

4.8 Simplifying Rational Expressions

Specific Expectations

- OC1.07U add, subtract, multiply, and divide rational expressions, and state the restrictions on the variable values

Evidence of Learning

Students should be able to

- rewrite a rational expression in factored form
- write a rational expression in reduced form
- identify any restrictions for a rational expression

Text Page 351

Time Required Functions and Relations or Functions: 75 min

Materials and Equipment

- graphing calculator
- software that allows manipulation of rational expressions (optional)

Overview

- Introduce simplifying rational expressions as a parallel method of expressing fractions in reduced form.
- Emphasize the need to write the numerator and denominator in factored form to reduce the fraction and to identify restrictions.

MANAGEMENT SUGGESTION

Show students a few examples of reducing fractions to lowest terms. Demonstrate a similar process with rational expressions.

TEACHER INTERVENTION

You might review factoring techniques. Develop a system for identifying the appropriate method of factoring for each case. For example,

1. Look for a common factor.
2. Look for a difference of squares. ...

Management Options

Teacher Demonstration – Remind students to reduce fractions to lowest terms and then demonstrate a parallel process for simplifying rational expressions. Emphasize that only a *factors* can be factored from an expression and that factors are multiplied.

Start with a simple example such as $\frac{3xy}{6xz}$.

Proceed to more difficult examples involving exponents and expressions that must be factored before they can be simplified.

Use coloured pens or chalk to highlight common factors.

Individual Work – Prepare a work sheet listing the steps for using the available software to help students develop skills in reducing expressions.

EXTENSION

Ask students to reconsider the relation between restrictions on a function and “discontinuities,” if they exist, in the corresponding graph.

Think, Do, Discuss, text page 351

1. Compare these two lists.

Expressions that are rational

$$\frac{x^2}{x^2-4} \quad x+2-\frac{4}{x}$$

$$4x^4+3x^2-2x-1$$

$$\frac{1}{h}\left(\frac{1}{x+h}-\frac{1}{x}\right) \quad \frac{x^2-2x+1}{x^2-x} \div \frac{x-1}{x}$$

Expressions that are not rational

$$\frac{x}{\sqrt{x-1}} \quad 1-x-\frac{2}{\sqrt{x}}$$

$$3x^4-2x^{\frac{3}{2}}+4x-1$$

$$\frac{1}{h}\left(\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}\right)$$

$$\frac{x^{\frac{1}{2}}-2}{x-4} \div \frac{x^{\frac{1}{2}}+2}{x^{\frac{1}{2}}+2}$$

Given these examples, describe a **rational expression**. Explain your answer.

Encourage students to use a mathematics dictionary to help express their observations. A rational expression can be written as a fraction where the numerator and denominator are polynomial expressions, without fractional exponents.

2. Rational expressions represent variable quantities and since they have variables in the denominator, you must ensure that the variables do not take values that would make the denominator equal to zero. Therefore, you must restrict the values of the variables.

For example, $\frac{x}{x+2}$ is defined only when $x \neq -2$.

2. What are the restrictions for $\frac{5y}{x(y-3)^2}$?

$x \neq 0$ and $y \neq 3$

3. Simplifying rational expressions is the same as simplifying rational numbers. For

example, $\frac{12}{16} = \frac{1\cancel{4} \times 3}{1\cancel{4} \times 4} = \frac{3}{4}$. How would you

simplify the expression $\frac{6x-9}{4x-6}$?

Factor the numerator and the denominator and then simplify.

$$\frac{6x-9}{4x-6} = \frac{3(2x-3)}{2(2x-3)} = \frac{3}{2}$$

4. When you simplify a rational number, the simplified number has exactly the same value as the original number. The two numbers are exactly equivalent and interchangeable. When you reduce a rational expression, the reduced expression must have the same restrictions as the original expression. For what values of x is the expression $\frac{(3x-5)}{(x-4)(3x-5)}$ not defined? Explain your answer. You must state all the restrictions on a variable before reducing a rational expression. What is the reduced expression?

The denominator cannot equal zero.

$$(x-4)(3x-5) = 0$$

$$x-4=0 \text{ and } (3x-5)=0$$

$$x=4 \text{ and } x=\frac{5}{3}$$

The restrictions are $x \neq 4, \frac{5}{3}$. The reduced expression is $\frac{1}{x-4}$.

