bacteria	3200	6400	12 800	25 600	

2. Write the number of bacteria after each hour for 8 h as a sequence. The first term is 100.
100, 200, 400, 800, 1600, 3200, 6400, 12 800, 25 600
3. Determine the first differences between pairs of consecutive terms of this sequence. Interpret the first differences.

Multiply each term by 2 to create the next term. The first differences are 100, 200, 400, 800, 1600, 3200, 6400, and 12 800. This relation is not linear but exponential.

4. Graph this sequence. Describe the rate of change between pairs of consecutive terms.

The rate of change is increasing, which is characteristic of an exponential relationship.



5. Divide each term, except the first, by the previous term. What do you notice? What is the significance of this value?

The quotient is always 2. The ratio of successive terms is constant, but the number of bacteria is increasing exponentially.

6. Rewrite each term of the sequence that you wrote in step 2 as an expression of the original 100 bacteria and the value that you calculated in step 5.

$$100(2)^0, 100(2)^1, 100(2)^2, 100(2)^3, 100(2)^4, 100(2)^5, 100(2)^6, 100(2)^7$$

7. Write the general term for this sequence.

$$t_n = 100(2)^{n-1}$$

8. Determine the number of bacteria after 24 h.

$$\begin{aligned} t_{24} &= 100(2)^{24-1} \\ &= 100(2)^{23} \\ &= 838\,860\,800 \end{aligned}$$

Part 2: Half-life

Think, Do, Discuss, text page 52

1. After a medical procedure, Harry's body has absorbed 100 mg of carbon-11. Copy and complete the table to show the mass of carbon-11 after each 20-min interval for 3 h.

20-Min Interval	0	1	2	3	4
Mass of Carbon-11 (mg)	100	50	25	12.5	6.25
20-Min Interval	5	6	7	8	9
Mass of Carbon-11 (mg)	3.13	1.57	0.78	0.39	0.20

2. Write the mass of carbon-11 after each 20-min interval for 3 h as a sequence. The first term is 100.

$$100, 50, 25, 12.5, 6.25, 3.13, 1.57, 0.78, 0.39, 0.20$$

3. Determine the first differences between pairs of consecutive terms of this sequence. Interpret the first differences.

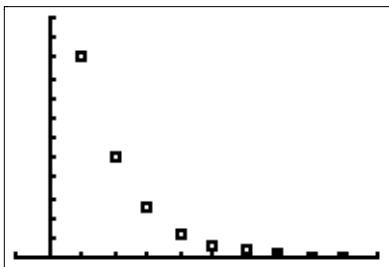
Divide each term by 2 to find the following term.

$$50, 25, 12.5, 6.25, 3.13, 1.57, 0.78, 0.39, 0.20$$

The relation is not linear but it is exponential.

4. Graph this sequence. Describe the rate of change between pairs of consecutive terms.

The rate of change is decreasing between pairs of consecutive terms.



5. Divide each term, except the first, by the previous term. What do you notice? What is the significance of this value?

The quotient is always $\frac{1}{2}$, which is the constant ratio between successive terms of the sequence.

6. Rewrite each term of the sequence that you wrote in step 2 as an expression of the original 100 mg and the value that you calculated in step 5.

$$100\left(\frac{1}{2}\right)^0, 100\left(\frac{1}{2}\right)^1, 100\left(\frac{1}{2}\right)^2, 100\left(\frac{1}{2}\right)^3, \\ 100\left(\frac{1}{2}\right)^4, 100\left(\frac{1}{2}\right)^5, 100\left(\frac{1}{2}\right)^6, 100\left(\frac{1}{2}\right)^7, \\ 100\left(\frac{1}{2}\right)^8, 100\left(\frac{1}{2}\right)^9$$

7. Write the general term for this sequence.

$$t_n = 100\left(\frac{1}{2}\right)^n$$

8. Determine the mass of carbon-11 remaining after 24 h.

After 24 h, there will be $3(24) = 72$ periods.

$$t_{72} = 100\left(\frac{1}{2}\right)^{72} \\ = 2.12 \times 10^{-20}$$

EXTENSION SUGGESTION

Ask students to search the Internet or visit a library to find some information about the following:

- nuclear accidents (Three Mile Island and Chernobyl)
- carbon dating of archaeological discoveries
- use of radioactive materials in medicine

Additional Example: Functions

For each sequence

- identify it as arithmetic, geometric, or neither
 - find the next three terms
 - find a general term
 - find the 100th term
- (a) 1, 2, 4, 8, ...
(b) 2, 4, 6, 8, ...
(c) 1, 4, 9, 16, ...

Solution

- (a) The sequence is geometric with common ratio 2. The next three terms are 16, 32, and 64. The general term is $t_n = 2^{n-1}$.

The 100th term is

$$t_{100} = 2^{99} \\ = 6.34 \times 10^{29}$$

- (b) The sequence is arithmetic with common difference 2. The next three terms are 10, 12, and 14. The general term is $t_n = 2n$.

