

2.5 Future Value and Amortization Tables

Specific Expectations

The following expectations are common to the Functions and Relations course and the Functions course.

- FA2.03 solve problems involving the amount and the present value of an ordinary annuity
- FA3.01 analyse the effects of changing the conditions in long-term savings plans (e.g., altering the frequency of deposits, the amount of deposit, the interest rate, the compounding period, or a combination of these) (Sample problem: Compare the results of making an annual deposit of \$1000 to an RRSP, beginning at age 20, with the results of making an annual deposit of \$3000, beginning at age 50)
- FA3.03 generate amortization tables for mortgages, using spreadsheets or other appropriate software
- OC3.05 use graphing technology effectively (e.g., use appropriate menus and algorithms; set the graph window to display the appropriate section of a curve)

Evidence of Learning

Students should be able to

- design and input formulas to create a report of the future value of an ordinary simple annuity
- design and input formulas to create an amortization schedule
- distinguish between an ordinary annuity and an annuity due
- distinguish between a simple annuity and a general annuity

Text Page 129

Time Required Functions and Relations or Functions: 75 min

Materials and Equipment

- spreadsheet software
- TI-83 Plus calculator

Overview

- Define ordinary annuity, annuity due, simple annuity, general annuity, amount of an annuity, and amortization period.
- Use a spreadsheet to show the growth of not one but many deposits (or payments).
- Use a spreadsheet to create an annuity, an amortization table, or a loan repayment schedule.
- In *Teacher Resource*, create a spreadsheet using the TI-83 Plus calculator.

Using Spreadsheets to Create Amortization Tables

In sections 1.8 and 2.4, the growth of one deposit was shown in a spreadsheet. In this section, the growth of many, regular deposits is shown in a spreadsheet.

Students may require access to computers after class for this section and section 2.12. Because students should have learned about the fundamentals of using a spreadsheet—entering data, copying data, and creating charts—in grades 9 and 10, many of them will be proficient. Some students may not be as skilled.

Discuss the following advantages of using a spreadsheet:

- The user can see most, if not all, of the amortization table.
- Calculating each line of the amortization table using algebra is repetitive and open to error; by copying data and formulas, it is much faster to create a spreadsheet.
- The information in a spreadsheet can be shared by several users.

- It is possible to change the parameters or variables, for example, interest rate, quickly.
- By using a spreadsheet, students can develop their skills in thinking algorithmically and may focus on organizing and analyzing information, rather than repetitive computation.
- Spreadsheets have many applications in real life, for example, business, science, and statistical analysis.

MANAGEMENT SUGGESTION

Section 2.9 explains how to use the TVM Solver of the TI-83 Plus, which may help the teacher to check answers.

Management Options

Groups – Pair an experienced spreadsheet user with a less experienced student. In this lesson, students must participate, rather than observe. Reconvene as a class to discuss the results of Part 1 before students proceed to Part 2. Prepare an overhead of the answers and show it to students while they are working so that you may assist students who need help.

Teacher Demonstration – If resources are limited, use one computer and a projection panel to demonstrate the lesson. Provide each student with a few copies of a spreadsheet template.

Independent Work – If there is an adequate number of computers, students who are experienced in using spreadsheets can work independently. Create cards so that each card shows a different interest rate, payment amount, and term. Invite students who are working independently to choose a card, to create an amortization table or repayment schedule, and to vary the parameters. Have them reconvene to discuss their findings. Collect copies of their work to be used in a student information forum.

Part 1: Future Value of Annuities

Think, Do, Discuss, p. 130

1. *Why is the interest \$0 for the first period?*

The first deposit is made at the *end* of the first period and does not collect interest for the first period.

2. *Explain how the new balance is calculated each year, or period.*

The new balance is the sum of the previous balance, interest, and the deposit. The interest is calculated on the previous balance at the end of the period.

