

## 3.7 Transformations and Function Notation

### Specific Expectations

These expectations are common to the Functions and Relations course and the Functions course.

- OC2.06 represent transformations (e.g., translations, reflections, stretches) of the functions defined by  $f(x) = x$ ,  $f(x) = x^2$ ,  $f(x) = \sqrt{x}$ ,  $f(x) = \sin x$ , and  $f(x) = \cos x$ , using function notation
- OC2.07 describe, by interpreting function notation, the relationship between the graph of a function and its image under one or more transformations
- OC2.08 state the domain and range of transformations of the functions defined by  $f(x) = x$ ,  $f(x) = x^2$ ,  $f(x) = \sqrt{x}$ ,  $f(x) = \sin x$ , and  $f(x) = \cos x$

### Evidence of Learning

*Students should be able to*

- describe transformations applied to a base function by interpreting the function's equation
- graph the transformation of a function, by interpreting the function's equation, rather than using a table
- distinguish transformations that affect the independent variable from transformations that affect the dependent variable
- describe transformations algebraically

### Text Page 278

**Time Required U:** 150 min

**Time Required U/C:** 200 min

### Materials and Equipment

- TI-83 Plus calculator or graphing software

### Overview

- Recognize and work with transformations of a general function. Students will build on their work with transformations of quadratic relations in grade 10.
- Describe the transformations of a function using graphs, diagrams, and algebraic expressions.
- Learn more about domain, range, inverse, horizontal and vertical translations, horizontal and vertical stretches, and reflections.

#### MANAGEMENT SUGGESTION

Review the different transformations that can be applied to a parabola, emphasizing the effect of the transformations on the  $x$ - and  $y$ -values.

#### MANAGEMENT SUGGESTION

Give students specific sets of relations to graph so that they may see the effect of individual transformations applied to a circle. The graphing software they use should accept equations in non-function form.

Here are two sets of relations.

$$x^2 + y^2 = 100$$

$$(x - 2)^2 + y^2 = 100$$

$$(x + 3)^2 + y^2 = 100$$

OR

$$x^2 + y^2 = 100$$

$$x^2 + (2y)^2 = 100$$

$$x^2 + (0.25y)^2 = 100$$

### Management Options

**Pairs** – Allow students to explore the effect of applying transformations to specific functions using graphing software or using a graphing calculator, as outlined in the *Think, Do, Discuss* sections of this lesson. At regular intervals, draw the class into a “summarizing” discussion to ensure that all pairs of students are reaching valid conclusions.

**Teacher Demonstration** – Using a TI-83 Plus and view screen or a projection of a graphing software screen, guide students through an exploration of the effects of various transformations applied to a given function. Using a circle as a “base” makes it very easy for students to recognize the effect of transformations. The next step is to read transformations from an equation. Frequently refer to whether the transformation is applied to the independent variable or to the dependent variable.

**Independent Work** – Provide each student with a graphing calculator, and allow time to for students to work through the *Think, Do, Discuss* sections. Invite students to submit a short, that is, two sentences, summary of what they have learned and understand. They could submit a summary every 20 min. Quickly scan the submissions as students hand them to you. For individual students, comment verbally or in writing, as necessary, to develop students’ understanding.

#### EXTENSION

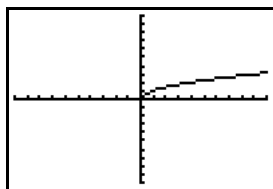
Ask students to consider the following questions:

1. Have you seen a television ad or a corporate logo that could have been produced using transformations? Explain.
2. What is an ellipse and how is it related to a circle?

## Part 1: The Graph of $y = af(x - p) + q$ Compared to the Graph of $y = f(x)$

**Think, Do, Discuss, text page 278**

1. Using a TI-83 Plus, press **ZOOM** **6** to set the standard window.
2. (a) Graph the base function  $y = \sqrt{x}$ . Enter this function into **Y1** in the equation editor.



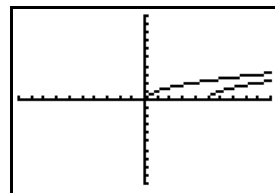
- (b) What is the domain? What is the range?

