

3.1 Investigating a Special Kind of Relationship

Specific Expectations

The following expectation is common to the Functions and Relations course and the Functions courses.

- OC2.01 define the term function

Text Page 222

Time Required Functions and Relations: 30 min

Time Required Functions: 50 min

Materials and Equipment

- TI-83 Plus calculator

Overview

- Explore the relation of height versus footprint length using a small set of data.
- A relation that consists of unique pairs of values is a function.
- Use technology to model data in different ways.

A Relationship

Using a small set of data, students examine a relationship and decide whether a specific value of the independent variable uniquely predicts the value of the dependent variable or not. They also experiment with several models to find an equation for the relation that will allow them to predict other values for the relation. The line of best fit predicts a specific value of the dependent variable for a value of the independent variable. Students must understand this concept of uniqueness to distinguish between a relation and a function.

Management Options

Groups – Post the instructions for graphing calculator keystrokes on the board or somewhere else in the class, or provide cue cards. Divide the class into groups and allow time for groups to do steps 1, 2, and 3 of the *Think, Do, Discuss*. Bring the class together and ensure that everyone understands that each value for the independent variable must

correspond to a unique value for the dependent variable, if the relation is a function. Then group members, working individually, can complete step 4 of the *Think, Do, Discuss* and compare responses with other members of their group.

Teacher Demonstration – If each student has a calculator, then students can work with the teacher to follow the steps in the *Think, Do, Discuss*. Emphasize the important characteristic of a function: determining or predicting a unique value of the dependent variable for each value of the independent variable.

TECHNOLOGY SUGGESTION

Pair students who are familiar with the TI-83 Plus with students who are not or have only some experience using the TI-83 Plus.

TEACHER INTERVENTION

Post keystroke sequences or provide cue cards for students who have trouble remembering the keystroke sequences. Students will not remember the keystrokes if they use the technology infrequently. See also the Technology Appendix that begins on text page 696.

Think, Do, Discuss, text page 222

1. (a) *This data illustrates a relationship between footprint length and height. Which variable is independent in this case?*

The independent variable is footprint length.

(b) Using a TI-83 Plus calculator, enter the data into lists and create a scatter plot. Zoom in, if needed, so that your screen looks like the third screen above.

(c) Someone in the group has a footprint that is 31 cm long. What can you say about that person's height?

The data in the first screen suggests that the person is 175 cm tall.

(d) Now you know one value of the independent variable. Does this value determine a unique value for the dependent variable?

One value of the independent variable does determine a unique value for the dependent variable in this case, because the data shows that one value of the independent variable matches one and only one value of the dependent variable.

A larger set of data would probably show that two students with the same footprint length would have different heights.

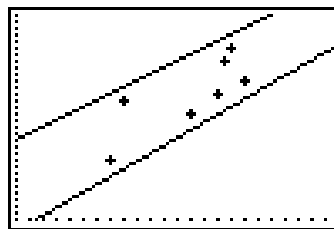
2. It appears that there might be a relationship between footprint length and height for all the students in the class. For this relationship, footprint [length] is the input and height is the output. It appears that a student's height can be found using the length of his or her footprint.

(a) Think of a line that runs just above all the data points. Think of another line that lies just below all the data points. Enter the equations for these lines into the equation editor and graph. It might be reasonable to expect that data about other students in the class may lie between these two lines. Write your two equations.

Students' responses will vary. Two possible equations are:

$$y = x + 153.5$$

$$y = x + 141.5$$



(b) Estimate a reasonable range for Asif's height if his footprint is 31 cm long.

i. Draw a vertical line through $x = 31$ by doing the following:

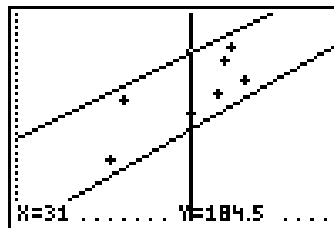
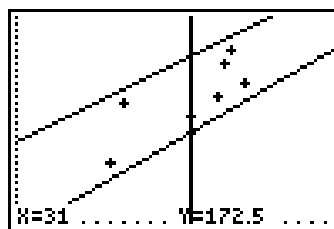
- Press **2nd** **PRGM** to open the DRAW menu.

- Select **4:Vertical**. Scroll left or right until $x = 31$.

ii. Use your graph to estimate a reasonable range for Asif's height by doing the following:

- Scroll up or down to move the cursor to each line.

- Record the y -coordinate for the top line as Asif's greatest possible height. Record the y -coordinate for the bottom line as Asif's least possible height.



- (c) What can you say about Asif's height? How could you make your prediction more accurate?

According to the information in the screens in (b), Asif's height is between 172.5 cm and 184.5 cm.

- (d) The two lines you have drawn and all of the points between these lines define a model for the relationship between footprint length and height for the whole class. Suppose that you are given a value of the independent variable. Does this model uniquely determine a value for the dependent variable?

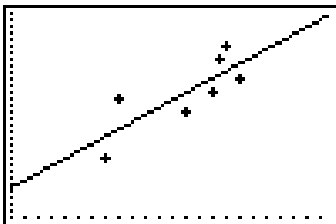
This model shows a range of values for the dependent variable (height) rather than a unique value.

- (e) A 31-cm long footprint is found in the snow. How can you use this model to predict the person's height? How might you change the model to make a more accurate prediction?

You can estimate a range for the person's height, but not an exact height. By decreasing the distance between the two lines and ignoring any extreme values, you can predict more accurately. Also, collecting more data will yield a more accurate model. Data should be collected from the population at large, because the class data will not reflect the footprint length of adults.