3. *Describe how the balance at the beginning of each period is determined.*

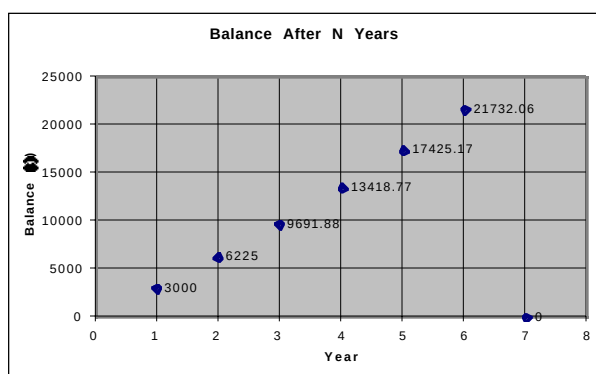
The balance at the beginning of each period is the same as the balance at the end of the last period.

4. *In cell D3, the expression =**\$D\$2** appears. Examine this column in the spreadsheet. What do you notice? Mr. Watts used the **Fill Down** command to complete the columns. What happens to the cell references in the spreadsheet? What effect do the \$ signs in =**\$D\$2** have on the values in the Payment column?*

All the values in column D are the same. The cell references do not change. The \$ sign makes the addresses of column D and row 2 *absolute*, so that the values in column D are all the same as the value in cell D2.

5. Reproduce Mr. Watts' spreadsheet and the graph.

	A	B	C	D	E
1	Period	Balance	Interest at 7.5%/a	Payment	New Balance
2	1	\$0.00	\$0.00	\$3,000.00	\$3,000.00
3	2	\$3,000.00	\$225.00	\$3,000.00	\$6,225.00
4	3	\$6,225.00	\$466.88	\$3,000.00	\$9,691.88
5	4	\$9,691.88	\$726.89	\$3,000.00	\$13,418.77
6	5	\$13,418.77	\$1,006.41	\$3,000.00	\$17,425.17
7	6	\$17,425.17	\$1,306.89	\$3,000.00	\$21,732.06
8	7	\$21,732.06	\$1,629.90	\$3,000.00	\$26,361.97



6. What is the balance in the account at the end of seven years? This balance is called the **amount**, or accumulated value, of an annuity. Describe two ways for finding the “accumulated” interest. Use the SUM function or the Σ toolbar button to determine the total interest earned.

The balance after seven deposits is \$26 361.97, which is the value in cell E8 of the spreadsheet. Calculate the accumulated or total interest by entering =SUM(C2:C8) into C9, using the Σ toolbar button, or by subtracting all the deposits from the final balance. The total interest earned is \$5361.97.

7. Change the payment in cell D2 to determine the payment that will yield

(a) \$30 000 after seven years

(b) \$15 000 after seven years

(a) To increase the balance to \$30 000, increase the payment. An annual payment of \$3200 yields a balance of \$28 119.43, an annual payment of \$3400 yields \$29 876.89, and an annual payment of \$3415 yields \$30 008.70, which is the amount closest to \$30 000.

(b) To decrease the balance to \$15 000, decrease the payment. An annual payment of \$2000 yields a balance of \$17 574.64, an annual payment of \$1750 yields \$15 377.81, and an annual payment of \$1705 yields \$14 982.38, which is the amount closest to \$15 000.

TECHNOLOGY TIP

Students will adjust only the value in cell D2, that is, the payment.

8. Change the interest rate to (a) 5%/a and (b) 10%/a. What is the balance in the account after seven years for each change in interest rate? Determine the total interest earned for each of these rates. Compare the interest rates to the total interest earned. What do you notice?