The domain is  $\{x \mid x \geq 0, x \in \mathbf{R}\}$ . The range is  $\{y \mid y \geq 0, y \in \mathbf{R}\}$ .

- (c) What transformation would you apply to the graph of  $y = \sqrt{x}$  to obtain the graph of  $y = \sqrt{x-5}$ ? Sketch your prediction on graph paper.

Translate the graph 5 units to the right.

- (d) Graph  $y = \sqrt{x-5}$  using a graphing calculator to check your answer. Enter this function into **Y2** in the equation editor.



- (e) What is the domain of  $y = \sqrt{x-5}$ ? Explain. What is its range? Explain.

The domain is  $\{x \mid x \geq 5, x \in \mathbf{R}\}$ . All points are translated five units to the right, so the  $x$ -coordinates for all points change. The range is  $\{y \mid y \geq 0, y \in \mathbf{R}\}$ . There is no vertical change, so the  $y$ -coordinates for all points do not change.

- (f) One point on the graph of  $y = \sqrt{x}$  is (4, 2). What is the corresponding point on the graph of  $y = \sqrt{x-5}$ ? Explain the transformation.

The corresponding point is (9, 2), because the original point was translated 5 units to the right,  $4 + 5 = 9$ . There is no change for the  $y$ -coordinate, since the point did not move up or down.

- (g) Evaluate  $y = \sqrt{x-5}$  for  $x = 9$ . How does this answer correspond to your answer to (f)?

For  $x = 9$ ,

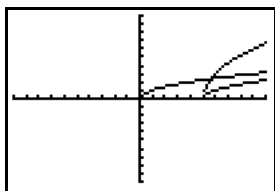
$$\begin{aligned} y &= \sqrt{x-5} \\ &= \sqrt{9-5} \\ &= 2 \end{aligned}$$

For  $x = 9$ ,  $y = 2$ . Point (9, 2) corresponds to the answer to (f).

3. (a) What transformations would you apply to the graph of  $y = \sqrt{x}$  to obtain the graph of  $y = 3\sqrt{x-5}$ ? Sketch your prediction on graph paper.

Translate the graph 5 units to the right and stretch it vertically by a factor of 3. It may be helpful to rewrite the equation as  $\frac{1}{3}y = \sqrt{x-5}$ .

- (b) Graph  $y = 3\sqrt{x-5}$  using a graphing calculator to check your answer. Enter this function into **Y3** in the equation editor.



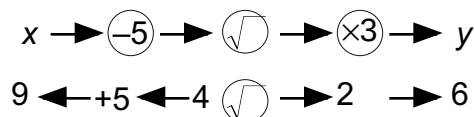
- (c) What is the domain of  $y = 3\sqrt{x-5}$ ? Explain. What is its range? Explain.

The domain is  $\{x \mid x \geq 5, x \in \mathbf{R}\}$ . The range is  $\{y \mid y \geq 0, y \in \mathbf{R}\}$ . After a horizontal translation of +5, the domain becomes  $\{x \mid x \geq 5, x \in \mathbf{R}\}$ . After the vertical stretch, the range does not change.

#### MANAGEMENT SUGGESTION

For (d) and (e), suggest that students draw input/output diagrams as well.

- (d) One point on the graph of  $y = \sqrt{x}$  is (4, 2). What is the corresponding point on the graph of  $y = 3\sqrt{x-5}$ ? Explain the transformation.



(9, 6), because the point is translated 5 units to the right, and it is stretched (multiplied) vertically by a factor of 3.

- (e) Evaluate  $y = 3\sqrt{x-5}$  for  $x = 9$ . How does this correspond to your answer to (e)?

For  $x = 9$ ,

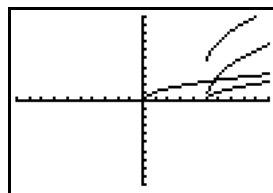
$$\begin{aligned} y &= 3\sqrt{x-5} \\ &= 3\sqrt{9-5} \\ &= 3(2) \\ &= 6 \end{aligned}$$

So the point is (9, 6), which is the same as the point in (d).

