

5.6 Investigating Transformations

Specific Expectations

The following expectations are common to both the Functions and Relations course and the Functions course.

- TF2.07 demonstrate facility in the use of radian measure in solving equations and in graphing
- TF3.02 determine, through investigation, using graphing calculators or graphing software, the effect of simple transformations (e.g., translations, reflections, stretches) on the graphs and equations of $y = \sin x$ and $y = \cos x$
- TF3.03 determine the amplitude, period, phase shift, domain, and range of sinusoidal functions whose equations are given in the form $y = a \sin(kx + d) + c$ or $y = a \cos(kx + d) + c$
- TF3.04 sketch the graphs of simple sinusoidal functions [e.g., $y = a \sin x$, $y = \cos kx$, $y = \sin(x + d)$, $y = a \cos kx + c$]
- TF4.01 determine, through investigation, the periodic properties of various models (e.g., the table of values, the graph, the equation) of sinusoidal functions drawn from a variety of applications
- TF4.02 explain the relationship between the properties of a sinusoidal function and the parameters of its equation, within the context of an application, and over a restricted domain
- TF4.03 predict the effects on the mathematical model of an application involving sinusoidal functions when the conditions in the application are varied
- TF4.04 pose and solve problems related to models of sinusoidal functions drawn from a variety of applications, and communicate the solutions with clarity and justification, using appropriate mathematical forms

Evidence of Learning

Students should be able to

- determine how the parameters of the equation of a primary trigonometric function are related to the transformations of the base function
- sketch the graph of a transformed trigonometric function by comparing it to the base curve of the trigonometric function
- calculate the period, amplitude, phase shift, and vertical shift from the equation of a function and verify them using the graph of the function
- solve problems represented by trigonometric equations

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Time Required Functions and Relations or Functions: 150 min

Materials and Equipment

- graph paper, ruler, TI-83 Plus calculator

Overview

- Investigate transformations of the base functions $y = \sin \theta$ and $y = \cos \theta$. These transformations include the following:

vertical stretch or compression: $y = a \sin \theta$ and $y = a \cos \theta$, where a is the amplitude

- horizontal stretch or compression: $y = \sin k\theta$ and $y = \cos k\theta$, where k determines the number of cycles
- horizontal translation: $y = \sin k(\theta + b)$ and $y = \cos k(\theta + b)$, where b is the phase shift
- vertical translation: $y = \sin \theta + d$ and $y = \cos \theta + d$
- Predict the graphs of $y = a \sin k(\theta + b) + d$ and $y = a \cos k(\theta + b) + d$.

Transforming Sinusoidal Functions

Students revisit the Ferris wheel from section 5.3 and other Ferris wheels to examine how the properties of sinusoidal functions change when certain aspects of the wheel change. In Part 1, they compare the radius of the wheel and the amplitude of the curve. In Part 2, they will see that creating a vertical distance between the horizontal diameter of the circle and a line tangent and parallel to this diameter results in a vertical translation of the curve. In Part 3, changing the time for one revolution of the wheel changes the period of the function and the number of cycles in a period. In Part 4, changing the boarding or starting point for the ride, that is, changing the angle's position from the standard position, results in a horizontal translation or shift of the curve.

Management Options

Groups – Explain to students the purpose of each investigation, as outlined in the paragraph above. Divide the class into small groups. Assign parts 1, 2, and 3 to each group and give the groups about 120 min to 150 min to complete the parts. While students are working and at appropriate times, select various groups to consolidate their learning for the class. When students have completed parts 1, 2, and 3, lead the class through each step on the board or overhead. Consolidate again, using the *Key Ideas*. Have original groups reconvene and read through examples 1 to 4. Remind students to record information in their notebooks as they work.

Teacher Lecture – For Part 1, do *Think, Do, Discuss* questions 1 and 2 at the board or on the overhead. Assign questions 3 and 4 as independent work. For Part 2, do *Think, Do, Discuss* questions 1 and 2 at the board or on the overhead. Assign question 3 as independent work. Students will need about 60 to 75 min.

