

Specific Expectations Addressed in the Chapter

The following expectations, unless otherwise noted, are common to both Functions and Relations and Functions courses:

- OC1.02 add, subtract, and multiply polynomials
- OC1.03 determine the maximum or minimum value of a quadratic function whose equation is given in the form $y = ax^2 + bx + c$, using the algebraic method of completing the square
- OC1.04 identify the structure of the complex number system and express complex numbers in the form $a + bi$, where $i^2 = -1$ (e.g., $4i$, $3 - 2i$)
- OC1.05 determine the real or complex roots of quadratic equations, using an appropriate method (e.g., factoring, the quadratic formula, completing the square), and relate the roots to the x -intercepts of the graph of the corresponding function
- OC1.06U add, subtract, multiply, and divide complex numbers in rectangular form
- OC1.07U add, subtract, multiply, and divide rational expressions, and state the restrictions on the variable values
- OC2.03 determine, through investigation, the properties of the functions defined by $f(x) = \sqrt{x}$ [e.g., domain, range, relationship to $f(x) = x^2$] and $f(x) = \frac{1}{x}$ [e.g., domain, range, relationship to $f(x) = x$]
- OC2.04 explain the relationship between a function and its inverse (i.e., symmetry of their graphs in the line $y = x$; the interchange of x and y in the equation of the function; the interchanges of the domain and range), using examples drawn from linear and quadratic functions, and from the functions $f(x) = \sqrt{x}$ and $f(x) = \frac{1}{x}$
- OC2.05 represent inverse functions, using function notation, where appropriate

What “big ideas” should students develop in this chapter?

Students who have successfully completed the work of this chapter and who understand the essential concepts and procedures will know the following:

- Many real-world situations may be modelled by quadratic or rational functions.
- A complete analysis of such a real-world relationship often requires graphical, numerical, and algebraic representations of the relationship.
- The graph of a quadratic function is a parabola and its equation has three useful, equivalent forms. The standard form reveals the y -intercept and direction of opening; the factored form reveals the x -intercepts and direction of opening; the vertex form reveals the position and size of the maximum or minimum value.
- The zeros of a function correspond to the x -intercepts of the graph of the function. In business, the zeros of a profit function represent break-even points.
- The roots of a quadratic equation correspond to the zeros of a quadratic function. In the real number system, a quadratic equation may have no roots, one root, or two roots, and the corresponding function has no x -intercepts (the graph does not meet or cross the x -axis), one x -intercept (the graph just touches the x -axis), or two x -intercepts (the graph crosses the x -axis).
- A complex number represents a real-life quantity that has two components. For the complex number $a + bi$, a and b are real numbers and i is an imaginary number such that $i^2 = -1$. The solution of a quadratic equation with no real roots has two complex roots, $a + bi$ and $a - bi$.
- Rational functions behave differently than linear or quadratic functions. Part or all of a rational expression may be a quotient. Because division by 0 is undefined, it is often necessary to restrict the values of the variables, if they appear in the denominator. The graph of a rational expression may have a vertical, horizontal, or oblique asymptote. An asymptote is a line that a function approaches but does not touch or intersect.
- The reciprocal function $f(x) = \frac{1}{x}$ models a relationship where two quantities vary inversely. In other words, one quantity increases as the other quantity decreases, and the product of the two quantities is constant. The graph of the reciprocal function is a hyperbola.

What do students already know about the work of this chapter?

In grade 10, students have

- expanded and simplified second-degree polynomials
- factored polynomials involving common factors, differences of squares, and trinomials
- solved quadratic equations by factoring and using graphing calculators or graphing software
- solved quadratic equations using the quadratic formula
- interpreted real and non-real roots of quadratic equations geometrically as the x -intercepts of the graph of a quadratic function
- determined, through investigation, the relationships between the graphs and the equations of quadratic functions
- identified the effect of simple transformations on the graph and the equation of $y = x^2$, using graphing calculators or graphing software;
- explained the roles of a , h , and k in the graph of $y = a(x - h)^2 + k$
- expressed the equation of a quadratic function in the form $y = a(x - h)^2 + k$ and rewrote it in the form $y = ax^2 + bx + c$ by completing the square (no fractions)
- sketched by hand the graph of the quadratic function $y = ax^2 + bx + c$, using a suitable method, for example, completing the square, locating the x -intercepts if the equation is factorable, or expressing the equation in the form $y = ax(x - s) + t$, to locate two points and deduce the vertex
- determined, through investigation, the basic properties of quadratic functions
- collected data that may be represented by quadratic functions, from secondary sources or from experiments, using technology
- fitted the equation of a quadratic function to a scatter plot, using an informal process, e.g., trial and error on a graphing calculator, and compared the results with the equation of a curve of best fit produced using graphing calculators or graphing software
- described the nature of change in a quadratic function, using finite differences (from a table), and compared the nature of change in a quadratic function with the nature of change in a linear function
- determined the zeros and the maximum or minimum value of a quadratic function, using algebraic techniques and using graphing calculators or graphing software
- solved problems related to an application, given the graph or the formula of a quadratic function, for example, given a quadratic function representing the height of a ball over elapsed time, determined the maximum height of the ball, the time the ball was in the air, or the time the height of the ball was greater than a given height