

5.4 Radian Measure

Specific Expectations

- TF2.01 define the term radian measure
- TF2.02 describe the relationship between radian measure and degree measure
- TF2.03 represent, in applications, radian measure in exact form as an expression involving π (e.g., $\frac{\pi}{3}$, 2π) and in approximate form as a real number (e.g., 1.05)
- TF2.07 demonstrate facility in the use of radian measure in solving equations and in graphing
- TF3.01 sketch the graphs of $y = \sin x$ and $y = \cos x$, and describe their periodic properties
- TF3.06 sketch the graph of $y = \tan x$; identify the period, domain, and range of the function; and explain the occurrence of asymptotes
- TF4.01 determine through investigation, the periodic properties of various models (e.g., the table of values, the graph, the equation) of sinusoidal functions drawn from a variety of applications
- TF4.04 pose and solve problems related to models of sinusoidal functions drawn from a variety of applications, and communicate the solutions with clarity and justification, using appropriate mathematical form

Evidence of Learning

Students should be able to

understand radian measure and convert between degree and radian measures

represent radian measure in exact and approximate forms

draw the graphs of the primary trigonometric functions using radian measure and interpolate from the graphs

solve simple trigonometric equations for angles measured in radians

make a table for an equation and draw the graph

Text Page 438

Time Required Functions and Relations or Functions: 75 min

Materials and Equipment

- compasses for using with board or overhead, graph paper, ruler

Overview

- Introduce the concept of radian measure and its relation to arc length and the radius of a circle.
- Convert between degree and radian measures.
- Draw graphs and solve problems involving radian measure.

Radian Measure

Radian measure is another way of measuring angles. The section begins with rotating a point about a circle and the definition of radian measure (arc length divided by radius). Students learn to convert between radian and degree measures and review the properties of periodic functions.

Management Options

Groups – Break up the class into six groups. Have each group read and discuss one of the following: the introduction, one of the four examples, or the *Consolidate Your Understanding*. After discussion, each group can present their section to the class. Groups can decide the type of presentation.

Teacher Demonstration – Work through the introduction and do examples 1 to 3 at the board or overhead. Ask students to read Example 4 and then take up this example orally. Invite students to go to the board or overhead and then explain various parts of the text solution for Example 4. Students can discuss the *Consolidate Your Understanding* in pairs. (The fact that π is 180° and is also about 3.14 can be confusing to students. Remind them that both values are constants, but one value is a degree measure and the other is a radian measure.) Summarize how to use the proportion for converting between radian and degree measures.

Independent Work – Have students read the introduction and examples 1 to 4. Ask them to create a visual that helps explain the connection between degree and radian measures, to record the proportion and give an example of how to apply it, and, finally, to show in a diagram how π can represent the same position on a circle, yet have a different numerical value.

TEACHER INTERVENTION

Visually show students the location of various points on the circumference of a circle. Indicate that $\pi = 180^\circ$ and that it is also the real number $\pi \doteq 3.14$. Show other real numbers from 0 to 2π . Students need to see that 360° equals 2π or about 6 (text page 438). Also reinforce examples such as

$\frac{\pi}{2} = 90^\circ$	$\pi = 180^\circ$	$\frac{3\pi}{2} = 270^\circ$
$\doteq 1.57$	$\doteq 3.14$	$\doteq 4.7$

Consolidate Your Understanding, text page 441

1. Show how degree measure and radian measure are related.

Students should show this difference both visually and algebraically.

Visually: For example, draw a circle with radius 1 cm and mark an angle measuring 30° and in standard position. Indicate the radian

measure of the angle as $\frac{\pi}{6}$ and highlight the

arc length as $\frac{\pi}{6} \doteq 0.52$. Draw similar diagrams for several other angles.

Algebraically: Write the proportion

$$\frac{\text{radians}}{\pi} = \frac{\text{degrees}}{180^\circ} \text{ and substitute values for}$$

any three parts of the proportion. Solve to get the fourth part. For example,

$$\frac{x}{\pi} = \frac{60}{180}$$

$$x = \frac{60\pi}{180}$$

$$x = \frac{\pi}{3}$$

2. Explain the difference between $\pi = 180^\circ$ and $\pi = 3.14$.

Both values are measures of angles at the centre of a circle. 180° is a degree measure. $\pi = 3.14$ means $\pi = 3.14$ radians. **One** radian is the measure of an angle at the centre of the circle such that the arc length of this angle equals the radius. In this way, a radian is a real number and has no units, unlike the degree measure, which has units. The circumference of a circle with radius 1 is 2π radians, and $\pi = 3.14$, representing half the circumference of the circle and the straight angle of 180° .

TECHNOLOGY

When solving a trigonometric equation such as $\sin \theta = 0.5$, for $0 \leq \theta \leq 2\pi$, which is expressed as real numbers, set the calculator's mode to radian. Remind students to look at the as the first step in a solution.

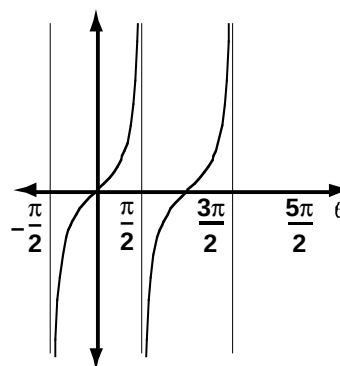
Additional Example: Functions and Relations

State the general equation that you could use to generate vertical asymptotes for the function

$$f(\theta) = \tan \theta, \theta \in \mathbf{R}.$$

Solution

Sketch the function.