For Part 3, lead students through *Think, Do, Discuss* questions 1 and 2 and assign questions 4 to 7 as independent work. For Part 4, lead students through *Think, Do, Discuss* questions 1 and 2. Have students work **in pairs** to complete questions 3 to 7 at their desks. Students will need about 60 to 75 minutes.

TECHNOLOGY SUGGESTION

Have students graph the family of trigonometric functions on the same screen. Start with the base curve and set the window so that the family of curves does not become cluttered; otherwise, it might be difficult to actually see the different transformations and how they compare to the base curve.

Part 1: The Graphs of $y = a \sin \theta$ and $y = a \cos \theta$

Think, Do, Discuss, text page 447

1. (a) Suppose that the centre of the wheel is at the origin of a graph. What are the coordinates of any point on the circumference of the Ferris wheel in Osaka?

$$(x, y) = (50 \cos \theta, 50 \sin \theta)$$

- (b) State the function that models the y-coordinate for any angle of rotation, θ . Graph the function for $0 \leq \theta \leq 4\pi$, and state its amplitude.

$$f(\theta) = 50 \sin \theta$$

The graph should show a typical sine curve with amplitude 50.

- (c) State the function that models the θ -coordinate for any angle of rotation, θ . Graph the function for $0 \leq \theta \leq 4\pi$, and state its amplitude.

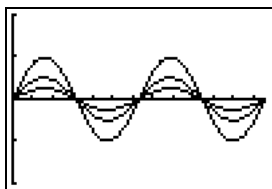
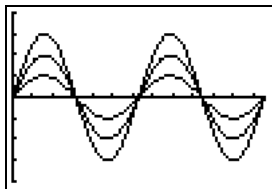
$$f(\theta) = 50 \cos \theta$$

The graph should show a typical cosine curve with amplitude 50.

2. (a) Predict the graph of $y = a \sin \theta$ for $0 \leq \theta \leq 4\pi$, for $a = 1, 2$, and 3 , and for $a = \frac{1}{2}$ and. Sketch each graph on the same axes. Verify your sketches with a graphing calculator.

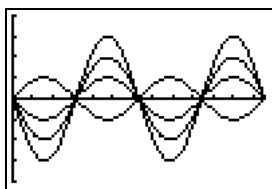
Have students predict and then sketch the graphs before they use a graphing calculator. Students should use the graphing calculator to confirm their predictions.

There will be two full cycles of the sine graph between 0 and 4π . Each graph will have the same zeros, but the amplitudes will be $1, 2, 3, 0.5$, and 0.25 , respectively.



- (b) Sketch, on a new set of axes, the graph of $y = a \sin \theta$ for $0 \leq \theta \leq 4\pi$, for $a = -1, -2$, and -3 . Verify your sketches with a graphing calculator. How are these graphs different from those for $a = 1, 2$, and 3 ?

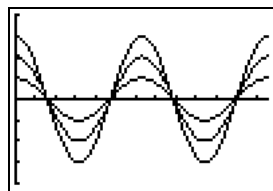
The graphs for the corresponding, negative amplitudes are a reflection in the θ -axis.



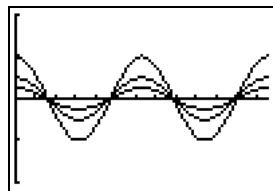
- (c) Compare the zeros for each function.
The zeros of each function are the same.
- (d) How is the amplitude of each graph affected by the value of a ?
The amplitude of each graph is the absolute value of a .
- (e) What are the maximum and minimum values for each graph?
The maximum and minimum values for each sine graph are 1 and -1 , 2 and -2 , 3 and -3 , 0.5 and -0.5 , 0.25 and -0.25 , 1 and -1 , 2 and -2 , and 3 and -3 , respectively.