4. (a) What transformations would you apply to the graph of  $y = \sqrt{x}$  to obtain the graph of  $y = 3\sqrt{x-5} + 4$ ? Sketch your prediction on graph paper.

The graph is translated 5 units to the right, stretched vertically by a factor of 3, and translated 4 units up. Any sequence of transformations that does not change the order of the vertical stretch is valid.

- (b) Graph  $y = 3\sqrt{x-5} + 4$  using the graphing calculator to check your answer. Enter this function into **Y4** in the equation editor.



- (c) What is the domain of  $y = 3\sqrt{x-5} + 4$ ? Explain. What is its range? Explain.

The domain is  $\{x \mid x \geq 5, x \in \mathbf{R}\}$ . The range is  $\{y \mid y \geq 4, y \in \mathbf{R}\}$ . The original domain is  $\{x \mid x \geq 0, x \in \mathbf{R}\}$ . After a horizontal translation of  $+5$ , the domain becomes  $\{x \mid x \geq 5, x \in \mathbf{R}\}$ . The original range is  $\{y \mid y \geq 0, y \in \mathbf{R}\}$ . After the vertical stretch, the range does not change. After the vertical translation of  $+4$ , the range becomes  $\{y \mid y \geq 4, y \in \mathbf{R}\}$ .

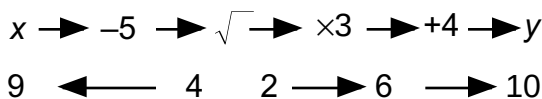
#### MANAGEMENT SUGGESTION

For (d) and (e), suggest that students draw input/output diagrams as well.

- (d) One point on the graph of  $y = \sqrt{x}$  is (4, 2). What is the corresponding point on the graph of  $y = 3\sqrt{x-5} + 4$ ? Explain the transformation.

The corresponding point is (9, 10).

- (e) Evaluate  $y = 3\sqrt{x-5} + 4$  for  $x = 9$ . How does this answer correspond to your answer to (d)?



For  $x = 9$ ,

$$\begin{aligned}
 y &= 3\sqrt{x-5} + 4 \\
 &= 3\sqrt{9-5} + 4 \\
 &= 3(2) + 4 \\
 &= 10
 \end{aligned}$$

So  $y = 10$ , and the point (9, 10) is the same as the point in (d).

You work backward from  $x = 4$  in (d) to get  $x = 9$ . In this part, you work forward from  $x = 9$  to get the same  $y$ -value, 10.

- (f) The “base” function in all of these examples is the square root. In (e), what operation did you do before taking the square root? Which transformation did this operation correspond to? Was this transformation vertical or horizontal?

Subtracted 5 before finding the root (applying the base function). This operation corresponds to the horizontal translation of 5 units to the right.

- (g) In (e), what did you do after finding the square root? Which transformations did these operations correspond to? Were the transformations vertical or horizontal? Multiplied by 3 and added 4. These operations correspond to a vertical stretch of factor 3 and a translation of 4 units up.

- (h) The input/output diagram shows how to evaluate  $y = 3\sqrt{x-5} + 4$  for  $x = 9$ . On this diagram, which operations correspond to vertical transformations? to horizontal transformations?

The operations corresponding to the vertical transformations are on the right of the base function. These operations affect the dependent variable or function value. The operations causing horizontal transformations are on the left of the base function. These operations affect the independent variable.

- (i) Notice that the coordinates of point (4, 2) appear in the diagram above. The 4 is to the left of the base function and the 2 is to the right of the base function. Why does this make sense?

This point is a pair of values, one independent variable and one for the dependent variable, of the base function. The diagram shows that the value 4 (independent variable) is acted upon by the base function to create the dependent value 2 (dependent variable).

- (j) Move to the right of the base function in this diagram. What is the order of the operations? In what order should you apply the transformations?

Multiplication, then addition. The vertical stretch comes before the vertical translation, in this case. Generally, the order of transformations should correspond to the order of operations.

- (k) Move to the left of the base function in this diagram. What is the operation? What transformation does this operation correspond to?

The operation is subtraction, and subtraction corresponds to a horizontal translation.

- (l) Find other values of  $x$  that would produce integer values of  $y$ . How can you get integral answers for  $y$  without guessing?