3. Another model for the relationship between footprint length and height is the line of best fit and its equation.

- (a) Let l be the length of the footprint for any student in the class. Then let h be the corresponding height. Think of an equation for the line of best fit and graph it on the scatter plot. Record your equation for the line of best fit.



$$h = 1.1l + 143.6$$

- (b) Use your equation to estimate the height of a student in the class whose footprint is 31 cm long.

$$\begin{aligned} h &= 1.1(31) + 143.6 \\ &= 177.7 \end{aligned}$$

- (c) Imagine that you know a value of the independent variable. Does this model uniquely determine a value for the dependent variable?

Yes. This model determines a unique value for the dependent variable for each value of the independent variable.

- (d) Determine the correlation coefficient, r , for the line. What does the correlation coefficient tell you about this model for the relationship?

The correlation coefficient is $r = 0.791\ 864\ 553\ 8$. This value indicates that the line of best fit reasonably fits the data, because it is close to 1.

4. For [each] set of data,

- i. create a scatter plot
- ii. determine a linear model and a quadratic model. Which model is more appropriate?
- iii. decide a reasonable range for the y -coordinate if the x -coordinate is 4
- iv. estimate the y -coordinate if the x -coordinate is 4

(a)

x	y
-1	5
2	-3
5	-4
8	0
9	3
12	15

(b)

x	y
-1	6
2	3
5	1
8	-2
9	-3
12	-5

(c)

x	y
-1	-4
2	2
5	7
8	11
9	12
12	15

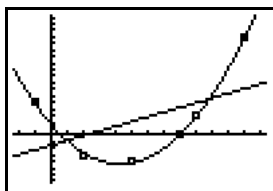
(d)

x	y
-1	0
2	1
5	3
8	4
9	5
12	6

(a)

```
LinReg
y=ax+b
a=.7198838897
b=-1.532656023
r2=.2465900617
r=.496578354
LinReg(ax+b) ■
```

```
QuadReg
y=ax2+bx+c
a=.3286306194
b=-2.829231407
c=1.698321939
R2=.9990234489
■
```



$$-4.4 \leq x \leq 1.4$$

The correlation coefficient indicates that the quadratic model is a better fit, so substitute 4 into the quadratic model.

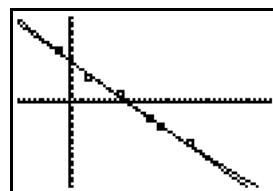
$$\begin{aligned} y &= 0.33x^2 - 2.83x + 1.7 \\ &= 0.33(4)^2 - 2.83(4) + 1.7 \\ &= -4.34 \end{aligned}$$

Alternatively, when you do the linear regression, enter {L1,L2,Y1}. When you do the quadratic regression, enter {L1,L2,Y2}. Then graph Y2 and calculate (2nd) [CALC] the value (1:value) of Y2 at 4.

(b)

```
LinReg
y=ax+b
a=-.8534107402
b=4.978229318
r2=.9956458636
r=-.9978205568
■
```

```
QuadReg
y=ax2+bx+c
a=.0061684276
b=-.9200279677
c=5.038875078
R2=.9964074847
■
```



If you look at the points, relative to the line of best fit, you will see that there is no clear pattern to the data above or below this line. There is no clear case for a nonlinear model. In the interval where there is data, the quadratic model is nearly linear.

$$\begin{aligned} y &= -0.85x + 5.0 \\ &= -0.85(4) + 5.0 \\ &= 1.6 \end{aligned}$$

Alternatively, when you do the linear regression, enter {L1,L2,Y1}. When you do the quadratic regression, enter {L1,L2,Y2}. Then graph Y2 and calculate (2nd) [CALC] the value (1:value) of Y2 at 4.

(c)

```

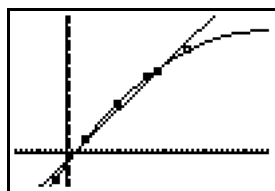
LinReg
y=ax+b
a=1.464441219
b=-1.375907112
r²=.9818080998
r=.9908623011

```

```

QuadReg
y=ax²+bx+c
a=-.0519318417
b=2.025290035
c=-1.886482287
R²=.9998861471

```



Consider the points relative to the line of best fit. All of the points in the “middle” of the interval are above the line, while there are some points at each end of the interval, which are below the line, suggesting a nonlinear model.

$$\begin{aligned}
 y &= -0.05x^2 + 2.03x - 1.89 \\
 &= -0.05(4)^2 + 2.03(4) - 1.89 \\
 &= 5.43
 \end{aligned}$$

Alternatively, when you do the linear regression, enter {L1,L2,Y1}. When you do the quadratic regression, enter {L1,L2,Y2}. Then graph Y2 and calculate (2nd) [CALC] the value (1:value) of Y2 at 4.

(d)

```

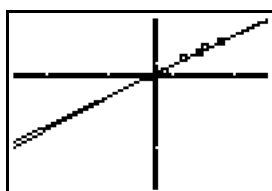
QuadReg
y=ax²+bx+c
a=-.0011601409
b=.4929355707
c=.3528899938
R²=.9877521241

```

```

LinReg
y=ax+b
a=.4804063861
b=.3642960813
r²=.9876677875
r=.9938147652

```



The correlation coefficients are both very close to 1, so use the linear model because it is simpler.

$$\begin{aligned}
 y &= 0.48x + 0.36 \\
 &= 0.48(4) + 0.36 \\
 &= 2.28
 \end{aligned}$$

Alternatively, when you do the linear regression, enter {L1,L2,Y1}. When you do the quadratic regression, enter {L1,L2,Y2}. Then graph Y2 and calculate (2nd) [CALC] the value (1:value) of Y2 at 4.

ASSESSMENT SUGGESTION

TECHNOLOGY APPLICATION

Provide students with a set of data or ask them to collect data. Then ask students to model the relation in two ways and to explain which model is better or more accurate.