

1.9 Rational Expressions

Specific Expectations

The following expectation is common to the Functions and Relations course and the Functions course.

- OC1.07 simplify and evaluate expressions containing integer and rational exponents, using the laws of exponents

Evidence of Learning

Students should be able to

- state laws for multiplication and division of powers with the same base
- find the power of a power
- express powers with fractional exponents in simplest form
- express powers with rational exponents in alternative forms

Text Page 73

Time Required Functions and Relations or Functions: 75 min

Materials and Equipment

- scientific calculator

Overview

- Explore partial conversion periods to learn about fractional exponents.
- Extend the rules for simplifying expressions containing powers to include powers with rational or fractional exponents.
- Express roots and powers in other ways, for example, $\sqrt[3]{x}$, $x^{\frac{1}{3}}$.

Rational Expressions

Begin with a discussion, drawing from students any “rules” they can remember for working with exponents. Create a display and post it on a bulletin board or above the board for future reference.

Management Options

Teacher-Led Discussion – Because some students will interpret fractional or rational exponents with difficulty, you may choose to closely monitor the lesson development. As a class, discuss the *Think, Do, Discuss* questions and work through a few specific examples. Then provide examples for students to

attempt on their own, giving immediate feedback.

Independent Work – Students may work at their own pace using specific software. Students may also work in a CAL lab, where possible.

EXTENSION SUGGESTION

Ask students to consider the following questions:

Use your calculator to find the square of 2.236 067 977.

Find the value of $\sqrt{5}$ or $5^{\frac{1}{2}}$.

What would you expect the value of $(\sqrt{5})^2$ to be?

Why is it useful to work with exponents instead of using estimated values?

Part 1: Investigating Fractional Conversion Periods

Think, Do, Discuss, page 73

1. Marjorie invested \$1000 in a mutual fund that earns interest at 21%/a, compounded annually. Copy and complete the table. Determine the values of the investment at the beginning of the period, at the end of the first year, and at the end of the second year.

Year	0	1	2
Amount (\$)	$1000(1.21)^0$ = 1000	$1000(1.21)^1$ = 1210	$1000(1.21)^2$ = 1464.10

2. Suppose Marjorie wishes to determine the value of her investment at the end of six months. How could six months be represented as a fraction of a year?

Six months is $\frac{1}{2}$ of one year.

3. In your table, enter the year and the expression for the amount of the investment at the end of six months.

Year	$\frac{1}{2}$
Amount (\$)	$1000(1.21)^{\frac{1}{2}}$

4. Use your calculator to evaluate $(1.21)^{\frac{1}{2}}$. What value do you get? Store this value in your calculator's memory.

1.1

5. Multiply 1000 by the number stored in memory. What does this value represent?

The value 1100 represents the amount after six months.

6. Multiply the product in step 5 by the value stored in memory. What does this value represent?

The value 1210 represents the amount after the second six months, that is, after one year.

7. Multiply the value stored in memory by itself. To what exponent must $(1.21)^{\frac{1}{2}}$ be raised to obtain 1.21? Describe the mathematical relationship between $(1.21)^{\frac{1}{2}}$ and 1.21.

$$1.1^2 = 1.21$$

$(1.21)^{\frac{1}{2}}$ must be raised to the exponent 2 to obtain 1.21.

$$\left[(1.21)^{\frac{1}{2}} \right]^2 = 1.21$$

8. Suppose Marjorie wishes to determine the value of her investment at the end of 18 months. How could 18 months be represented as a fraction of a year?

18 months is $\frac{3}{2}$ years.

9. In the table you copied from page 73, enter the year and the expression for the amount of the investment at the end of 18 months.

Students should enter $\frac{3}{2}$ and $1000(1.21)^{\frac{3}{2}}$.

10. Evaluate $1000 \times (\text{number stored in memory}) \times (\text{number stored in memory}) \times (\text{number stored in memory})$. What does the product represent?

The product 1331 represents the amount after 18 months.

11. To what exponent must $(1.21)^{\frac{1}{2}}$ be raised to obtain $(1.21)^{\frac{3}{2}}$? Describe the mathematical relationship between $(1.21)^{\frac{1}{2}}$ and $(1.21)^{\frac{3}{2}}$.

The exponent is 3.

$$\left[(1.21)^{\frac{1}{2}} \right]^3 = (1.21)^{\frac{3}{2}}$$

12. Anthony invests \$1000 at 9.2727%/a, compounded annually. Determine an expression that represents the amount of the investment at the end of 4 months, 8 months, and 12 months, respectively. Use your calculator to evaluate each expression.

At the end of four months, the amount is

$$1000(1.092727)^{\frac{4}{12}} = 1000(1.092727)^{\frac{1}{3}} \\ = 1030$$

At the end of eight months, the amount is

$$1000(1.092727)^{\frac{8}{12}} = 1000(1.092727)^{\frac{2}{3}} \\ = 1060.90$$

At the end of 12 months, the amount is

$$1000(1.092\,727)^{\frac{12}{12}} = 1000(1.092\,727)^1 \\ = 1092.73$$

13. Use your calculator to evaluate

$(1.092\,727)^{\frac{1}{3}}$. How many times must this result be multiplied by itself to get 1.092 727?