TECHNOLOGY TIP

Students must adjust the formula in cell C2, or suggest that students do the following:

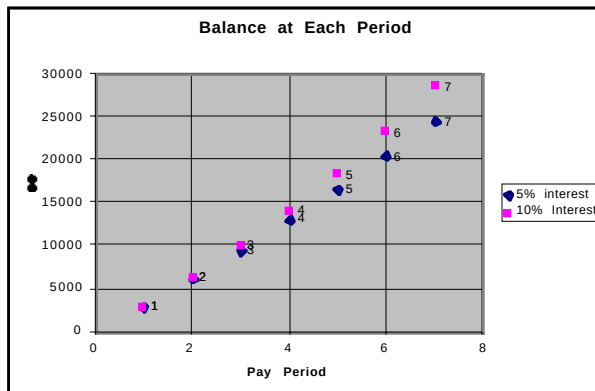
Enter the interest rate in a cell outside of the range, for example, H1, and then use $\$H\$1$ as an absolute cell reference in the formula in cell C2, so $=B2*\$H\$1$. By doing this, students do not need to copy cells to adjust the interest rate. Similarly, when the payment or deposit is constant, enter the payment into cell H2 and enter the formula $=\$H\$2$ in cell D2. This method is similar to stating a formula with variables before substituting.

(a) When the interest rate is 5%/a, the balance is \$24 426.03 and the total interest is \$3426.03.

	A	B	C	D	E	F	G	H
1	Period	Balance	Interest at 5%/a	Payment	New Balance		interest rate	0.05
2	1	\$0.00	\$0.00	\$3,000.00	\$3,000.00		payment	3000
3	2	\$3,000.00	\$150.00	\$3,000.00	\$6,150.00			
4	3	\$6,150.00	\$307.50	\$3,000.00	\$9,457.50			
5	4	\$9,457.50	\$472.88	\$3,000.00	\$12,930.38			
6	5	\$12,930.38	\$646.52	\$3,000.00	\$16,576.89			
7	6	\$16,576.89	\$828.84	\$3,000.00	\$20,405.74			
8	7	\$20,405.74	\$1,020.29	\$3,000.00	\$24,426.03			
9			\$3,426.03					

(b) When the interest rate is 10%/a, the balance is \$28 461.51 and the total interest is \$7461.51.

	A	B	C	D	E	F	G	H
1	Period	Balance	Interest at 10%/a	Payment	New Balance		interest rate	0.10
2	1	\$0.00	\$0.00	\$3,000.00	\$3,000.00		payment	3000
3	2	\$3,000.00	\$300.00	\$3,000.00	\$6,300.00			
...	...	...	...	...	...		...	...
8	7	\$23,146.83	\$2,314.68	\$3,000.00	\$28,461.51			
9			\$7,461.51					



10% is twice 5%. \$7461.51 is not twice \$3426.03. This relationship is not linear.

9. The annual payment is \$3000. What must be the minimum annual interest rate, to one decimal place, so that the balance in the account after seven years is \$30 000?

When the interest is 10%/a, the balance is \$28 461.51, from step 8 (b), so increase the interest rate to increase the balance to \$30 000. When the interest rate is 11.5%/a, the balance is \$29 804.76; at 11.7%/a, the balance is \$29 988.79; and at 11.75%/a, the balance is \$30 034.98. Then the interest rate should be about 11.7%/a so that the balance in the account after seven years will be \$30 000.

	A	B	C	D	E	F	G	H
1	Period	Balance	Interest at 11.7%/a	Payment	New Balance		interest rate	0.117
2	1	\$0.00	\$0.00	\$3,000.00	\$3,000.00		payment	3000
3	2	\$3,000.00	\$351.00	\$3,000.00	\$6,351.00			
...	...	...	...	...	...		...	...
8	7	\$24,161.85	\$2,826.94	\$3,000.00	\$29,988.79			
9			\$8,988.79					

10. Describe how you would modify the spreadsheet to find the annual balance for seven years if Mr. Watts made each payment at the beginning of the year. This type of annuity is called an **annuity due**. Modify your spreadsheet and compare it with your original spreadsheet.

