

4.10U Skill Builder: Dividing Complex Numbers

Although this section does cover an expectation common to the Functions course, the section itself is intended for the Functions and Relations course only.

Specific Expectations

Unless otherwise stated each expectation is common to both the Functions and Relations course and the Functions courses.

- OC1.04 identify the structure of the complex number system and express complex numbers in the form $a + bi$, where $i^2 = -1$ (e.g., $4i$, $3 - 2i$)
- OC1.06U add, subtract, multiply, and divide complex numbers in rectangular form

Evidence of Learning

Students should be able to

- create the conjugate for a complex number
- divide a complex number, expressed as a quotient, by multiplying the numerator and denominator by the complex conjugate of denominator
- multiply complex numbers and simplify the product

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Time Required Functions and Relations:

75 min

Overview

- The product of any complex number and its conjugate is a real number.
- If the numerator and denominator of a fraction are identical, the fraction's value is 1. Multiplying another expression by this type of fraction does not change the value of the expression.
- Build on the skill of multiplying binomials or use the distributive property.

MANAGEMENT SUGGESTIONS

- Have students play a ticktacktoe game. Make game boards out of Bristol board. Each game board should have nine squares (3 by 3). In each square, write a different step, but not the first or final step, for dividing a complex number or finding a reciprocal (see the Examples). Also make some Xs and Os. To play the game, student 1 picks a square. If student 1 can simplify or reduce the expression, he or she can place an X or O on the square. Then student 2 can pick a square and do the same. The student who first places three Xs or three Os in a row, column, or diagonal wins the game.

- Prepare a set of 18 pairs of cards, writing nine expressions on nine cards, one equation to a card. Write an equivalent, simplified expression for each of the nine equations on the other cards. The cards should be turned face down, and a student picks two cards. If the expressions on the cards are equivalent, the student keeps those cards; otherwise, the student returns the cards to the "deck" and another student can take a turn. When the players have picked all the matching pairs, the student with the highest number of cards wins.
- Students can play a version of a children's game, using the deck of cards described above, or a larger set, depending on how many students want to play. All cards are dealt to players who keep their cards face down. The first player places one card face up on the table so that the other players can see it. Then each player in turn places a card on the table, always placing the card face up on top of their individual pile. When a player places a card down, the first player who sees two equivalent expressions says "equal" (or another agreed-upon word) and takes the piles of cards. When there are no cards left to play, the student with the most cards wins the game.

Management Options

Teacher Demonstration – Remind students about multiplying binomials (FOIL) and demonstrate how to create a conjugate. Point to the result of multiplying a complex number, expressed as a quotient, by the conjugate of the denominator: the denominator becomes a real number.

Independent Work – Invite students to read the examples quietly, try several of the *Practise, Apply, Solve* questions, and then select a question which they think might be difficult to solve. If the student is confident, he or she can write the question and the solution on the board or the student can write just the question on the board and ask for help from another student to solve the question. This technique encourages students to take responsibility for their own learning, but use this technique when the material is easily demonstrated by a few examples.

EXTENSION

Ask students to consider the following questions:

1. Why is it helpful to re-express a quotient showing $a + bi$ form?
2. What practical uses do complex numbers have?
3. Research complex numbers in fractal geometry.
4. Compare and contrast dividing complex numbers with multiplying by the reciprocal.

Additional Example: Functions and Relations

Simplify $\frac{5+3i}{2-i} \div \frac{3+i}{4i}$.

Solution

$$\begin{aligned} & \frac{5+3i}{2-i} \div \frac{3+i}{4i} \\ &= \frac{5+3i}{2-i} \times \frac{2+i}{2+i} \times \frac{5i}{3+i} \times \frac{3-i}{3-i} \\ &= \frac{10+5i+6i+3i^2}{4+2i-2i-i^2} \times \frac{15i-5i^2}{9-3i+3i-i^2} \\ &= \frac{10+11i+3(-1)}{4-(-1)} \times \frac{15i-5(-1)}{9-(-1)} \\ &= \frac{7+11i}{5} \times \frac{5(3i+1)}{10} \\ &= \frac{(7+11i)(3i+1)}{10} \\ &= \frac{21i+7+33i^2+11i}{10} \\ &= \frac{32i+7+33(-1)}{10} \\ &= \frac{-26+32i}{10} \text{ or } \frac{-13+16i}{5} \end{aligned}$$

Practise, Apply, Solve 4.10

Questions with a Functions and Relations Focus:

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15. Check Your Understanding, p. 363

Management Suggestion

Provide a supply of coloured paper and invite students to prepare a “model response.” You might post a few models on the bulletin board.