Select values for  $x$  so that  $(x - 5)$  is a perfect square. In other words, find perfect squares and add 5 to each of them.

5. Using graphing technology, explore the effect of varying  $a$ ,  $p$ , and  $q$  in

$$y = a\sqrt{x - p} + q \text{ on the graph of } y = \sqrt{x}.$$

For each graph, record the following in your notebook:

- the equation
- the values of  $a$ ,  $p$ , and  $q$
- the domain and range of the function
- a description of the transformations that must be applied to the graph of

$$y = \sqrt{x} \text{ to obtain the graph of}$$

$$y = a\sqrt{x - p} + q$$

Create an organized exploration for each parameter.

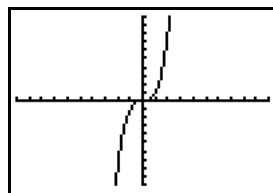
For example, graph the equations  $y = \sqrt{x}$ ,  $y = \sqrt{x - 2}$ ,  $y = \sqrt{x + 3}$ , and  $y = \sqrt{x - 1}$  on the same axes.

A change in  $p$  causes the entire graph to be translated left or right, without changing the shape of the graph. A positive value of  $p$  causes a translation to the left and a negative value causes a translation to the right.

A change in  $q$  causes the entire graph to be translated up or down, without changing the shape of the graph.

A positive value of  $a$  causes the graph to shrink or stretch and a negative value not only causes the graph to shrink or stretch but also to be reflected vertically in the  $x$ -axis.

6. (a) Use graphing technology to graph  $y = x^3$ . What is the domain? What is the range? Is this relation a function?



The domain is  $\{x \mid x \in \mathbf{R}\}$ . The range is  $\{y \mid y \in \mathbf{R}\}$ . Yes, this relation is a function.

- (b) Explore the effects of varying  $a$ ,  $p$ , and  $q$  in  $y = a(x - p)^3 + q$  on the graph of  $y = x^3$ . For each graph, record the following in your notebook:

- the equation
- the values of  $a$ ,  $p$ , and  $q$
- the domain and range of the function
- a description of the transformations that must be applied to the graph of  $y = x^3$  to obtain the graph of  $y = a(x - p)^3 + q$

The transformations resulting from changing each parameter will mimic the changes described in question 5.

#### ASSESSMENT SUGGESTION

##### Initiative, Work Habits

Students may be initially confused by the concepts in this lesson, but using technology outside of class time to explore and learn is evidence of positive work habits and initiative. It is important that technology be accessible for all students in a library or lab, or during designated hours in the math classroom.

## Part 2: The Graph of $y = f(kx)$ Compared to $y = f(x)$

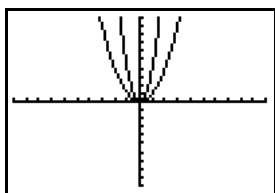
Think, Do, Discuss, text page 280

1. (a) Clear all the equations from the equation editor. Graph the function base  $y = x^2$ . Enter this equation into Y1. What is the domain? What is the range?

The domain is  $\{x \mid x \in \mathbf{R}\}$ . The range is  $\{y \mid y \geq 0, y \in \mathbf{R}\}$ .

- (b) On the same set of axes, graph  $y = (2x)^2$ . Enter this equation into Y2. How does the new graph compare to the graph of  $y = x^2$ ? What is the domain? What is the range?

The new graph is a result of shrinking the original graph horizontally. The new graph is half as wide as the original graph. The domain is  $\{x \mid x \in \mathbf{R}\}$ . The range is  $\{y \mid y \geq 0, y \in \mathbf{R}\}$ .



- (c) The input/output diagram shows how to evaluate  $y = (2x)^2$  for  $x = 3$ . What is the corresponding point on the graph of  $y = x^2$ ? Explain.

This point  $(3, 36)$  on the new graph corresponds to  $(6, 36)$  on the original graph. Because the graph shrinks horizontally, the  $x$ -coordinate is half the original  $x$ -coordinate. The  $y$ -coordinate remains the same.