In the ordinary annuity, no interest was earned during the first year, because the money was deposited in the account at the end of the year and there was no money in the account until the end of the year. For an annuity due, the payment is made at the beginning of the year, so interest is earned on the payment and on the balance from the end of the previous period. Therefore, switch the payment and interest columns. In cell D2, change the formula from $=B2*0.075$ to $=(B2+C2)*0.075$. For the ordinary annuity, the balance is \$26 361.97 and the total interest is \$5361.97; for the annuity due, the balance is \$28 339.11 and the total interest is \$7339.11.

	A	B	C	D	E	F	G	H
1	Period	Balance	Payment	Interest at 7.5%/a	New Balance		interest rate	0.075
2	1	\$0.00	\$3,000.00	\$225.00	\$3,225.00		payment	3000
3	2	\$3,225.00	\$3,000.00	\$466.88	\$6,691.88			
...	...	...	...	...	...		...	...
8	7	\$23,361.97	\$3,000.00	\$1,977.15	\$28,339.11			
9				\$7,339.11				

11. What factors would affect the total future value of an annuity? List as many factors as possible.

The size of the payment, the rate of interest, and when the payment is deposited are all factors affecting the total future value. (Later, students will see that the frequency of compounding has some effect and that the time when the money is deposited, or withdrawn, can have a serious effect.)

MANAGEMENT SUGGESTION

Post this list of factors along with students' spreadsheets that verify these conclusions. Start collecting these summaries to share with other classes who may use the same classroom, to display on Parents' Night, or to publish in the school's newsletter.

LEARNING SKILLS ASSESSMENT

Ensure that students know when and how to ask for assistance. Independence does not mean isolation.

Part 2: The Amortization Table

Think, Do, Discuss, p. 132

1. Enter the information in the spreadsheet in a new spreadsheet. What expression would you enter in cell A3 to increase the payment number?

Enter $=A2+1$ to increase the payment number.

2. The interest rate is 9%/a, compounded monthly. What is the interest rate of the interest conversion period? Does the payment interval correspond to the interest conversion period? Explain.

The interest rate for the conversion period is $9\% \div 12 = 0.0075$. Yes, the payments are monthly and the interest is compounded monthly.

3. Calculate the interest paid for the first month on the principal. Write an expression for this calculation and enter it in cell C3. How is the interest calculated for subsequent periods?

The interest paid for the first payment is $\$9500 (0.0075) = \$71.25$. Enter **=E2*0.0075** in cell C3. The interest for each subsequent period is calculated on the outstanding balance in the previous row, column E.

4. Subtract the interest for the first month from the payment. What remains? Write an expression to represent the remaining value and enter it in cell D3.

$$\begin{aligned}\text{principal paid} &= \text{payment} - \text{interest} \\ &= \$830.79 - \$71.25 \\ &= \$759.54\end{aligned}$$

Enter **=B3-C3** in D3.

5. What is the outstanding balance at the end of the first month? Describe how to calculate the outstanding balance for a given period. Write an expression for the outstanding balance and enter it in cell E3.

$$\begin{aligned}\text{outstanding balance} &= \text{previous outstanding balance} - \text{principal paid} \\ &= \$9500 - \$759.54 \\ &= \$8740.46\end{aligned}$$

Enter **=E2-D3** in E3.

6. Enter the values and formulas for the third row in your spreadsheet. Should any of these values remain constant? Explain. Use the **Fill Down** command to complete the amortization table.

The values in column B should remain constant, because the monthly payment does not change, unless the final payment is adjusted slightly.

