

5.2 Skill Builder: Understanding Angles

Specific Expectations

The following expectation is common to both the Functions and Relations course and the Functions course.

- TF1.01 determine the sine, cosine, and tangent of angles greater than 90° , using a suitable technique (e.g., related angles, the unit circle), and determine two angles that correspond to a given single trigonometric function value

Evidence of Learning

Students should be able to

- sketch positive and negative angles in standard position
- determine in which quadrant an angle terminates
- determine related acute angles for a standard angle and use related acute angles and their specific quadrant location to calculate standard angles
- calculate principal angles

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Time Required Functions and Relations or Functions: 75 min

Materials and Equipment

No special materials are required.

Overview

- Introduce angles in the x - y plane.
- Examine standard position, positive and negative angles, principal angles, coterminal angles, and related acute angles.
- Name the four quadrants of the x - y plane and their corresponding domains in degrees.

Angles in the x - y plane

The section introduces students to the terminology of angles in the x - y plane and begins by defining the standard position of an angle. Positive (ccw) and negative (cw) describe the rotation of an angle. The relation between a principal angle, a coterminal angle, and a related acute angle is explained. The x - y plane is divided into four quadrants by the x - and y -axes.

Management Suggestion: Functions and Relations Students could do this section as an independent learning assignment at home. Use the period allotted for this section by taking up answers to the *Practise, Apply, Solve* questions and consolidating the *Key Ideas*.

Management Options

Groups – Have students work in pairs to discuss the terminology and the given examples. Each student can then summarize the definitions in words and in a graphic form. Divide the class into three groups and assign one example to each group. Ask students to change either the angle or the coordinates in the question of each example and rework the solution. Select several of these questions and their new solutions to display on the board or the overhead. The students who wrote the questions can explain them orally. With the class, consolidate the concepts by reviewing *Key Ideas*.

Teacher Demonstration – Write terms on the board or overhead transparencies and ask students to name example of the terms orally or in writing. Work through the examples with students and consolidate by referring to the *Key Ideas*.

Independent Work – Students can read the first two pages of this section and then record illustrated definitions in their notebooks. Have students read examples 1 and 2 and then write Example 3 in their notebooks, with this additional question: (d) Determine the first positive coterminal angle and the first negative coterminal angle. Take up the responses to (d) with the class.

Additional Example: Functions and Relations:

State all values of $\theta = -52^\circ + 360^\circ n$ for $-2 \leq n \leq 1$, where $n \in I$.

Solution

Substitute $n = -2, -1, 0$, and 1 .

$$\begin{aligned}\theta &= -52^\circ + 360^\circ(-2) \\ &= -772^\circ\end{aligned}$$

$$\begin{aligned}\theta &= -52^\circ + 360^\circ(-1) \\ &= -412^\circ\end{aligned}$$

$$\begin{aligned}\theta &= -52^\circ + 360^\circ(0) \\ &= -52^\circ\end{aligned}$$

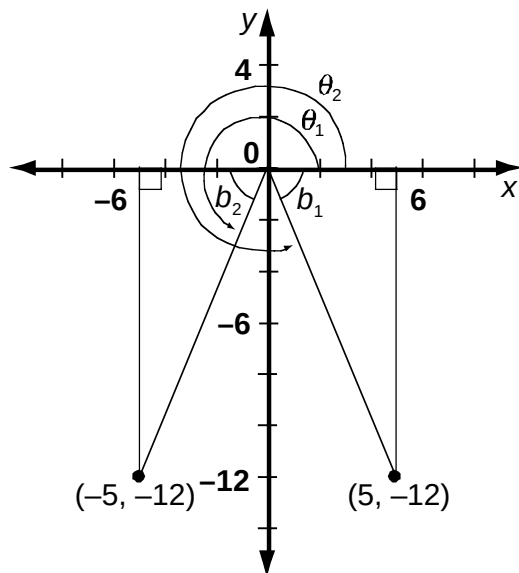
$$\begin{aligned}\theta &= -52^\circ + 360^\circ(1) \\ &= 308^\circ\end{aligned}$$

Additional Example: Functions

The point $(5, -12)$ is on the terminal arm of angle θ_1 in standard position. The point $(-5, -12)$ is on the terminal arm of angle θ_2 in standard position. Determine $\theta_1 - \theta_2$.

Solution

Plot the points and draw right triangles.



β_1 is the acute angle related to θ_1 . β_2 is the acute angle related to θ_2 .

$$\tan \beta_1 = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan \beta_1 = \frac{12}{5}$$

$$\beta_1 = \tan^{-1}\left(\frac{12}{5}\right)$$

$$\beta_1 \doteq 67.4^\circ$$

$$\begin{aligned}\theta_1 &= 360^\circ - 67.4^\circ \\ &= 292.6^\circ\end{aligned}$$

Similarly,

$$\tan \beta_2 = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan \beta_2 = \frac{12}{5}$$

$$\beta_2 = \tan^{-1}\left(\frac{12}{5}\right)$$

$$\beta_2 \doteq 67.4^\circ$$

$$\begin{aligned}\theta_2 &= 180^\circ + 67.4^\circ \\ &= 247.4^\circ\end{aligned}$$

$$\begin{aligned}\theta_1 - \theta_2 &= 292.6^\circ - 247.4^\circ \\ &= 45.2^\circ\end{aligned}$$

Questions with a Functions and Relations:

Focus:

1–10, 12–15

Questions with a Functions Focus:

1–10, 12–14