

3.2 Functions: Concept and Notation

Specific Expectations

These expectations are common to the Functions and Relations course and the Functions course.

- OC2.01 define the term function
- OC2.02 demonstrate facility in the use of function notation for substituting into and evaluating functions

Evidence of Learning

Students should be able to

- describe the characteristics of a function
- interpret function notation to find the value of a function for a specific value of the variable
- use the vertical line test to determine whether a relation is a function or not
- describe a function as a set of ordered pairs, a graph, an equation, a description in words, an arrow diagram, or $f(x)$
- determine the domain and range of a function from a graph, an equation, or the context of a problem

Text Page 225

Time Required Functions and Relations: 120 min

Time Required Functions: 150 min

Overview

- Describe a function in terms of input, output, independent variable, and dependent variable.
- Introduce $f(x)$ notation.
- Model a function as a diagram, a graph, or an algebraic expression.
- Introduce the vertical line test as a quick visual check to determine if a relation is a function.

Functions

A variety of examples provide students with opportunities to understand the idea of a function and to recognize and use different methods for describing functions.

MANAGEMENT SUGGESTION

Encourage students to recognize several different models of the same function by having a short game. Before class, prepare two sets of cards. Each card in set A should have a different “blank” model, for example, $f(x)$ notation, a table, a graph, or an input/output diagram. (There will be only one function at a time.) On each card in set B write a letter of the alphabet, the number of cards depending on the number of members in each team. Group students into teams and assign each team member a letter. Draw one card from set A for each team. Write a description of the function in words on the board or the overhead. Pause so that students can consider how to use the model to describe this function and then draw a few cards from set B to determine which members of each team will play this round. Let the designated players put their responses on the board.

Management Options

Groups – Allow each group of students to discuss Part 1. When each group is finished discussing this part, ask them to give you one or two written questions that they may have about this part. This activity will allow students to clarify understanding and will provide a basis for class discussion about the concepts.

After each group has had time to read Part 2, you might present a question on the overhead for students to answer individually. Show a model solution for self-evaluation.

Ask students to read Part 3 to themselves. Pair students and give them a few minutes to ensure that each partner understands the vertical line test. Then present a group of graphs of relations and ask students to select any graphs that do not represent functions. Students should draw vertical lines to justify their choices.

Teacher Demonstration – You may choose to present the ideas of this section using other examples, for example, $f(x)$ is like a mould for jelly in that you can use different liquids as input but each liquid will fill the mold in exactly the same way.

Independent Work – If resources are available, students might use software to individually develop understanding of functions. Discuss as a class to consolidate understanding and create a bulletin board display (mind map) which will remind everyone of the key ideas about functions.

EXTENSION

Ask students to consider the following questions:

1. Are all linear relations functions?
2. Why might a mathematician say that the only useful relations are functions?

Part 3: The Vertical Line Test Think, Do, Discuss, text page 229

1. (a) *Why is it no longer possible to identify every person in the group only by the length of his or her footprint?*

For the 33 cm long footprint, there are two people with different heights.

- (b) *How do you know this relation is not a function?*

One value for the independent variable, 33, gives two different values, 178 and 183, of the dependent variable.

2. (a) *Place a ruler on the vertical axis (the height axis). Move the ruler slowly to the right. As you move the ruler to the right, note the number of points on the graph of the relation that lie along the edge of the ruler at any one time.*

(Students should notice just one point as they move their rulers until the ruler passes through the points with x -value 33.)

- (b) *Which two points lie on the same line? Explain why these two points show that this relation is not a function.*

(33, 178) and (33, 183) lie on the same vertical line. For the same value of the independent variable, there are two values of the dependent variable, so the relation does not produce a unique value of the dependent variable for each value of the independent variable and it is not a function.

- (c) *Draw a line through these two points. Describe the direction of this line.*

The line is vertical.

- (d) *In your own words, describe a test to determine whether a relation is a function using its graph.*

The relation is not a function if a vertical line crosses the graph of the relation at more than one point.

Consolidate Your Understanding, text page 230

1. *How can you tell if a relation is a function?*

Each value for the independent variable must correspond to, or predict, a unique value for the dependent variable.

2. *Why is it useful when a relation is a function?*

You may predict the unique value of a function.

3. How can you determine the domain and range of a function from its graph?

Draw a rectangle around the function and identify the length (range) and width (domain) of the rectangle.

4. A function is given by $f(x) = 3x - 5$. Describe this function as a rule.

The output is five less than three times the input.