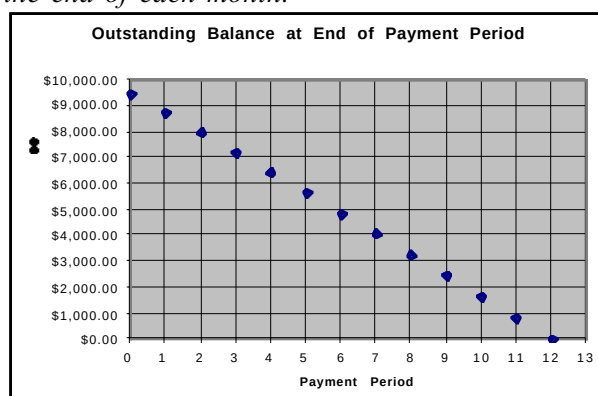
7. What is the outstanding balance after the 12th payment? How do you know that the loan has been completely paid? How can you adjust the last payment so that the outstanding balance will be \$0? Make the adjustment.

The outstanding balance is $-\$0.01$, that is, Lise overpaid by one cent. Lower the last payment to \$830.78.

	A	B	C	D	E	F	G	H
1	Payment Number	Payment	Interest Paid	Principal Paid	Outstanding Balance		payment	830.79
2	0	—	—	—	=H\$3		interest rate	0.0075
3	=A2+1	=H\$1	=E2*H\$2	=B3-C3	=E2-D3		outstanding balance	9500
4	=A3+1	=H\$1	=E3*H\$2	=B4-C4	=E3-D4		no. of periods	12
...	...	...	...	...	...			
13	=A12+1	=H\$1	=E12*H\$2	=B13-C13	=E12-D13			
14	=A13+1	\$830.78	=E13*H\$2	=B14-C14	=E13-D14			
15	Total	=SUM(B2:B14)	=SUM(C3:C14)	=SUM(D3:D14)				

	A	B	C	D	E	F	G	H
1	Payment Number	Payment	Interest Paid	Principal Paid	Outstanding Balance		payment	830.79
2	0	—	—	—	\$9,500.00		interest rate	0.0075
3	1	\$830.79	\$71.25	\$759.54	\$8,740.46		outstanding balance	9500
4	2	\$830.79	\$65.55	\$765.24	\$7,975.22		no. of periods	12
...	...	...	...	...	...			
13	11	\$830.79	\$12.32	\$818.47	\$824.59			
14	12	\$830.78	\$6.18	\$824.60	\$0.00			
15	Total	\$9,969.47	\$469.47	\$9,500.00				

8. Use the features of the spreadsheet to obtain a scatter plot that shows the outstanding balance at the end of each month.



9. Use the **SUM** function or the Σ toolbar button to find the total payments, the total interest paid, and the total principal paid. Do these values make sense? Explain.

The total of the payments is \$9969.47. The total interest paid is \$469.47. The total principal paid is \$9500.00. The sum of the interest and the principal equals the total of the payments. See the spreadsheet in step 7, above.

10. Estimate how long it would take Lise to repay the loan if she reduced each payment by almost one-half to \$415.40. Modify the spreadsheet to determine how long it would take her to repay the loan.

After the 12th payment, she still owes \$5195.51, more than half the original balance. After the 25th payment, she still owes \$75.49, which accrues interest of \$0.57. Therefore, her 25th payment should be $\$415.40 + \$75.49 = \$490.89$ or her 26th payment should be $\$75.49 + \$0.57 = \$76.06$. It would take her 25 months or 26 months to repay the loan if she reduces the payments.

	A	B	C	D	E	F	G	H
1	Payment Number	Payment	Interest Paid	Principal Paid	Outstanding Balance		payment	415.40
2	0	—	—	—	\$9,500.00		interest rate	0.0075
3	1	\$415.40	\$71.25	\$344.15	\$9,155.85		outstanding balance	9500
4	2	\$415.40	\$68.67	\$346.73	\$8,809.12		no. of periods	?
...	...	...	...	...	...			
14	12	\$415.40	\$41.77	\$373.63	\$5,195.51			
...	...	...	...	...	...			
27	25	\$415.40	\$3.65	\$411.75	\$75.49			
28	26	\$415.40	\$0.57	\$414.83	-\$339.34			

11. Suppose each payment is still \$830.79, but the interest rate changes to 12%/a, compounded monthly. How much more would Lise have to pay in interest? What would be her last payment?

