

2.12 Using Technology to Analyze Canadian Mortgages

Specific Expectations

The following expectations are common to the Functions and Relations course and the Functions course.

- FA3.02 describe the manner in which interest is calculated on a mortgage (i.e., compounded semi-annually but calculated monthly) and compare this with the method of interest compounded monthly and calculated monthly
- FA3.04 analyse the effects of changing the conditions of a mortgage (e.g., the effect on the length of time needed to pay off the mortgage of changing the payment frequency or the interest rate)
- FA3.05 communicate the solutions to problems and the findings of investigations with clarity and justification
- OC3.02 present problems and their solutions to a group, and answer questions about the problems and the solutions
- OC3.03 communicate solutions to problems and to findings of investigations clearly and concisely, orally and in writing, using an effective integration of essay and mathematical forms

Evidence of Learning

Students should be able to

- create a mortgage amortization schedule algebraically, in a spreadsheet, or using the TI-83 Plus calculator
- select the appropriate tool to solve a problem
- anticipate the effects of changing the conditions of a mortgage or annuity
- gather information from a variety of sources, analyze the information, and present the findings either orally or in writing, for example, in a display or as an essay

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Time Required Functions and Relations:

75 min

Time Required Functions: 110 min

Materials and Equipment

- TI-83 Plus calculator
- spreadsheet software

Overview

- Apply concepts from, and skills learned in, sections 2.7, 2.8, and 2.10.
- Change the variables—interest rate, early down payment, payment frequency—of a mortgage or a long-term savings plan.
- Recognize the effects of changing the conditions of a mortgage or a long-term savings plan.

Analyzing Canadian Mortgages

Students will analyze typical financial situations, such as mortgages, car loans, and savings plans, using appropriate technology.

Management Options

Groups – Set up three workstations as follows: one with a computer with spreadsheet software, another with one or two TI-83 Plus calculators, and yet another with paper, pencil, and two scientific calculators. Students should do at least one example, if not both examples, at each station.

Teacher Demonstration – Discuss the questions in the *Consolidate Your Understanding*, before students record their responses to these questions in their notebooks.

Independent Work – Independent work would be appropriate for those students who are very independent learners, but individuals could rotate through the stations as well.

Consolidate Your Understanding, p. 190

1. Compare using the **TVM Solver** and using a spreadsheet for solving mortgage and general annuity problems. Does one form of technology have more advantages than the other?

The **TVM Solver** is easier to use than a spreadsheet if you need to calculate only specific information, for example, the future value of an annuity. There are only seven pieces of information to enter in the **TVM Solver**. When creating a spreadsheet, you must calculate the equivalent nominal interest rate and enter all the appropriate formulas, as well as other data. (See also the response to question 2.)

2. Compare using a spreadsheet and using the TI-83 Plus calculator for producing repayment schedules and amortization tables. Does one form of technology have more advantages than the other?

TVM Solver	Spreadsheet Software
completely portable	computer can be portable
can be used anywhere	students may have limited access to a computer
the user may see only portion of a spreadsheet at a time	the user can see most of or all of a spreadsheet when viewing the spreadsheet
the user cannot add descriptive information to results produced by the TVM solver	the user can enter descriptive information, for example, titles for columns
the calculator calculates the equivalent nominal interest rate	the user calculates the equivalent nominal interest rate
the user enters only 7 pieces of information	the user enters all formulas and some data and must perform commands to copy data
to find the sum of, for example, payments, takes several steps	easy to find the sum of any column or row
easy to adjust data	easy to adjust formulas
printing facility may be less accessible, and several steps needed to print screen	if printer available, then easy to print copy of spreadsheet
difficult to compare different values for the same parameter	easy to compare different schedules in the same spreadsheet

3. Discuss some of the factors that could affect the time it might take for someone to amortize a mortgage.

Some of the factors that affect the time it takes to amortize a mortgage are the size of the down payment, the interest rate, the size of each payment, the cost of mortgage insurance, and paying down the principal. Increasing the original down payment decreases the principal, which decreases the amortization period. A lower interest rate will also decrease the amortization period. If each payment is larger, the principal will be repaid faster, and the amortization period will decrease. If mortgage insurance is not required, more money can be applied to each payment. If the mortgagor can afford to pay up to 20% of the original mortgage every year, then the principal will be dramatically reduced, which also decreases the amortization period.