5. What are the restrictions for $\frac{12xy^3}{9x^2y}$?

Simplify the expression.

The restrictions are $x \neq 0$ and $y \neq 0$.

The simplified expression is $\frac{4y^2}{3x}$.

6. What are the restrictions for $\frac{18x(x-2y)}{12x(2x-y)}$? Simplify the expression.

The restrictions are $x \neq 0, \frac{y}{2}$. The

simplified expression is $\frac{3(x-2y)}{2(2x-y)}$.

7. What is the first step in deciding whether the expression $\frac{3x^2 - 7x + 4}{x^2 - 1}$ is simplified?

Simplify the expression, if possible, and state any restrictions.

The first step is to write the numerator and denominator in factored form.

$$\begin{aligned}\frac{3x^2 - 7x + 4}{x^2 - 1} &= \frac{(3x - 4)(x - 1)}{(x + 1)(x - 1)} \\ &= \frac{3x - 4}{x + 1}, x \neq 1, -1\end{aligned}$$

ASSESSMENT SUGGESTION

Knowledge and Understanding

Provide several opportunities for students to demonstrate mastery of this material in preparation for subsequent lessons.

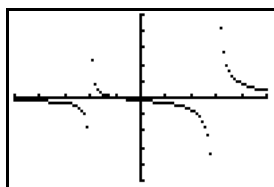
Additional Example: Functions and Relations

Examine the function $y = \frac{(x-1)(x+1)}{(x-3)(x-1)(x+2)}$.

Use a graphing calculator to identify important points on the graph (restrictions) and compare the original expression to the simplified expression.

Solution

The graph of the original and the graph of the simplified version are identical, because the expressions are exactly equal. There are no points on the graph when $x = 3$ and $x = -2$, but there are asymptotes.



Additional Example: Functions

Simplify the expression and state any restrictions.

$$\frac{x^3 - x}{x^2 + 2x + 1}$$

Solution

$$\begin{aligned}\frac{x^3 - x}{x^2 + 2x + 1} &= \frac{x(x^2 - 1)}{(x + 1)(x + 1)} \\ &= \frac{x(x + 1)(x - 1)}{(x + 1)(x + 1)} \\ &= \frac{x(x - 1)}{x + 1}, x \neq -1\end{aligned}$$

Practise, Apply, Solve 4.8

Questions with a Functions and Relations Focus:

1, 3, 8–18

Questions with a Functions Focus:

1–11, 14–16, 18

9. Knowledge and Understanding, p. 354

Management Suggestion

Provide opportunities for students to seek extra help outside of class time. Sometimes more practice will be required.

Achievement Chart Connection

Students perform algorithms by hand, using their ability to write expressions in factored form.

Level 3 Performance

Students should

- correctly write expressions in factored form
- demonstrate ability to differentiate a factor from a term in an expression
- identify restrictions
- write rational expressions in reduced form

11. Communication, p. 354**Management Suggestion**

Invite a volunteer to respond orally and then ask for other explanations or additional comments.

Achievement Chart Connection

Students communicate the mathematical meaning of a common word, restriction.

Level 3 Performance

Students should refer to the fact that division by 0 is undefined and use the factored form of the denominator to identify restrictions.

15. Thinking, Inquiry, Problem Solving, p. 355**Management Suggestion**

Students may need to do some research. State ahead of time whether the can has a top, bottom, or both. Encourage students to use tools such as *The Geometer's Sketchpad* and ask for a written response. Discuss expectations for this inquiry.

Achievement Chart Connection

By simplifying the ratio of surface area to volume, students develop an expression to justify their answers.

Level 3 Performance

Students must find a simplified expression and generate data, which will support their conclusions.

16. Check Your Understanding, p. 355**Management Suggestion**

Use the question to initiate a class discussion before discussing any homework difficulties. Reinforce the importance of careful factoring and self-checking.

18. Application, p. 355**Management Suggestion**

Divide students into groups of four and ask two people in each group to argue each position.

Achievement Chart Connection

Students use a mathematical model in an unfamiliar context.

Level 3 Performance

Students should correctly simplify the expression and state restrictions. They might justify their answers using a graph.

The Chapter Problem—Fundraising

The Chapter Problem Answers are on p. 4-63 of the *Teacher Resource*.

Challenge 1**Challenge 2**

The Challenge Answers are on p. 4-68 of the *Teacher Resource*.