

4.3 Zeros of Quadratic Functions

Specific Expectations

The following expectation is common to both the Functions and Relations and Functions courses.

- OC1.05 determine the real or complex roots of quadratic equations, using an appropriate method (e.g., factoring, the quadratic formula, completing the square), and relate the roots to the x -intercepts of the graph of the corresponding function

Evidence of Learning

Students should be able to

- interpret the x -intercepts of a graph as the zeros of an equation
- use factoring when appropriate to determine the x -intercepts, or zeros, of a quadratic function
- relate the number of x -intercepts to the nature of the zeros of the function
- use the vertex form of the equation to predict the number of zeros for a quadratic equation

Text Page 318

Time Required Functions and Relations:

75 min

Time Required Functions: 105 min

Materials and Equipment

- graphing calculator or graphing software

Overview

- Build on grade 10 work on quadratic functions.
- Use various methods to determine the zeros of a quadratic function.
- Interpret a quadratic function graphically.

Management Options

Groups – Because much of the material in this section has been discussed in grade 10, students might work in groups of three to recall concepts and solidify understanding. At the end of each part, reserve some time for checking and ask each group to briefly summarize the salient points. Collect these summaries for assessment. When possible, check students' understanding while they are working.

Alternatively, use the jigsaw strategy. Place students in “home groups” of five in each group. Then organize “expert tables” such that each expert table will discuss one of these topics:

- using a graph to find zeros
- using the factored form to find zeros
- using the vertex form to find zeros
- using the standard form and the quadratic formula to find zeros
- number of roots

When each expert group is competent, members return to their home groups and “teach” what they have learned. You might test the understanding of each group.

Teacher Demonstration – Present this lesson using a whole-part-whole technique. Present a complete analysis of a function on the overhead, highlighting each component and the information in each component. Give students time to practice each skill separately.

Independent Work – In some classes, capable students might enjoy the challenge of preparing one part for presentation to the class. Partners can prepare a presentation, but limit preparation time to 15 min. After each presentation, discuss as a whole class.

EXTENSION

Ask students to consider the following questions:

1. Predict the graph of $y = x(x - 3)(x + 2)$.
2. Predict the graph of $y = x(x^2 - 4x + 4)$.

Consolidate Your Understanding, p. 323

1. What are the zeros of a function? How are the zeros related to the x -intercepts of the graph of the function?

The zeros of a function are the values of x that will produce a y -value of 0. The zeros are the x -intercepts.

2. How many zeros does a quadratic function have? How can you determine the number of zeros.

(a) from the graph?

(b) from the factored form?

(c) from the vertex form?

(d) from the standard form?

A quadratic function may have two, one, or no zeros. Some people prefer to say that a quadratic always has two roots, although the roots may be equal.

- From the graph, identify the x -intercepts.
- From the factored form, set each factor equal to zero and solve for x .
- From the vertex form, identify the direction of opening and the location of the vertex.
- From the standard form, apply the quadratic formula or calculate the discriminant.

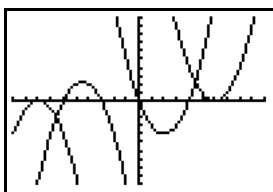
if $b^2 - 4ac < 0$, the function has no zeros

if $b^2 - 4ac = 0$, the function has one zero

if $b^2 - 4ac > 0$, the function has two zeros

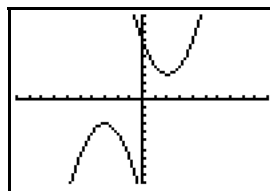
3. (a) Draw a sketch that shows all the ways that $f(x) = ax^2 + bx + c$ can intersect the x -axis.

The x -axis can be intersected in many ways.



- (b) Draw a sketch that shows all the ways that $f(x) = ax^2 + bx + c$ cannot intersect the x -axis.

Sometimes the graph does not cross the x -axis, or the graph crosses at imaginary points.