Additional Example: Functions and Relations

Ms. Cvar takes out a mortgage of \$98 000 to purchase a house. She will repay the mortgage with equal monthly blended payments over 20 years. The initial term is five years at 8.5%/a, compounded semiannually. At the end of the five years, she renews the mortgage for two years at 9.5%/a, compounded semiannually. During this time, interest rates fall substantially. She then renews the mortgage for ten years at 6.8%/a, compounded semiannually. The interest rate for the final three-year renewal term is 8%/a, compounded semiannually.

- (a) Determine the monthly payment for the first term.
- (b) At the end of each term, determine the outstanding balance, the amount of the principal paid, and the amount of interest paid. Use this information to determine the payment for each subsequent term.
- (c) What is the total cost of the house, including the finance charge?

Solution

- (a) Note, for the first term, even though it is only five years, the mortgage is amortized for 20 years, therefore, $N = 20 \times 12$ or 240.

The monthly payment for the first term is \$841.39.

```
N=240
I%=8.5
PV=98000
PMT=-841.38801...
FV=0
P/Y=12
C/Y=2
PMT: [ ] [ ] [ ] BEGIN
```

- (b) At the end of the first term, the outstanding balance is \$86 193.85, a total of \$11 806.15 has been paid on the principal, and \$38 677.13 has been paid in interest.

```
ΣInt(1,60,6)
-38677.12704
ΣPrn(1,60,6)
-11806.15386
bal(60,6)
86193.84614
```

At this time, she owes \$86 193.85 on the principal and the amortization period is now 15 years, so $N = 15 \times 12$ or 180. She renews the mortgage for two years at 9.5%/a, compounded semiannually. The payments are still monthly.

The monthly payment for the second term is \$890.58.

```
N=180
I%=9.5
PV=86193.85
PMT=-890.57739...
FV=0
P/Y=12
C/Y=2
PMT: [ ] [ ] [ ] BEGIN
```

At the end of the second term, the outstanding balance is \$80 379.34, a total of \$5814.51 has been paid on the principal, and \$15 559.35 has been paid in interest.

```
ΣInt(1,24,6)
-15559.35213
ΣPrn(1,24,6)
-5814.505378
bal(24,6)
80379.34462
```

At this time, she owes \$80 379.34 on the principal. The amortization period is now 13 years, so $N = 13 \times 12$ or 156. She renews the mortgage for 10 years at 6.8%/a, compounded semiannually.

The monthly payment for the third term is \$773.41.

```
N=156
I%=6.8
PV=80379.35
PMT=-773.40671...
FV=0
P/Y=12
C/Y=2
PMT: [ ] [ ] [ ] BEGIN
```

At the end of the third term, the outstanding balance is \$25 157.42, a total of \$55 221.92 was paid on the principal, and \$37 586.88 was paid in interest.

```
ΣInt(1,120,6)
-37586.88153
ΣPrn(1,120,6)
-55221.92391
bal(120,6)
25157.42609
```

At this time, she owes \$25 157.42 on the principal. The amortization period is now 3 years, so $N = 3 \times 12$ or 36. She renews the mortgage for 3 years, at 8%/a, compounded semiannually.

The monthly payment for the third term is \$786.83.

```
N=36
I%=8
PV=25157.42
PMT=-786.83235...
FV=0
P/Y=12
C/Y=2
PMT: [ ] [ ] [ ] BEGIN
```

At the end of term 4, the outstanding balance was $-\$0.000\ 016$ ($1.60\text{E}-5$), a total of \$25 157.42 has been paid on the principal, and \$3168.54 has been paid in interest.

```
ΣInt(1,36,6)
-3168.544652
ΣPrn(1,36,6)
-25157.41998
bal(36,6)
1.6E-5
```

- (c) The total interest paid is \$38 677.13
 + \$15 559.35 + \$37 586.88 + \$3168.54
 = \$94 991.90. Therefore, the total cost of
 the house including interest is
 \$98 000 + \$94 991.90 = \$192 991.90.

Additional Example: Functions

In Example 4 of section 2.5, Mina repays a \$75 800 mortgage by making equal monthly payments of \$653.02 each over 20 years. The interest rate is 8.4%/a, compounded monthly. She must increase her final payment by \$0.25 to close the mortgage.

- (a) Calculate, to the nearest cent, the new monthly payment if the interest rate is 8.4%/a, compounded semiannually.
 (b) Explain why the monthly payment in (a) is less than \$653.02.
 (c) Determine the annual mortgage rate, compounded semiannually, that is equivalent to 8.4%/a, compounded monthly.

Solution

- (a) The new monthly payment is \$646.18.

