

2.2 Exploration: Investigating a Sequence Using the TI-83 Plus Calculator and the TI Calculator-Based Ranger (CBR)

Text Page 117

Time Required Functions and Relations or Functions: 30 min

Materials and Equipment

- TI CBR (Calculator-Based Ranger)
- TI-83 Plus calculator
- ball (a racquetball or a large rubber ball)

Overview

- Investigate a geometric sequence.
- Investigate the sum of a geometric sequence, i.e., a series.
- Describe the conditions of a real situation in mathematical form.

Investigating a Sequence

Students may have investigated the relationship between the height of a ball and its velocity in grade 9 (*Mathematics 9*, section 1.15, page 84). Students may have also investigated the relationship between height, time, and the coefficient, a , in the quadratic equation in grade 10 (*Mathematics 10*, section 4.2). The purpose of this exploration is to investigate the sum of a geometric sequence. This experiment is good practice in following instructions, a technology skill that students will need working in a dynamic environment.

Management Options

Groups – Pairs or teams of three or four students could do this exploration with little teacher intervention. If they do not grasp the connection to a geometric sequence, suggest they read Example 4 in section 2.3 for a hint.

Teacher Demonstration – Students who feel comfortable using this equipment could demonstrate the experiment to the class. A class discussion about the variability of the

results could help students who believe there is only one correct answer to a mathematical problem.

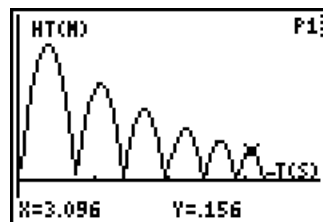
Independent Work – This experiment requires teamwork and careful planning, but steps 7 and 8 in the *Procedure* can be answered individually.

Procedure

Remind students that the ball is released, not thrown.

This data was obtained using a Ping-Pong ball on a hardwood floor. Recording a golf ball bouncing on a ceramic floor would produce erratic data. Answers will vary. Example 4 in section 2.3 outlines a similar situation, but, in this example, the “experiment” starts at the moment when the ball first hits the floor, not when the ball is dropped.

7. *Your graph should have at least five bounces. If this is not the case or you are not satisfied with the results of your experiment, press **ENTER**. Then select **5:REPEAT SAMPLE**, and try the experiment again.*



8. *When you are satisfied with your data, sketch the graph of height versus time.*

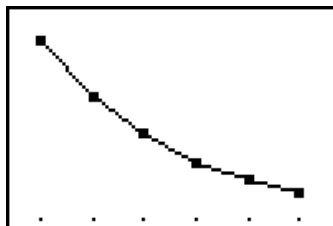
See the screens for question 3 in Analyzing the Data.

Analyzing the Data

Bounce, n	Maximum Height of the Ball After Bounce n , y (m)	$\frac{\text{Maximum Height After this Bounce}}{\text{Maximum Height After Previous Bounce}} \times 100\%$	Total Vertical Distance Travelled by the Ball Between this Bounce and the Next Bounce (m)	Total Vertical Distance Travelled Since $t = 0$ (m)
1	0.718		$2(0.718) = 1.436$	1.436
2	0.512	$0.512 \div 0.718 = 71.3\%$	$2(0.512) = 1.024$	$1.436 + 1.024 = 2.46$
3	0.373	$0.373 \div 0.512 = 72.8\%$	$2(0.373) = 0.746$	$2.46 + 0.746 = 3.206$
4	0.270	$0.270 \div 0.373 = 72.4\%$	$2(0.270) = 0.540$	$3.206 + 0.540 = 3.746$
5	0.203	$0.203 \div 0.270 = 75.2\%$	$2(0.203) = 0.406$	$3.746 + 0.406 = 4.152$
6	0.156	$0.156 \div 0.203 = 76.8\%$		

- Use the right arrow and **TRACE** to find the approximate maximum height of the ball, y , after the first bounce. After each bounce, n , record the maximum height of the ball, y , in your notebook, using a table. Use these values to calculate the total vertical distances, as described in the table, travelled by the ball.
- Plot the data points representing the maximum height of the ball on a scatter plot. Press **STAT**, select **1:Edit** to enter the bounce number, n , in **L1** and the maximum height of the ball after each bounce, y , in **L2**. Press **2nd** **Y=**. Press **ENTER** to select **Plot1**. Press **ENTER** again. For **Type**, select the second graph; for **Xlist**, select **L1**; for **Ylist**, select **L2**; and select the square for **Mark**. To see a plot of this data, press **ZOOM** **9**.

L1	L2	L3	Z
1	.718	-----	
2	.512		
3	.373		
4	.27		
5	.203		
6	.156		
-----	-----		
L2(?) =			

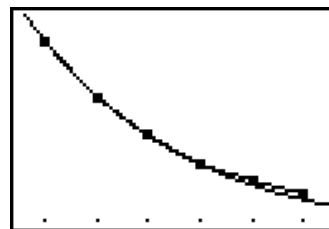


- Write the sequence that represents the maximum height of the ball after each bounce. Describe this type of sequence. Use the third column in your table to help you determine the general term, t_n , that models this sequence.

The sequence is 0.718, 0.512, 0.373, 0.270, 0.203, 0.156 The sequence is similar to a geometric sequence with a common ratio of about 0.72. A possible model is $t_n = (0.718)(0.72)^{n-1}$.

- Rewrite your model for t_n by substituting x for n . Press **Y=** and enter the equation of your model. Press **ENTER** and then **GRAPH**. Describe how well the model fits the data.

$y = (0.718)(0.72)^{n-1}$. The model is insignificantly different from the data.



6. Use **TRACE** and the graph of your model to predict the maximum height after each bounce from $n = 1$ to $n = 8$. Record your predictions in your notebook.

Use **ZOOM** 2 to see this data. The data in the second column are rounded to three decimal places.

x	y
1	0.725
2	0.517
3	0.373
4	0.268
5	0.193
6	0.139
7	0.100
8	0.071

7. Use your model to calculate the total vertical distance travelled by the ball after the first

(a) five bounces

(b) eight bounces

(c) Compare the answer you found in (a) to the total vertical distance that you calculated for the table.

The total vertical distance travelled after five bounces is 4.152 m (twice the sum of the distances above). After eight bounces, this distance is 4.772 m. The total distance after five bounces in the table is exactly the same. The data for eight bounces was not gathered.

8. Suppose you dropped a large super ball from a great height. Explain the difficulty of using this method to calculate the total vertical distance travelled by the ball after the first 20 bounces.

The CBR collects data for only 15 s. The height of a bouncing super ball is very high. Fifteen seconds would pass before a super ball could bounce 20 times. Also, a super ball might bounce higher than the height from which the ball was dropped, which would make collecting data difficult.

9. Repeat this exploration to determine how the maximum height and the total vertical distance travelled by the ball is affected by different surfaces, for example, a concrete floor, different balls, or different initial heights.

Some surfaces absorb more shock, for example, a hardwood floor is more flexible than a concrete floor. A maximum height of a bouncing ball would be less if it were bouncing on this type of floor, and the total vertical distance travelled would be less. Different balls have different “bounce,” for example, a Ping-Pong ball has more bounce than a squash ball. A Ping-Pong ball would probably bounce higher and travel a greater distance than a squash ball. Using a higher initial height would increase the total distance the ball travels.

TEACHER INTERVENTION

Conclude with a discussion about the variability of results.

ASSESSING LEARNING SKILLS

Use the Learning Skills Tracking Sheet to assess all aspects of students' work during this activity. Students could use the Learning Skills Self-Assessment Checklists.