- (d) Create a table for  $y = (2x)^2$ . Create another table of corresponding points for  $y = x^2$ .

| $x$ | $y = (2x)^2$ |
|-----|--------------|
| 3   | 36           |
| 0   | 0            |
| 1   | 4            |
| 2   | 16           |
| 4   | 64           |
| 5   | 100          |

corresponds  
to

| $x$ | $y = x^2$ |
|-----|-----------|
| 6   | 36        |
| 0   | 0         |
| 2   | 4         |
| 4   | 16        |
| 8   | 64        |
| 10  | 100       |

- (e) Suppose that you had started with the table for  $y = x^2$ . Describe how you would find the corresponding points on the graph of  $y = (2x)^2$ . This transformation is called a horizontal stretch of factor  $\frac{1}{2}$ .

Multiply the  $x$ -coordinate of each point by  $\frac{1}{2}$ . The  $y$ -coordinates remain the same.

- (f) Press **2nd** **GRAPH** to examine the table for both graphs. Is your work for (d) and (e) correct?

- (g) Explore what happens to the graph and table when you change the coefficient  $k$  in  $y = (kx)^2$  from 2 to some other number. Try numbers between 0 and 1, greater than 1, and less than 0.

For numbers between 0 and 1, the graph stretches horizontally. For numbers greater than 1,  $k > 1$ , the graph stretches horizontally by a fractional factor,  $\frac{1}{k}$ . For negative numbers,  $k < 0$ , the graph is reflected in the  $x$ -axis.

2. Explore the effect of varying  $k$  in  $y = \sqrt{kx}$  on the graph of  $y = \sqrt{x}$ . For each graph, record the following:
- the equation
  - the value of  $k$  (use numbers between 0 and 1, greater than 1, and less than 0)
  - an input/output diagram for the equation
  - a table for  $y = \sqrt{x}$  and the corresponding points for  $y = \sqrt{kx}$
  - the domain and range of the function
  - a description of the transformations that must be applied to  $y = \sqrt{x}$  to obtain the graph of  $y = \sqrt{kx}$

The results will mimic those found for step 1.

3. Explore the effect of varying  $k$  in  $y = (kx)^3$  on the graph of  $y = x^3$ . For each graph, record the following:
- the equation
  - the value of  $k$
  - an input/output diagram for the equation
  - a table for  $y = x^3$  and the corresponding points for  $y = (kx)^3$
  - the domain and range of the function
  - a description of the transformations that must be applied to  $y = x^3$  to obtain the graph of  $y = (kx)^3$

The results will mimic those for steps 1 and 2. The stretch factor is  $\frac{1}{k}$ .

### Part 3: The Graph of $y = af[k(x - p)] + q$ Compared to $y = f(x)$

**Think, Do, Discuss, text page 282**

1. (a) Clear all the equations from the equation editor. Graph the function  $y = \sqrt{x}$ . Enter this equation into **Y1**.

- (b) The input/output diagram shows how to evaluate  $y = 3\sqrt{-2(x - 5)} + 4$  for  $x = 3$ . What is the corresponding point on the graph of  $y = \sqrt{x}$ ? Explain.

This input/output diagram creates the point (3, 10), which corresponds to (4, 2) on the graph of  $y = \sqrt{x}$ . The operations on the left side of the diagram are applied to the independent variable, the  $x$ -coordinate. Subtract 5 corresponds to a translation of 5 units to the right. Multiply by 2 corresponds to a horizontal stretch of  $\frac{1}{2}$ . Multiply by  $-1$  corresponds to a reflection in the vertical line  $x = 5$ . So  $x = 4$ .

The positive square root of 4 is 2.

The operations on the right side of the diagram are applied to the dependent variable, the  $y$ -coordinate. Multiply by 3 corresponds to a vertical stretch of factor 3. Add 4 corresponds to a vertical shift up of 4 units. So  $y = 10$ .

- (c) Notice that the coordinates of point (4, 2) appear in this diagram. The 4 is to the left of the base function and the 2 is to the right of the base function. Why does this make sense?

The values 4 and 2 are also the corresponding coordinates of a point for the base function.

- (d) Move to the right of the base function in this diagram. What is the order of the operations? In what order should you apply the transformations?

Multiplication and then addition, in this case. It is usually easier to apply stretches and reflections before translations. The order of the transformations must correspond to the order of operations in the input/output diagram.