If Lise repays the loan in 12 payments, her total interest is \$637.82 (\$639.50 – \$1.68) at the new rate and it was \$469.47 at the old rate, from step 9. Therefore, she pays $\$637.82 - \$469.47 = \$168.35$ more in interest. After her 12th payment, she still owes \$168.34, which accrues \$1.68 interest. Her 12th payment should be $\$830.79 + \$168.34 = \$999.13$.

	A	B	C	D	E	F	G	H
1	Payment Number	Payment	Interest Paid	Principal Paid	Outstanding Balance		payment	830.79
2	0	—	—	—	\$9,500.00		interest rate	0.01
3	1	\$830.79	\$95.00	\$735.79	\$8,764.21		outstanding balance	9500
4	2	\$830.79	\$87.64	\$743.15	\$8,021.06		no. of periods	12
...	...	...	...	...	...			
14	12	\$830.79	\$9.89	\$820.90	\$168.34			
15	13	\$830.79	\$1.68	\$829.11	–\$660.77			
16	Total		\$639.50					

12. Lise may have chosen to purchase a new car of the same make. To buy the new car, she would have had to borrow \$28 000. How long would it take her to repay this loan under the original terms? How much interest altogether would she pay?

After the 38th payment, she still owes \$821.92, which accrues \$6.16 in interest. She could increase her 38th payment, but this would almost be double the amount. Her 39th payment should be $\$821.92 + \$6.16 = \$828.08$. It would take 39 months to repay the loan, and she would pay a total of \$4398.11 in interest.

	A	B	C	D	E	F	G	H
1	Payment Number	Payment	Interest Paid	Principal Paid	Outstanding Balance		payment	830.79
2	0	—	—	—	\$28,000.00		interest rate	0.0075
3	1	\$830.79	\$210.00	\$620.79	\$27,379.21		outstanding balance	28 000
4	2	\$830.79	\$205.34	\$625.45	\$26,753.76		no. of periods	?
...	...	...	...	...	...			
40	38	\$830.79	\$12.30	\$818.49	\$821.92			
41	39	\$830.79	\$6.16	\$824.63	–\$2.70			
42	Total		\$4,398.11					
43	40	\$830.79	–\$0.02	\$830.81	–\$833.51			

13. Suppose the monthly payment is \$415.40. How long would it take for Lise to repay this \$28 000 loan? How much total interest would Lise pay?

After the 94th payment, she still owes \$105.87. Her 94th payment should be $\$415.40 + \$105.87 = \$521.27$. It would take 94 months to repay the loan, and she would pay a total of \$11 153.47 in interest. **Note:** It is unlikely that a financial institution would allow such a long repayment time.

1	Payment Number	Payment	Interest Paid	Principal Paid	Outstanding Balance		payment	415.40
2	0	—	—	—	\$28,000.00		interest rate	0.0075
3	1	\$415.40	\$210.00	\$205.40	\$27,794.60		outstanding balance	28 000
4	2	\$415.40	\$208.46	\$206.94	\$27,587.66		no. of periods	?
...	...	...	...	...	...			
96	94	\$415.40	\$3.88	\$411.52	\$105.87			
97	Total		\$11,153.47					
98	95	\$415.40	\$0.79	\$414.61	-\$308.74			

14. What would you tell someone who is purchasing a car about minimizing the total interest paid on a car loan?

Suggest that the purchaser increase the monthly payment, make as large a down payment as possible, and seek a lower rate of interest.

MANAGEMENT SUGGESTION

You might collect the answers to this step to check students' communication skills. You could read to the class (anonymous) solutions at levels 2, 3, and 4 at the beginning of the next class.

Post the collected suggestions along with the spreadsheets that verify these conclusions. Add this list to the collection of information to be used in a forum.

Key Ideas

An artistic student could create a poster showing the terms and the definitions, but not matching. You could quiz students by asking them to choose a term and name its matching definition. The first student to name two matching terms and definitions "wins."