In the graph, the equations for the asymptotes are $\theta = -\frac{\pi}{2}$, $\theta = \frac{\pi}{2}$, $\theta = \frac{3\pi}{2}$, and $\theta = \frac{5\pi}{2}$. Then the general equation is $\theta = -\frac{\pi}{2}(2n-1)$, $n \in \mathbf{I}$.

Additional Example: Functions and Relations and Functions

Determine the values of θ , to two decimal places, if $\cos \theta = -0.75$ and $0 \leq \theta \leq 2\pi$.

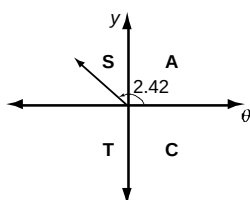
Solution

Set the calculator to radian mode.

$$\cos \theta = -0.75$$

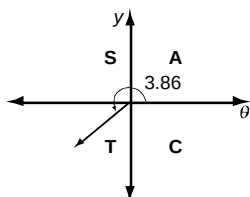
$$\theta = \cos^{-1}(-0.75)$$

$$\theta = 2.42 \text{ radians}$$



The cosine function is negative in quadrant II.

The related acute angle is $\pi - 2.42 = 0.72$.



The cosine function is also negative in quadrant III. Then $\theta = \pi + 0.72$, or 3.86.

ASSESSING LEARNING SKILLS

Teamwork

Provide two copies of the Group Participation: Peer-Assessment Rating Scale to each student. Pair students. At the end of any group work, have students fill out the scale for themselves and for their partners. Have students share results.

Practise, Apply, Solve 5.4

5. Knowledge and Understanding, p. 442

Management Suggestion

This question is straightforward. Send a few students to work at the board on the same question, while other students work at their desks. When students have completed the

question, send a different group of students to the board. Continue the process.

Achievement Chart Connection

Students must know the relationship between degrees and radians to convert from degrees to radians.

Level 3 Performance

Students should substitute in the proportion relating radians and degrees (text page 440) and then accurately solve the equation. All answers should be simplified to an exact radian measure.

16. Application, p. 442

Management Suggestion

Have students work in small groups to answer the question. Collect one full solution from each group for evaluation. Then select one member of each group and ask him or her to answer orally one part of the question. For each student, the solution should count for half the mark, the oral answer should count for the other half. In this way, each member of the group will be prepared.

Achievement Chart Connection

Students apply the cosine function to the periodic nature of wave action. This situation will be unfamiliar in that the cosine function is different from the base cosine function $y = \cos x$.

Level 3 Performance

Students should recognize that the real-number values in the domain are seconds. Students should draw a correctly labelled graph and use the graph to determine the period. As well, they must evaluate and solve simple trigonometric equations. Algebraic and graphic solutions are required.

17. Communication, p. 444

Management Suggestion

Students could work independently on the question for a specific time. At the end of this time, ask students to share solutions with a partner. Have students discuss whether all aspects of the question were answered with sufficient detail and explanation.

Achievement Chart Connection

Students should use mathematical language, symbols, visuals, and conventions associated with the graph and properties of the cosine function. Their reasoning should be communicated orally and in writing.

Level 3 Performance

The table and graph should be correctly labelled and neat. The explanation about the periodic behaviour of the function should be clear and well justified.

18. Check Your Understanding, p. 444

Management Suggestion

Ask students to discuss the question in pairs and then write their answers independently. Pairs can exchange and discuss their answers.

The same quality can be measured in either degrees or real numbers called radians. For instance, a seat on a Ferris wheel can be rotated through a 45° angle or a $\frac{\pi}{4}$ angle. In either case, the angle is the same.

However, there are quantities that can be measured using only real numbers. For instance, the height of the buoy in question 16 or the displacement of the spring in question 17 are functions of time and are not directly related to degree measure.

19. Thinking, Inquiry, Problem Solving, p. 444

Management Suggestion

Assign the question when there are about ten minutes remaining in the class. Students can work in groups or pairs. Ask students to discuss what is required for each part. Have students clarify with each other, as required. They can complete the question as a homework assignment. Start the next class by asking students to work in the same pairs or groups. Have students make any corrections. Either collect responses for evaluation or take up the answers to this question with the class.

Achievement Chart Connection

Before students can answer the questions, they need to visualize the relationship between the two connected and moving gears. Students should select strategies and tools to help clarify the question. As well, they are required to represent the situation in mathematical form, interpret information, and draw conclusions.

Level 3 Performance

Students should be able to reason how one gear drives the other and how the gears move in opposite directions. They should also note the physical differences between the gears. In their solutions, students should apply the steps of problem solving effectively.

Questions with a Functions and Relations:

Focus:

1–11, 13–19

Questions with a Functions Focus:

1–11, 13–19

The Chapter Problem—How Much Daylight?

By the end of this section, students should be able to show the periodic nature of the data. That is, $f(t) = f(t + c)$. As well, they should be able to use this concept to extend the graph of the data in either direction and to extrapolate information from the graph.

The Chapter Problem Answers are on p. 5-61 of the *Teacher Resource*.

Challenge 1

Remind students that they should be able to

- verify their graphs for the two-year period, where the independent variable is a real number
- use this graph to interpolate and extrapolate

Challenge 2

Remind students that they should be able to

- predict that the wave produced at one end of the wave action pool is different from the wave at the opposite end
- determine the relationship between the maximum and minimum heights of the initial wave and the motion of the apparatus that produces the wave
- determine the properties of the sinusoidal function that represents the wave action for the first wave
- hypothesize what effect the shape of the pool has on the properties of the initial wave
- sketch a graph that represents the wave action
- begin constructing a working model of a wave action pool

The Challenge Answers are on p. 5-68 of the *Teacher Resource*.