- (e) Move to the left of the base function in this diagram. What is the order of operations? What transformations do these operations correspond to? In what order should you apply these transformations?

In this case, multiplication, division, and then subtraction. Start with the base function,  $\sqrt{-2(x-5)}$ , because in this part, you work from right to left. Multiply by  $-1$  corresponds to a reverse reflection in the vertical line  $x = 5$ , and the graph opens to the right. Multiply by 2 now corresponds to the operation divide by 2 and the horizontal stretch of factor 2. Subtract 5 now corresponds to the operation add 5 and a horizontal translation of 5 units to the right. The base function is now  $\sqrt{x}$ .

- (f) Find other values of  $x$  that would produce integer values of  $y$ . How can you get integral answers for  $y$  without guessing?

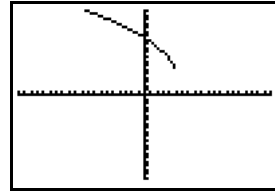
Divide any even, perfect square by  $-2$ , and add 5 to get a value for  $x$ .

- (g) What transformations are applied to the graph of  $y = \sqrt{x}$  to obtain the graph of  $y = 3\sqrt{-2(x-5)} + 4$ ? Sketch your prediction on graph paper.

- horizontal stretch of factor  $\frac{1}{2}$  and a horizontal reflection in the vertical line  $x = 5$
- vertical stretch of factor 3
- horizontal translation of 5 units right
- vertical translation of 4 units up

(Any sequence of these transformations that does not change the order of the horizontal or vertical translations is correct.)

- (h) Graph  $y = 3\sqrt{-2(x-5)} + 4$  using a calculator to check your answer.



- (i) What is the domain of  $y = 3\sqrt{-2(x-5)} + 4$ ? Explain. What is its range? Explain.

The domain is  $\{x \mid x \leq 5, x \in \mathbf{R}\}$ . The range is  $\{y \mid y \geq 4, y \in \mathbf{R}\}$ . The value of the expression under the radical sign, that is, the radicand, must be positive or 0, which means the value of the expression  $x - 5$  must be negative or 0.

$$x - 5 \leq 0$$

$$x \leq 5$$

For  $x = 5$ ,

$$\begin{aligned} y &= 3\sqrt{-2(5-5)} + 4 \\ &= 3\sqrt{-2(0)} + 4 \\ &= 0 + 4 \\ &= 4 \end{aligned}$$

Here is another explanation. For  $y = \sqrt{x}$ ,  $x \geq 0$ . After the horizontal stretch of factor  $\frac{1}{2}$ ,  $x \geq 0$ . After the reflection in the  $y$ -axis,  $x \geq 0$ .

After the horizontal translation of 5 units to the right,  $x \leq 5$ . Therefore, the domain is  $\{x \mid x \leq 5, x \in \mathbf{R}\}$ .

For  $y = \sqrt{x}$ ,  $y \geq 0$ . After the vertical stretch of factor 3,  $y \geq 0$ . After the vertical translation of 4 units up,  $y \geq 4$ . Therefore, the range is  $\{y \mid y \geq 4, y \in \mathbf{R}\}$ .

#### EXTENSION SUGGESTION

Point  $(x, y)$  of a function is translated to a new position,  $(u, v)$ , where  $u = x + 2$  and  $v = y$ . Then you can replace  $x$  in the equation of the original function with the expression  $u - 2$ .

Explain how this reasoning could be related to the direction of change caused by a horizontal translation.

#### Additional Example: Functions and Relations

The function  $y = \sqrt{x-3}$  is shifted 4 units to the right and 2 units down. It is then stretched vertically by a factor of 3 and reflected in the  $x$ -axis.

- (a) Find the equation of the transformed function.
- (b) Sketch the transformed function.

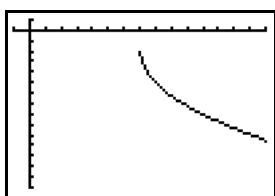
#### Solution

- (a) After the horizontal and vertical translations,  $y = \sqrt{x-7} - 2$ .