5. What does $f(2)$ mean?

the value of the dependent variable when the independent variable has a value of 2

6. How can you tell from the graph of a relation whether it is a function or not? Explain.

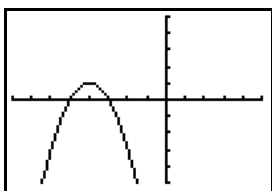
The relation is not a function if you can identify more than one y -value for a single x -value or if a vertical line crosses the graph of the relation in more than one place.

Additional Example: Functions

- (a) Graph the relation $y = -x^2 - 8x - 15$.
 (b) Is this relation a function? Explain your answer.
 (c) What is the range of the relation?

Solution

- (a)



- (b) A vertical line does not cross the graph in more than one place, so this relation is a function.
 (c) $y \leq 1$

Additional Example: Functions and Relations

Find an expression for $f(x + 4)$, given $f(x) = x^2 + 3x - 1$.

Solution

$$\begin{aligned} f(x) &= x^2 + 3x - 1 \\ &= (x + 4)^2 + 3(x + 4) - 1 \\ &= x^2 + 8x + 16 + 3x + 12 - 1 \\ &= x^2 + 11x + 27 \end{aligned}$$

ASSESSMENT SUGGESTION

KNOWLEDGE AND UNDERSTANDING

Ask students to work with a partner. On the overhead, present one of the following as a model of a relation: a graph, a table, a diagram, or an algebraic equation. After students have had a moment to consider the model, ask partners "A" or "B" to tell their partners whether the model represents a function or not and to explain why.

Practise, Apply, Solve 3.2

Questions with a University Focus:

2, 4–11, 15–22, 24, 25

Questions with a University/College Focus:

1–7, 9, 10, 13–16, 18, 19, 22, 24

9. Knowledge and Understanding, p. 236

Management Suggestion

Give students a specific time, for example, 3 min, to sketch the relation (i) for the first relation. Then show an overhead with the sketch so that students can self-check, and discuss any questions. Repeat these steps for ii and iii until students seem confident.

Achievement Chart Connection

Students must demonstrate understanding of the definition of a function and the domain and range of a relation. They must show that they can draw a graph to describe a function.

Level 3 Performance

Students must draw accurate graphs, quickly decide whether a relation is a function, and promptly identify the domain and range from a graph.

19. Communication, p. 238

Management Suggestion

Pair students. With the class, develop a model solution so that students can judge their own work.

Achievement Chart Connection

Students must explain their decisions and justify their reasoning. They should use mathematical language, graphs (visuals), and conventions.

Level 3 Performance

Students must draw accurate graphs that are clearly labelled and that have reasonable scales. Students should correctly identify the domain and range and express them using conventional notation. They should provide evidence that they understand the concept of a function.

21. Thinking, Inquiry, Problem Solving, p. 238

Management Suggestion

Give individuals or pairs time to consider the problem, in class or as an assignment, and then provide an opportunity in class for students to present solutions.

Achievement Chart Connection

Students are required to interpret information and form conclusions, judge the reasonableness of results, and make arguments of some complexity.

Level 3 Performance

Students should provide reasonable answers for the domain, the range, and the importance of a function. They should also draw accurate and clearly labelled graphs, which show that employees' earnings increase as sales increase.

22. Application, p. 238

Management Suggestion

Have students work independently. Tell them beforehand that you will collect solutions and respond if students need "a hint." Write hints on the student's work—the presence of hints might influence assessment.

Achievement Chart Connection

Students are required to apply their knowledge of Chapter 2 (compound interest) and identify the function in this context of compound interest.

Level 3 Performance

Students' graph should include appropriate scales and accurate values. Students should identify a reasonable range and domain.

24. Check Your Understanding, p. 239

Management Suggestion

Ask students to provide a written response to this question at the end of class, ensuring that they have been given sufficient time to review and edit their responses. Before the following class, provide individuals with comments or suggestions if students' misunderstanding is evident in their responses.

The Chapter Problem—Cryptography

Encourage students to think about what the possible remainders would be when any positive integer is divided by 26. What is the smallest possible value? the largest possible value? What are the possible values after 1 is added? Have students relate these values back to original conversion from a letter to a number.

The Chapter Problem Answers are on p. 3-45 of the *Teacher Resource*.

Challenge 1

Remind students that they should be able to

- create a frequency table for letters of the alphabet in any paragraph and for the coded text.

Challenge 2

Encourage students to create an input/output diagram to illustrate this model of an error-correcting code.

The Challenge Answers are on p. 3-48 of the *Teacher Resource*.