3. Repeat step 2 for $y = a \cos \theta$.

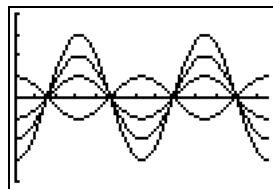
For $a = 1, 2$, and 3 ,



For $a = 0.5$ and 0.25 ,



For $a = -1, -2$, and -3 ,



The graphs for the corresponding, negative amplitudes are a reflection in the θ -axis. The zeros of each function are the same. The amplitude of each graph is the absolute value of a .

The maximum and minimum values for each cosine graph are 1 and -1 , 2 and -2 , 3 and -3 , 0.5 and -0.5 , 0.25 and -0.25 , 1 and -1 , 2 and -2 , and 3 and -3 , respectively.

4. Explain how the value of a affects $y = a \sin \theta$ and $y = a \cos \theta$.

Every y -coordinate of the base curve is multiplied by factor a .

Each base curve is stretched vertically when $a > 1$ or $a < -1$. When $0 < a < 1$ or $-1 < a < 0$, each base curve is compressed vertically. When $a < 0$, each base curve is reflected in the θ -axis.

Part 2: The Graphs of $y = a \sin \theta + d$ and $y = a \cos \theta + d$

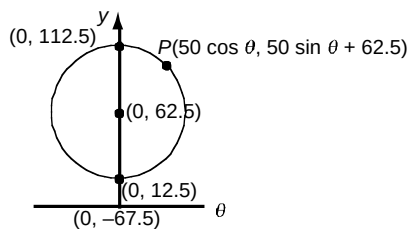
Think, Do, Discuss, text page 448

1. (a) Sketch a model of the Osaka Ferris wheel so that the θ -axis represents the ground and the centre of the wheel is on the y -axis.

See the graph in the response to (b).

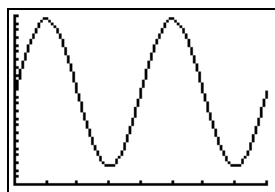
- (b) Describe the transformation of any point, $P(x, y)$, on the circumference of the circle if the centre of the Ferris wheel is not at the origin, but at the point in (a).

The y -coordinate of any point $P(x, y)$ is vertically translated up by 62.5 m.



- (c) State the function that models the y -coordinate for any angle of rotation, θ . Graph the function for $0 \leq \theta \leq 4\pi$ and state its amplitude.

$$f(\theta) = 50 \sin \theta + 62.5$$



The amplitude is 50.

- (d) Describe the relation between the amplitude and both the maximum and minimum values.

The amplitude is the distance from the axis of the curve to the maximum value, or to the minimum value.

- (e) Suppose that Sarah and Billy were riding on the Ferris wheel in Osaka. What is their height above the ground when the angle, in standard position, of the Ferris wheel is $\frac{2\pi}{3}$?

Evaluate $f(\theta) = 50 \sin \theta + 62.5$ for $\theta = \frac{2\pi}{3}$. Be sure to set the calculator to radian mode.

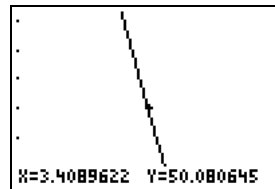
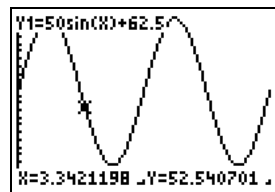
$$f\left(\frac{2\pi}{3}\right) = 50 \sin\left(\frac{2\pi}{3}\right) + 62.5 \\ \doteq 105.8$$

Their height above the ground is about 105.8 m.

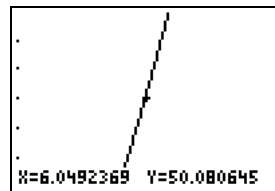
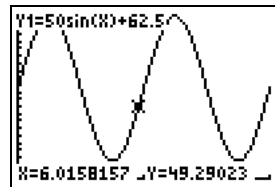
- (f) Sarah and Billy are 50 m above the ground. What is the angle of rotation, in standard position, at this point?

