

4.12 Skill Builder: Extending Algebra Skills: Working with Polynomials

Specific Expectations

- OC1.02 add, subtract, and multiply polynomials

Evidence of Learning

Students should be able to

- give examples of monomials, binomials, trinomials, and other polynomials
- state the degree of a polynomial
- add and subtract polynomials by adding and subtracting like terms
- use the distributive property to multiply polynomials and simplify the result by combining like terms

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Time Required Functions and Relations and Functions: 75 min

Materials and Equipment

- chart paper and markers (optional)
- overhead transparencies and markers (optional)

Overview

- Review the vocabulary of polynomials, useful special products, and exponent laws.
- Review and extend adding, subtracting, and multiplying by a monomial.
- Multiply binomials, trinomials, and other polynomials.

Working with Polynomials

The skills developed in this section are needed for future work with limits, derivatives, and polynomial functions in the Advanced Functions and Introductory Calculus Functions course. These skills are equally important for students in Functions and Relations and Functions courses.

MANAGEMENT SUGGESTION

Students have already multiplied binomials in working with quadratic functions and equations at the start of this chapter. You might quickly review the distributive property, as well as the correct order of operations.

Management Options

It is important to spend some time familiarizing students with the concept of polynomials and the associated vocabulary before students work with polynomials.

Groups – Form students into groups of six and assign each group one of these review topics:

- exponent laws
- adding and subtracting algebraic expressions
- multiplying by monomials
- order of operations
- products of binomials
- new terminology

Allow groups 3 to 5 min to brainstorm and prepare a 2-min presentation.

Alternatively, use a jigsaw strategy for review and to check terminology. Then have groups study examples 1 and 2. Give each group a sheet of chart paper and several different coloured markers. Assign *Practise*, *Apply*, *Solve* 1(b), 2(b), 3(b), 4(b), 5(b), and 6(b) to the groups, that is, one question for each group. In each group, students can take turns writing a line of the solution until the solution is complete. Have groups present their solutions to the class and then post these solutions on the walls for the next few days.

TEACHER INTERVENTION

Students become easily frustrated when they are unsuccessful with this algebraic manipulation. As they work on the questions, circulate and provide help. Watch for errors with signs and with the order of operations, especially at the step of removing brackets.

ASSESSMENT LEARNING SKILLS

Teamwork and Initiative

Observe students as they work in groups. Is every student doing a fair share of the work? Do they encourage each other? Are they sharing ideas? Do they know where to look for help with the task?

ASSESSMENT SUGGESTION

Communication

As groups present their review topics and solutions, note how clearly students explain the processes and the extent to which they use correct mathematical language and good form.

Teacher Demonstration – Review the topics mentioned above by brainstorming as a whole class. Have a student record the results on the board or overhead. Then go through and discuss examples 1 and 2 on an overhead or on the board. Keep the students engaged by asking them questions and obtaining their input frequently.

Independent Work – Students with good algebra skills should be able to study and learn from the examples working on their own.

ASSESSMENT SUGGESTION

Knowledge and Understanding

Learning Skill: Work Habits

If students have worked independently on this section, give a homework quiz the next day. The data collected from marking this quiz would also provide information about work habits.

Additional Example: Functions and Relations

Factorial notation, $n!$, which is read “ n factorial” is a short form for a product.

$$n! = 1 \times 2 \times 3 \times \dots \times n$$

$$\begin{aligned} 5! &= 1 \times 2 \times 3 \times 4 \times 5 \\ &= 120 \end{aligned}$$

$$\begin{aligned} 7! &= 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \\ &= 5040 \end{aligned}$$

Simplify the expression.

$$\frac{(n+3)!}{n!} - \frac{n!}{(n-2)!}$$

Solution

$$\begin{aligned} &\frac{(n+3)!}{n!} - \frac{n!}{(n-2)!} \\ &= \frac{n(n+1)(n+2)(n+3)!}{n!} - \frac{(n-2)(n-1)n!}{(n-2)!} \\ &= (n+3)(n+2)(n+1) - n(n-1) \\ &= (n+3)(n^2 + 3n + 2) - (n^2 - n) \\ &= n^3 + 6n^2 + 11n + 6 - n^2 + n \\ &= n^3 + 5n^2 + 12n + 6 \end{aligned}$$

Additional Example: Functions

Simplify $(1 - 2x + x^3)^2 - 5x(1 - 2x)^2$ and state the degree of the resulting polynomial.

Solution

It is important to model a successful strategy for expanding expressions like this one.

$$\begin{aligned} &(1 - 2x + x^3)^2 - 5x(1 - 2x)^2 \\ &= (1 - 2x + x^3)(1 - 2x + x^3) \\ &\quad - 5x(1 - 4x + 4x^2) \\ &= 1 - 2x + x^3 - 2x + 4x^2 - 2x^4 \\ &\quad + x^3 - 2x^4 + x^6 - 5x + 20x^2 - 20x^3 \\ &= 1 - 9x + 24x^2 - 18x^3 - 4x^4 + x^6 \end{aligned}$$

Since the highest power of x is 6, the resulting polynomial has degree 6.

Practise, Apply, Solve 4.12

It may not be necessary for students to do every part of questions 1, 5, 6, or 7. Begin by assigning alternate parts. Assign the remaining parts if students have had difficulties and you feel they need extra practice.

11. Check Your Understanding, p. 376 Management Suggestion

Have students work in pairs. Each partner should create an expression that satisfies the required criteria and then challenge the other partner to simplify the expression.

Questions with a Functions and Relations Focus:

4–7 (every other part), 9, 11, 13–16

Questions with a Functions Focus:

1–7 (every other part), 11