Example 3

Have students investigate the interest rates that retail stores and financial institutions charge for their credit cards. Add this information to the collection to be presented at a forum.

Additional Example: Functions and Relations

This is an alternative solution for Example 1, text page 134.

Samantha has a part-time job and is saving money for a trip to Europe. She deposits \$600 at the end of every three months in a savings plan that pays 8%/a, compounded quarterly. Determine the quarterly balances for the first 30 months using the TI-83 Plus calculator.

Solution

The new balance at the end of each quarter is the sum of the balance at the beginning of the quarter, plus 2% interest, and a \$600 payment. Therefore, calculate each new balance using a recursive equation. Put the calculator in sequence graphing mode by pressing **MODE** and then scrolling down and across to **Seq**. Press **ENTER**.

At the end of the first period ($n\text{Min}=1$), the balance, $u(n\text{Min})$, will be 600. For each subsequent month, the new balance will be the sum of the balance at the beginning of the period, plus 2% interest, and the \$600 periodic payment. Therefore, $u(n)=600+1.02u(n-1)$. Press **Y=** and enter this information. Then change any values in the TABLE SETUP screen so that it is like the one shown (**2nd WINDOW**).

```

Plot1 Plot2 Plot3
nMin=1
u(n)=600+1.02u(
n-1)
u(nMin)=600
v(n)=
v(nMin)=
w(n)=

```

TABLE SETUP		
TblStart=1	■	
ΔTbl=1		
IndEnt:	Auto	Ask
Defend:	Auto	Ask

Press **2nd** **GRAPH** to see the quarterly balances for the first seven quarters. Scroll down to see more of the table. Notice that the fixed decimal mode has been set to two decimal places.

n	u(n)	
1.00	600.00	
2.00	1212.0	
3.00	1836.2	
4.00	2472.0	
5.00	3122.4	
6.00	3784.9	
7.00	4460.6	
u(n)=3784.872578		

n	u(n)	
5.00	3122.4	
6.00	3784.9	
7.00	4460.6	
8.00	5149.8	
9.00	5852.8	
10.00	6569.8	
11.00	7301.2	
u(n)=6569.8326		

Additional Example: Functions

This is an alternative solution for Example 4 (b), text page 137.

Mina took out a mortgage of \$75 800 to purchase a house in Massey. She agrees to repay the mortgage over 20 years with equal monthly blended payments of \$653.02 each. The interest rate is 8.4%/a, compounded monthly. Her final payment must be increased by \$0.25 to pay off the mortgage. Determine the outstanding balance for each of the first ten months, using the TI-83 Plus calculator.

Solution

As in the Additional Example: Functions and Relations above, select the sequence graphing mode. In this case, you will also use a recursive equation to calculate the outstanding balance after each payment.

To have the numbers rounded to two decimal places, press **MODE**, scroll down to **Float**, scroll right to 2, and press **ENTER** **2nd** **MODE**.

Press **7** **5** **8** **0** **0** **ENTER**.

Update the balance by multiplying the previous balance by 1.007, that is, the previous balance, plus 0.7% interest. Then subtract the payment.

Press **1** **1** **0** **0** **7** **×** **2nd** **(-)** **6** **5** **3** **0** **2**.

Press **ENTER** ten times to see the next nine subsequent balances.

75800	75800.00
1.007*Ans-653.02	75677.58

75305.15
75179.27
75052.51
74924.85
74796.31
74666.86
74536.51

Practise, Apply, Solve 2.5

Questions with a University Focus:

1–5, 7–10, 12, 13, 15, 17–21

Questions with a Functions Focus:

1–9, 11, 13, 15–17, 19

Management Suggestion

Before beginning any problems dealing with money, students should ask themselves the following questions: Are there one or more deposits or payments? Do I know, or want to know, the sum now, or later? Is the interest conversion period the same as the payment period? Is the annuity an ordinary annuity or an annuity due?

