

4.9 Multiplying and Dividing Rational Expressions

Specific Expectations

- OC1.07U add, subtract, multiply, and divide rational expressions, and state the restrictions on the variable values

Evidence of Learning

Students should be able to

- rewrite products of rational expressions in factored form
- rewrite quotients of rational expressions in factored form as products
- simplify products or quotients of rational expressions
- state restrictions

Text Page 356

Time Required Functions and Relations:
75 min

Time Required Functions: 150 min

Overview

- Simplify the products or quotients of rational expressions.
- Draw a parallel between multiplying and dividing rational expressions and the same operations for fractions.

MANAGEMENT SUGGESTION

For some students, these questions will be somewhat overwhelming. Encourage them to focus on one step at a time. Assign each person in a group of four a numerator or a denominator to factor. Then the group can remove the common factors and state restrictions.

Management Options

Groups – Small groups or partners might begin working together and check solutions, but encourage students to work independently if they wish to do so.

Teacher Demonstration – Begin the class with a simple example of multiplying or dividing fractions. Show how to rewrite the fraction as factors and to remove the common factors. Note that the denominator should never be 0. Beside this example, demonstrate the same process for a

simple rational expression, for example,

$$\frac{x^2 + 2x}{x^2}.$$
 Provide other examples of varied

complexity, emphasizing the importance of factoring before reducing and restrictions. Invite volunteers to put sample solutions on the board, allowing a time lag to ensure that most students are ahead of the board work.

EXTENSION

Ask students to consider the following questions:

1. What would you expect if you substitute the same variable in the original question and in the reduced expression? Try it!
2. Provide each pair of students with a full solution, which has been cut into several, but distinct pieces. The task is to sequence the pieces to re-create a reasonable solution.

Think, Do, Discuss, text page 356

1. *How do you multiply two rational numbers? Explain using the product $\frac{5}{48} \times \frac{72}{85}$ as an example. Give the answer as a rational number in lowest terms.*

Some students may suggest multiplying numerators and then multiplying denominators. While this method is not incorrect, it is better to identify

common factors and remove them before multiplying.

$$\begin{aligned}\frac{5}{48} \times \frac{72}{85} &= \frac{5}{(3)(2)(8)} \times \frac{(3)(3)(8)}{(5)(17)} \\ &= \frac{3}{(2)(17)} \\ &= \frac{3}{34}\end{aligned}$$

2. At what point is it easier to remove the common factors, before multiplying or after? Explain.

It is easier to remove the common factors before multiplying because multiplying fewer items will be easier and other factors will be more obvious.

3. Apply the same steps to find the product $\frac{x(x+1)}{x^2-1} \times \frac{2(x-1)^2}{5x^2}$. Look for common factors and reduce, where possible. Since you are working with rational expressions, what must you be careful to state at each step?

$$\begin{aligned}\frac{x(x+1)}{x^2-1} \times \frac{2(x-1)^2}{5x^2} \\ &= \frac{x(x+1)}{(x+1)(x-1)} \times \frac{2(x-1)(x-1)}{5x(x)}, x \neq -1, 1, 0 \\ &= \frac{2(x-1)}{5x}\end{aligned}$$

Restrictions must be considered at each step.

4. How do you divide two rational numbers? Explain using the quotient $\frac{25}{28} \div \frac{10}{21}$ as an example. Give the answer as a rational number in lowest terms.

Multiply by the reciprocal.

$$\begin{aligned}\frac{25}{28} \div \frac{10}{21} &= \frac{25}{28} \times \frac{21}{10} \\ &= \frac{15}{8}\end{aligned}$$

5. Apply the same steps to find the quotient $\frac{4x^2}{x^2+6x-7} \div \frac{x}{x^2-1}$. As before, look for common factors and reduce, where possible. Why are there more restrictions to include

when dividing rational expressions?

$$\begin{aligned}\frac{4x^2}{x^2+6x-7} \div \frac{x}{x^2-1} \\ &= \frac{2(2)(x)(x)}{(x+7)(x-1)} \div \frac{x}{(x-1)(x+1)}, \\ &\quad x \neq -7, 1, -1 \\ &= \frac{2(2)(x)(x)}{(x+7)(x-1)} \times \frac{(x-1)(x+1)}{x}, x \neq 0 \\ &= \frac{4x(x+1)}{(x+7)}\end{aligned}$$

Restrictions must be considered both before and after taking the reciprocal of the divisor.

Additional Example

Simplify.

$$\frac{x^2-4}{x^2+5x+6} \div \frac{x^2-x-2}{x^3+4x^2+3x}$$

Solution

$$\begin{aligned}\frac{x^2-4}{x^2+5x+6} \div \frac{x^2-x-2}{x^3+4x^2+3x} \\ &= \frac{(x-2)(x+2)}{(x+3)(x+2)} \div \frac{(x-2)(x+1)}{x(x^2+4x+3)}, x \neq -3, -2, 0 \\ &= \frac{(x-2)(x+2)}{(x+3)(x+2)} \times \frac{x(x+3)(x+1)}{(x-2)(x+1)}, x \neq -1, 2 \\ &= x\end{aligned}$$

ASSESSMENT SUGGESTION

Communication

Invite students to create a product or quotient using expressions in factored form, expand parts of the problem, and present the result to the class or to a small group whose responsibility it is to find the simplified version.

Practise, Apply, Solve 4.9

Questions with a Functions and Relations

Focus:

1, 2, 4–7, 9, 10, 12, 13, 16, 17

Questions with a Functions Focus:

1–6, 7, 9–13, 16

6. Communication, p. 359

Management Suggestion

Find time as you circulate to ask individual students questions like this one to assess their abilities.

Achievement Chart Connection

Students justify their reasoning and provide clear explanations.

Level 3 Performance

Students should refer to the denominator, that division by 0 is undefined, and that creating a reciprocal may add another restriction.

10. Knowledge and Understanding, p. 360

Management Suggestion

Collect and mark this question. Provide opportunity for remediation, if needed, and a second chance to demonstrate mastery.

Achievement Chart Connection

Students are required to demonstrate understanding of the steps for multiplying and dividing rational expressions.

Level 3 Performance

Students should

- factor all expressions
- multiply by the reciprocal
- list the appropriate restrictions
- express the final answer in simplified form

12. Application, p. 360

Management Suggestion

Briefly discuss how simplifying expressions first saves time when you must substitute several values in the same expression.

Achievement Chart Connection

Students simplify rational expressions in an unfamiliar context. They will recognize how much time and effort is saved by simplifying the expression before substituting each value for the variable.

Level 3 Performance

Students should simplify the formula correctly before substituting.

13. Check Your Understanding, p. 360

Management Suggestion

Students can record their answers in their math journals. Collect answers for assessment and respond to individual students.

16. Thinking, Inquiry, Problem Solving, p. 360

Management Suggestion

Allow students to react to the question and then later discuss the wisdom of “trying something” when all steps to a final solution may not yet be obvious.

Achievement Chart Connection

Students should combine simplifying products and quotients of rational expressions with solving an equation.

Level 3 Performance

Students might

- write all expressions in factored form
- remove common factors and state restrictions
- solve a simple equation
- express one function as a multiple of the other or express the functions as a proportion

ASSESSMENT SUGGESTION

Knowledge and Understanding

Provide daily opportunities for the class to demonstrate mastery.