Interpolate θ from the graph when $f(\theta) = 50$. Use **TRACE** and **ZOOM**.

There are four values of θ when $f(\theta) = 50$, for $0 \leq \theta \leq 4\pi$.



$$\theta = 3.4 \text{ and } \theta = 3.4 + 2\pi \text{ or } 9.68$$



$$\theta = 6.04 \text{ and } \theta = 6.04 + 2\pi \text{ or } 12.3$$

2. (a) Predict the graph of $y = \sin \theta + d$ for $d = -2, -1, 1, \text{ and } 2$. Sketch each graph on the same axes. Verify each sketch with a graphing calculator.

The graph of the base sine function will be vertically translated up or down by d units. All graphs will be congruent to $y = \sin \theta$ and parallel.

Use the domain $0 \leq \theta \leq 2\pi$.



- (b) How can you determine the maximum and minimum values without drawing the graphs?

The maximum and minimum values of $y = \sin \theta$ are 1 and -1 , respectively. Then add the value of d to determine the maximum or minimum of the base curve.

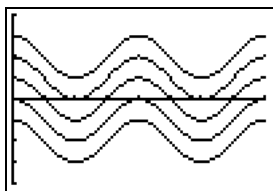
- (c) State the maximum and minimum values for $y = 4 \sin \theta - 9$. Explain how you calculated the values.

The maximum and minimum values of $y = 4 \sin \theta$ are 4 and -4 , respectively.

Then $4 - 9 = -5$ and $-4 - 9 = -13$ are the maximum and minimum values of $y = 4 \sin \theta - 9$.

3. (a) Sketch, on the same axes, the graph of $y = \cos \theta + d$ for $0 \leq \theta \leq 4\pi$ for $d = -2, -1, 1, \text{ and } 2$. Verify each sketch with a graphing calculator.

The base cosine function will be translated up or down d units. All graphs will be congruent to $y = \cos \theta$ and will be parallel to each other.



- (b) State the maximum and minimum values for $y = -3 \cos \theta + 5$. What is the equation of the axis of the curve? How did you find this equation?

The maximum and minimum values of $y = -3 \cos \theta$ are 3 and -3 , respectively.

Then $3 + 5 = 8$ and $-3 + 5 = 2$ are the maximum and minimum values of $y = -3 \cos \theta + 5$.

The axis of the curve is halfway between the maximum and minimum values, $y = 5$. The value 5 is also the value of d , the vertical translation.

- (c) Explain how the value of d affects $y = a \sin \theta + d$ and $y = a \cos \theta + d$.

For each function, the graph is vertically translated d units up if $d > 0$, or the graph is vertically translated d units down if $d < 0$.

Part 3: The Graphs of $y = a \sin k\theta + d$ and $y = a \cos k\theta + d$

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1. (a) What is Sarah and Billy's height on this Ferris wheel at any angle of rotation, θ ?

The radius, r , of the Ferris wheel is 5 and the (axis of the) curve is translated up 6 units. Then their height at any angle θ is $f(\theta) = 5 \sin \theta + 6$.

- (b) Through how many degrees does the Ferris wheel rotate in the first second? How many degrees has the Ferris wheel rotated after 2 s? 3 s? 4 s? Through how many degrees has it rotated after t seconds?

$$2 \text{ min} = 120 \text{ s}$$

$$\text{Then } \frac{360^\circ}{120 \text{ s}} = 3^\circ / \text{s}.$$

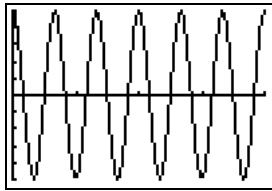
After 2 s, the Ferris wheel has rotated 6° . After 3 s, it has rotated 9° . After 4 s, it has rotated 12° . So, after t seconds, the Ferris wheel has rotated $(3t)^\circ$.