5. Communication, p. 138

Management Suggestion

Let pairs discuss possible scenarios and select the most reasonable one. Each partner should solve the question and then the other partner can peer-edit the solutions, which should encourage students to strive for clarity. Use the TVM Solver to check answers quickly (see section 2.9). When students see the teacher using the calculator, their interest will rise.

Achievement Chart Connection

Students must understand that each payment is a blend of interest and principal, and that each payment reduces the principal and therefore the interest. Students must correctly use the terms and apply the concepts.

Level 3 Performance

Responses should indicate a clear understanding of the concepts, present a reasonable, logical explanation, and integrate writing and mathematical expressions. Students should use terms correctly and predict a reasonable estimate, more than $6500 \div 16$, but less than

$$(6500 \div 16) + \frac{10\%}{4} \times 6500.$$

7. Knowledge and Understanding, p. 139

Management Suggestion

Students must know how to solve this problem manually, without relying on technology. Students must be skilled with their scientific calculators and should be able to construct an amortization table in (a), without clearing the screen at any step. One partner can use the calculator, while the other records the values. When partners switch roles, change the payment to \$650 or the interest rate to 10%/a, compounded semiannually. Comment on the difference between terms, (b) and (c). When you ask students to redo or practise similar problems, make sure the term is not so long that calculation becomes tedious, but is long enough so that students may see a pattern.

Achievement Chart Connection

Students are expected to calculate the interest on the outstanding balance, subtract this interest from the payment, and subtract the result from the balance. They should repeat these steps until the balance is less than or equal to 0. Students can use the scientific calculator.

Level 3 Performance

The repayment schedule must be in a conventional format and calculations must be accurate. Students should identify variables and explain the solutions for (b) and (c).

Direct students to assist their classmates, but help them to understand the difference between solving the question for a classmate and supporting a classmate. When students assist others, they should encourage, not condescend.

13. Thinking, Inquiry, Problem Solving, p. 140

Management Suggestion

Teams should first discuss possible approaches. A whole class discussion should produce an efficient approach. Record the plan in abbreviated form on the board for reference.

Achievement Chart Connection

Students must select the appropriate tools and organize a plan to compare both accumulating amounts.

Level 3 Performance

Responses should include

- variables for both cases
- a table, or a spreadsheet showing, the values of Bev's deposits from her 15th to 26th birthday and the values of Cindy's deposits from her 26th birthday to her 90th birthday
- a logical conclusion based on the data in the spreadsheets

15. Application, p. 140

Management Suggestion

Teams should first discuss possible approaches. A whole class discussion should produce an efficient approach. Suggest they ignore the down payment at first, since it is better to work backward. Record the plan in abbreviated form on the board for reference.

Achievement Chart Connection

Students apply the concepts in an unfamiliar situation where the original balance is unknown and the down payment is given as a percent. The balance represents 90% of the cost of the boat.

Level 3 Performance

Responses should include

- a repayment schedule in conventional format
- identification of variables
- an initial estimate of the original price of the boat that is less than $\$225(36) = \$8100$
- an accurate calculation of the original price of the boat

19. Check Your Understanding, p. 142

Management Suggestion

See the Management Suggestion for question 7. Though students cannot use a spreadsheet, suggest they review step 10 of the *Think, Do, Discuss*, Part 1 for the design of the repayment schedule.

The Chapter Problem Answers are on p. 2-67 of the *Teacher Resource*.

Challenge 1

Students could respond to this challenge at any time through research, but the answers will make more sense if they do this challenge when they do section 2.12.

When students research the Chapter Problem, remind them to read Challenge 1 and to research mortgages as well.

Challenge 2

The challenge will make sense after they have become familiar with the financial terminology, although section 1.8 provides the tools for calculating. Assign this challenge after section 2.8.

The Challenge Answers are on p. 2-82 of the *Teacher Resource*.