4. How are the roots of a quadratic equation related to the zeros of a quadratic function?

The roots of a quadratic equation correspond directly to the zeros of a quadratic function.

- if $b^2 - 4ac < 0$, the function has no real roots
- if $b^2 - 4ac = 0$, the function has one double root
- if $b^2 - 4ac > 0$, the function has two distinct real roots

Additional Example: Functions and Relations

Develop the graph and the equation for a quadratic function that has a maximum value of 30 when x is 10 and a value of 0 when x is 5.

Solution

The graph must have a maximum at $(10, 30)$, so use the vertex form.

$$y = -a(x - 10)^2 + 30$$

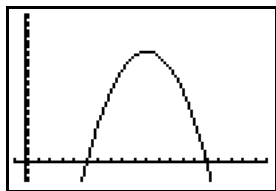
$$y = -ax^2 + 20ax - 100a + 30$$

When $x = 5$, $y = 0$. So,

$$0 = -25a + 100a - 100a + 30$$

$$a = \frac{6}{5}$$

Then the equation is $y = -\frac{6}{5}(x-10)^2 + 30$.



Additional Example: Functions

Why are the zeros of a function interesting to mathematicians?

Solution

Students should provide examples of how zeros may be used to determine the graph of a function.

Practise, Apply, Solve 4.3

Questions with a Functions and Relations

Focus:

1–6, 9–18

Questions with a Functions Focus:

1–9, 12–16

6. Knowledge and Understanding, p. 326

Management Suggestion

Arrange students in groups of three. Each student should analyze the cost for one machine and check the work of one other partner. Then the group should compare results and recommend one machine.

Achievement Chart Connection

Students must demonstrate understanding of the lesson's concepts.

Level 3 Performance

Students should

- state an accurate profit function
- find the zeros of the profit function
- make a recommendation based on the calculations

12. Communication, p. 326

Management Suggestion

While students are working, you might have a one-on-one discussion with several individual students. On another day, you might discuss this question again with other students.

Achievement Chart Connection

Students justify their reasoning using mathematical language.

Level 3 Performance

Students should clearly state a conclusion, that is, the outcome of appropriate reasoning, and explain.

13. Application, p. 326

Management Suggestion

Divide students into groups of four and assign each student a different revenue amount, that is, \$40, \$50, \$60, \$70, or \$80. Students will have the group for support, but each group member should do the steps.

Achievement Chart Connection

Students apply concepts and procedures to an unfamiliar situation.

Level 3 Performance

Students should

- create an accurate profit function
- find the zeros of the profit function
- reasonably interpret the results, referring to the question's context

15. Thinking, Inquiry, Problem Solving, p. 327

Management Suggestion

Insist that students begin with a diagram.

Achievement Chart Connection

Students must develop a reasonable approach to a considerably complex situation.

Level 3 Performance

Solutions should include a well-labelled diagram. Students must treat the two equations as a system and solve the system to find the intersection points, which will involve manipulating the quadratic equation. Solutions should also include a reasonable interpretation referring to the question's context.

ASSESSMENT SUGGESTION

Provide a set of three functions named A, B, and C and a list of possible tasks, for example,

1. Find the zeros.
2. Draw a sketch.
3. Describe the roots.

Ask two students to pick a task and a function. While all students attempt the task for the chosen function, prepare an overhead solution. When students have completed the task, lead a whole-class discussion to compare results and discuss form and methods.

16. Check Your Understanding, p. 327

Management Suggestion

Students are required to read and understand the described situation. They must recognize that the break-even points occur where costs equal revenue, that is, the points of intersection in the graph. Students should explain the number of reasonable solutions as well as how to find them.

The Chapter Problem—Financial Planning

The Chapter Problem Answers are on p. 4-63 of the *Teacher Resource*.

Challenge 1

Challenge 2

The Challenge Answers are on p. 4-68 of the *Teacher Resource*.