A calculator screen displaying the following values: N=240, I%=8.4, PV=75800, PMT=-646.18287..., FV=0, P/Y=12, C/Y=2, and PMT:BEGIN.

- (b) The more frequently interest is compounded, the more interest is calculated. The amortization period and the principal do not change. The payments are a blend of interest and principal. Therefore, the less frequently interest is compounded, the less interest is paid by each blended payment.
 (c) annual rate, i ,
 compounded semiannually = 8.4%/a,
 compounded monthly

$$(1+i)^2 = \left(1 + \frac{8.4\%}{12}\right)^{12}$$

$$(1+i)^2 = (1.007)^{12}$$

$$1+i = (1.007)^6$$

$$1+i \doteq 1.042\,741\,896$$

$$i = 0.042\,741\,896$$

$$4.2741\,896\%/half\ year$$

$$= 8.548\,379\,22\%/a, \text{ compounded semiannually}$$

Therefore, 8.548%/a, compounded semiannually, is approximately equivalent to 8.4%/a, compounded monthly.

Practise, Apply, Solve 2.12

Questions with a Functions and Relations Focus:

1–8, 12, 13–17, 19

Questions with a Functions Focus:

1–8, 12, 13, 16, 17, 19

EXTENSION #3 and #5

Include selected solutions to these questions in the student forum.

6. Knowledge and Understanding, p. 192

Management Suggestion

Individual partners can work through each part of the question and then exchange solutions after completing each part to check calculations and discuss before doing the next part.

Achievement Chart Connection

Students are expected to match the data with the appropriate variables and choose a technological tool.

Level 3 Performance

Responses should include

- a justification for the variables used
- a sketch or printed copy of the results produced by technology
- a correct calculation and explanation of the interest saved
- conclusions supported by evidence
- clear communication

13. Communication, p. 194

Management Suggestion

By this point, students should already have a comprehensive list of questions about mortgages, from their work for the Chapter Problem and Challenge 1. Students should verify their research findings by comparing the information to what they have learned from this chapter. Students should share information, but the report must be

individual. Allow class time for peer-editing.

Achievement Chart Connection

Students can gather information from mathematical sources, from their own personal classroom experience, and from financial institutions. They must analyze the information and write a report that integrates mathematical and narrative forms.

Level 3 Performance

Responses should include accurate, current information and the correct use of financial terminology. Students' suggestions for minimizing the total cost of paying for a home should include the following: maximize the down payment, maximize the payments, increase the frequency of payments, maximize early additional payments to reduce the principal, negotiate the interest rate, and reduce real estate fees.

Students should clearly communicate their ideas in student-friendly language.

13. Have students actually create and print an amortization schedule for a family member or a friend.

16. Application, p. 194

Management Suggestion

Individual partners can solve each part and then exchange their solutions for peer-editing. Review calculating percent increase with the class before student do (c).

Achievement Chart Connection

Students apply mortgage calculations to examine an interesting historical perspective.

Level 3 Performance

Responses should include

- a justification for the variables used in (a) and (b)
- sketches of the results produced using technology in (a) and (b)
- an explanation and correct calculation of the percent increase
- clear communication of the conclusion

17. Thinking, Inquiry, Problem Solving, p. 194

Management Suggestion

Ensure students understand the financial vocabulary in this question, including the

meaning of "3% cash back mortgage offer." Discuss methods for calculating the interest and the balance. Remind students that they must compare the total costs after five years for both options; they should not compare any single factor for the options.

Achievement Chart Connection

Students are expected to identify two cases and to organize the data accordingly. They must understand the connection between the 25-year amortization period and the 5-year term and how to enter this information in the chosen technological tool.

Level 3 Performance

- justify the variables used in both options
- calculate the payment, interest, and balance for both options
- explain the calculation of total cost
- clear communication of the conclusion

19. Check Your Understanding, p. 195

Management Suggestion

Conduct a class brainstorming session about the factors that determine the monthly payment on a mortgage. The resulting list of factors should also explain how each factor determines the monthly payment on a mortgage. Add this information to the collection for the student forum.

The Chapter Problem Answers are on p. 2-67 of the *Teacher Resource*.

Challenge 1

Students could respond to this challenge at any time through research, but the answers will make more sense once they have learned about mortgages. Ask them to do this challenge before they start this section or during this section.

Challenge 2

The challenge will make sense after they have become familiar with the financial terminology, although section 1.8 provides the tools for calculating. If you have not already assigned this challenge, you could assign it now.

The Challenge Answers are on p. 2-82 of the *Teacher Resource*.