After the vertical stretch and reflection in the  $x$ -axis,

$$\begin{aligned} y &= -3(\sqrt{x-7} - 2) \\ &= -3\sqrt{x-7} + 6 \end{aligned}$$

- (b)



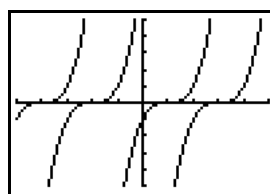
#### Additional Example: Functions

A graphic artist wants to draw a curve that repeats itself at regular intervals, using illustration software. Use transformations to generate a sample of possible designs.

#### Solution

Start with a curve, for example,  $y = x^3$ , and translate it horizontally or vertically. For example,

$$\begin{aligned} y &= (x + 2)^3 \\ y &= (x - 3)^3 \\ y &= (x - 1)^3 \\ y &= (x + 4)^3 \end{aligned}$$



#### ASSESSMENT SUGGESTION

##### Knowledge and Understanding

Ask a single, quick question and then ask students to hand in their responses. For example, ask them to sketch the graph of a given transformed function, or create an algebraic function to match a given graph. Treat the first responses as formative assessment and “count” several of the later responses.

#### Practise, Apply, Solve 3.7

##### Questions with a University Focus:

1–11, 14–18, 20–22, 24

##### Questions with a University/College Focus:

1–8, 10–18, 21, 22

#### 1. Communication, p. 285

##### Achievement Chart Connection

Students must make a conjecture and use mathematical language and symbols to explain their reasoning.

**Management Suggestion**

Invite students to consider situations where the shape is identical or where the shape has changed a little but might be called the same “shape.” Asking students to present responses may encourage them to consider alternative responses.

**Level 3 Performance**

Responses should include a clearly explained position, supported by mathematical evidence. Students should use mathematical symbols, graphs, or conventions appropriately.

**15. Knowledge and Understanding, p. 289****Achievement Chart Connection**

Students are required to demonstrate understanding of horizontal and vertical translations and of horizontal and vertical stretches.

**Management Suggestion**

Provide a specific length of time in class for students to prepare a response, which shows each step on a graph, and collect responses.

**Level 3 Performance**

Students should correctly interpret the direction of translation, up, down, left, or right. They should also correctly apply a vertical or horizontal stretch and label the graph at each step.

**16. Application, p. 289****Achievement Chart Connection**

Students must apply procedures to increasingly complex functions.

**Management Suggestion**

Encourage students to sketch each step. When students have completed their work, they should check their answers using the solutions provided. If students do not demonstrate proficiency, suggest they discuss questions with a partner or the teacher and then try another example.

**Level 3 Performance**

Responses should include an accurate interpretation of an algebraic description of a function. Students should also demonstrate understanding of the differences between transformations applied to the dependent and the independent variables and of the differences between a translation and a stretch.

**17. Thinking, Inquiry, Problem Solving, p. 289****Achievement Chart Connection**

Students are required to develop a hypothesis, create a model, and adjust the model.

**Management Suggestion**

Allow students to work in pairs or small groups

**Level 3 Performance**

Responses should include a reasonable model of the data and transformations that improve the accuracy of the model.

**21. Check Your Understanding, p. 290****Management Suggestion**

Students might prepare a summary diagram, a flow chart, or a mind map, indicating the steps for analyzing the transformations of a function. The value of this exercise is to allow students to practise communicating in student friendly language.

**The Chapter Problem—Cryptography**

This series of questions formalizes the development of ideas in the previous sets by introducing the correct notation. In CP15(d), encourage students to systematically try “all” possible values. This idea is important in many codes— finding the inverse boils down to finding the value so that  $17x = 1 \pmod{26}$ . If this is difficult, then decoding is difficult. Naturally, this code is too simple for any practical use. Encourage students to think of other operations that might be harder to invert. Exponentiation is commonly used in encryption techniques.

The Chapter Problem Answers are on p. 3-48 of the *Teacher Resource*.

**Challenge 1**

Challenge the students to determine a formula for the coding function as well as its inverse.

**Challenge 2**

Suggest students explore making their coders more efficient by playing with transformations of the input by reducing the number of wasted output characters.

The Challenge Answers are on p. 3-45 of the *Teacher Resource*.