The 100th term is

$$t_{100} = 2(100) \\ = 200$$

- (c) The sequence is neither arithmetic nor geometric. The next three terms are 25, 36, and 49. The general term is $t_n = n^2$.

The 100th term is

$$t_{100} = 100^2 \\ = 10\,000$$

Additional Example: Functions

For each sequence,

- i. identify it as arithmetic, geometric, or neither
 - ii. create a general term and a recursive formula
- (a) 1, 2, 4, 8, ...
- (b) 2, 4, 6, 8, ...
- (c) 1, 4, 9, 16, ...

Solution

- (a) The sequence is geometric with common ratio 2. The general term is $t_n = 2^{n-1}$. The recursive formula is $t_n = 2t_{n-1}$, $t_1 = 1$.
- (b) The sequence is arithmetic with common difference 2. The general term is $t_n = 2n$. The recursive formula is $t_n = t_{n-1} + 2$, $t_1 = 2$.
- (c) The sequence is neither arithmetic nor geometric. The general term is $t = n^2$. The recursive formula is $t_n = (\sqrt{t_{n-1}} + 1)^2$, $t_1 = 1$.

Practise, Apply, Solve 1.7

Questions with a Functions and Relations Focus

Questions 4 and 18 refer specifically to recursive formulas.

1–4, 6, 8–9, 13–14, 16, 18–20, 23–25

Questions with a Functions Focus

57–60: 1–3, 5–12, 16–17, 19–23

9. Knowledge and Understanding, p. 58

Management Suggestion

Ask students to list the steps for identifying and describing a geometric sequence.

Achievement Chart Connection

The introduction of fractions and negative signs tests students' understanding of the concepts. The common ratio is still critical.

Level 3 Performance

Some students may initially require some prompting but determining a common ratio or a common difference should become routine.

For example,

$$t_2 - t_1 = ?, t_3 - t_2 = ?$$

Does the sequence appear to be arithmetic?

$$\frac{t_2}{t_1} = ?, \frac{t_3}{t_2} = ?$$

Does the sequence appear to be geometric?

For an arithmetic sequence, how do you create a general term? For a geometric sequence, how do you create a general term?

16. Communication, p. 59

Management Suggestion

Ask students to exchange and edit a first draft of their responses. Then a polished draft can be retained in a portfolio.

Achievement Chart Connection

Students are expected to use tables, graphs, algebraic descriptions, or precise language to explain how the situation can be modelled with a geometric sequence.

Level 3 Performance

Students should clearly explain the decreasing value using appropriate vocabulary. The response should include a sequence and a general term.

ASSESSMENT SUGGESTION [formative]

Create a sequence that is

(a) arithmetic (b) geometric (c) neither

Pass your sequences to a partner who will find:

i. the next four terms

ii. a general term

iii. the 75th term

Discuss any questions with your partner, or teacher if you cannot solve the problems.

19. Application, p. 60

Management Suggestion

Students can get information from advertisements in a newspaper or a car-trading magazine about current values of vehicles. This information should help students to predict future values.

Achievement Chart Connection

Students will apply their knowledge of geometric sequences in a possibly unfamiliar situation. In many cases, students will need to recognize mathematical concepts in the real world.

Level 3 Performance

Some students may have to create a table to find the terms of the sequence. For students who might have little or no knowledge of automobile values, clarify the calculation of depreciation.

21. Check Your Understanding, p. 60

Management Suggestion

Near the end of the class, ask students to answer this question and hand in their work. Assess students' work using an analytic scheme.

23. Thinking, Inquiry, Problem Solving, p. 60

Management Suggestion

Encourage the use of dynamic geometry software or a computer to predict values.

Achievement Chart Connection

Students should follow the steps of the problem-solving model to create a sequence that seems to be geometric.

Level 3 Performance

Students are expected to draw a diagram, collect data (terms of the sequence), find a general term from the pattern (make a model), and use the model to predict the sixth term. At a less sophisticated level, students might calculate six successive elements of the sequence. If technology is available, encourage students to use dynamic geometry software to find a solution.

ASSESSMENT OF LEARNING SKILLS

Work Habits / Homework

On a regular basis, collect one or two questions from the homework assigned and mark it using a

0 – no reasonable attempt

1 – reasonable effort

2 – all questions attempted

The Chapter Problem—Controlling Non-Native Plant Populations

By the end of this section, students should be able to recognize and describe sequences where successive terms have a common ratio and to create a general term for a geometric sequence.

The answers to the Chapter Problem are on p. 1-54 of the *Teacher Resource*.

Challenge 1

Remind students that they should be able to

- recognize a geometric sequence
- use a general term to describe a geometric sequence
- note that the question deals with a total mass rather than the mass on any one square

Challenge 2

Remind students that they should be able to

- recognize a geometric sequence
- use a general term to describe a geometric sequence
- attempt to create an expression for the shaded area