The value is 1.03. This result must be multiplied three times.

14. To what exponent must $(1.092\,727)^{\frac{1}{3}}$ be raised to obtain 1.092 727? Describe the mathematical relationship between

$(1.092\,727)^{\frac{1}{3}}$ and 1.092 727.

The exponent is 3.

$$\left[(1.092\,727)^{\frac{1}{3}}\right]^3 = 1.092\,727$$

15. Determine the value of Anthony's investment after 32 months.

$$1000(1.092\,727)^{\frac{8}{3}} = 1266.77$$

Part 2: Defining Rational Exponents

Part 1 gives a context for rational expressions. Part 2 formalizes the mathematics and explains how to express powers with rational exponents by using radical signs.

Consolidate Your Understanding

1. What exponent can you use to represent the n th root of a number?

The exponent is $\frac{1}{n}$.

2. Use the exponent law for power of a power

to express $27^{\frac{2}{3}}$ in two ways.

$$27^{\frac{2}{3}} = \left(27^2\right)^{\frac{1}{3}}$$

$$= \left(27^{\frac{1}{3}}\right)^2$$

$$27^{\frac{2}{3}} = \sqrt[3]{27^2}$$

$$= \left(\sqrt[3]{27}\right)^2$$

Additional Example: Functions and Relations

Simplify.

(a) $\left(\sqrt[3]{x^{3n+1}}\right)\left(\sqrt[3]{x^{-1}}\right)$

(b) $\frac{\left(x^2y^4\right)^{\frac{1}{2}}\left(x^4y^2\right)^{\frac{1}{2}}}{\left(x^{\frac{1}{2}}y^{\frac{1}{2}}\right)^4}$

Solution

(a) x^n (b) xy

Additional Example: Functions

Simplify.

(a) $\frac{81^{\frac{1}{4}} + 81^{\frac{3}{4}}}{81^{\frac{3}{4}} - 81^{\frac{1}{4}}}$ (b) $\frac{64^{\frac{5}{6}} \times 16^{\frac{3}{4}}}{27^{\frac{2}{3}}}$

Solution

(a) $\frac{5}{4} = 1.25$ (b) $\frac{256}{9}$

ASSESSMENT SUGGESTION

APPLICATION

Ask students to write the equation for several of the situations in this section but not to calculate. Conversely, provide a complicated numerical expression for students to enter into their calculators. In this way, students can identify where they may make entry errors.

Practise, Apply, Solve 1.9

Questions with a Functions and Relations Focus:

3–5, 7–8, 11–23

Questions with a Functions Focus:

79–82: 1–10, 12, 14–18, 21–22

12. Knowledge and Understanding, p. 80

Management Suggestion

Discuss situations where depreciation is used, for example, accounting and operating a business (the worth of capital).

Achievement Chart Connection

Students substitute values into a given formula and must know how to perform calculations involving fractional exponents.

Level 3 Performance

Students should substitute values appropriately and determine the correct depreciation rate.

15. Communication, p. 81

Management Suggestion

Ask students to work in pairs. A *silent* partner must exactly follow written or verbal instructions.

Achievement Chart Connection

Students use flow-chart symbols to clearly describe the sequence. Students must reason carefully to choose the correct order of keystrokes.

Level 3 Performance

The sequence of keystrokes must lead to the correct answer. Repeated attempts are not necessary. Students should recognize and accept alternatives.

18. Application, p. 81

Management Suggestion

Collect students' work and assess it by making constructive suggestions, but do not record marks.

Achievement Chart Connection

For this complex problem, students must adjust the compound-interest formula.

Level 3 Performance

Students substitute correctly into the formula.

21. Check Your Understanding, p. 81

Management Suggestion

Discuss the task with the class. Collect students' suggestions for the methods and record them on the board.

22. Thinking, Inquiry, Problem Solving, p. 82

Management Suggestion

Set aside time for students to focus on this question with the expectation that their work will be collected and assessed. Allow students to "purchase" a helpful suggestion. Deduct a mark for each purchase. Give direct feedback, for example, try drawing a time line.

Achievement Chart Connection

The solution to this question is complex.

Level 3 Performance

Students carefully organize the given information and recognize intermediate steps.

Students should provide evidence of valid, logical thinking.

ASSESSING LEARNING SKILLS

Post a checklist for students to follow when they are having difficulty with a problem. This will allow students to self-assess. For example,

1. Did you draw the time line?
2. Did you identify unknown information?

The Chapter Problem—Controlling Non-Native Plant Populations

Students might first attempt to apply rational exponents, but advise them to examine the context before they do so.

The answers to the Chapter Problem are on p. 1-54 of the *Teacher Resource*.

Challenge 1, Challenge 2

In both chapter challenges, a fractional exponent has little meaning. However, capable students might like to use the second challenge as a "jumping off" point to begin examining iterative processes and the creation of fractal images, or the idea of fractional dimensions.