- (c) Write the angle of rotation, θ , in terms of time, t .

$$\theta = (3t)^\circ$$

- (d) Let time be the independent variable. Describe their position by writing two trigonometric equations. Use a graphing calculator to graph the equations for a complete ride. What mode should you set for the graphing calculator? Explain.

The equation for their height is $f(t) = 5 \sin (3t)^\circ + 6$ and the equation for their distance from the vertical diameter of the wheel is $f(t) = 5 \cos (3t)^\circ$, where t is the time in seconds.



Note that the length of the ride, or the domain, is $0^\circ \leq 3t \leq 720^\circ$.

Set the mode for degrees for graphing, because $3t$ is a degree measure.

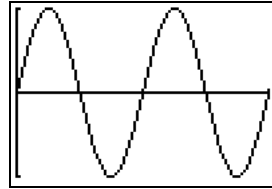
- (e) What is the period of each graph? How might you determine the period from each equation?

In each graph, there are six cycles in 720° or three cycles in 360° . Then the period of each graph is 120° .

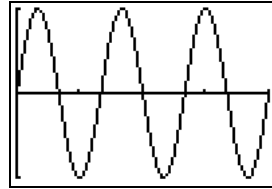
From each equation, divide the period of the base curve by 3, the value by which t is multiplied.

2. Predict the period of the graph of $y = \sin k\theta$ for $k = 2, 3$, and 4 . Confirm each period by drawing each graph. Clear the previous equation from the graphing calculator before entering another equation.

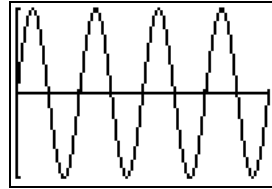
For $k = 2$, there should be two full cycles over 360° . Then the period is $\frac{360^\circ}{|2|} = 180^\circ$.



For $k = 3$, there should be three cycles over 360° . The period is 120° .



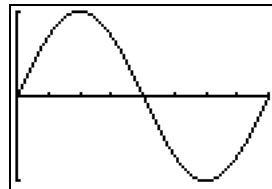
For $k = 4$, there should be four cycles over 360° . The period is 90° .



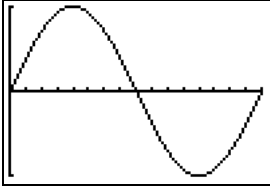
3. Repeat step 2 for $k = \frac{1}{2}$ and $\frac{1}{4}$. Set the window on the graphing calculator so that you may see one complete cycle of each graph.

For $k = \frac{1}{2}$, the period is 4π or 720° .

Set **Xmin** to 0 and **Xmax** to 720.



For $k = \frac{1}{4}$, the period is 8π or 1440° . Set **Xmin** to 0 and **Xmax** to 1440.



4. How may you find the period of $y = \sin k\theta$ from the period of $y = \sin \theta$?

Let $k\theta = 360^\circ$ (or 2π) and solve for θ .

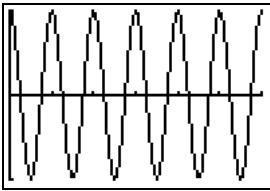
5. What is the period of $y = \cos 6\theta$? Draw the graph and visually verify the period.

$$6\theta = 360^\circ$$

$$\theta = \frac{360^\circ}{6}$$

$$\theta = 60^\circ$$

The period of $y = \cos 6\theta$ is 60° .



6. State the period of $y = -3 \cos (2\theta) + 7$.

The factor by which θ is multiplied determines the period. Then $\frac{360^\circ}{2} = 180^\circ$. The period is 180° or π .

7. Explain how the value of k affects $y = a \sin k\theta + d$ and $y = a \cos k\theta + d$.

k determines the number of cycles in the period. If $k > 1$, there will be more than one cycle. If $0 < k < 1$, there will be less than one cycle. The period for either $y = a \sin k\theta + d$ or $y = a \cos k\theta + d$ is $\frac{360^\circ}{|k|}$ or $\frac{2\pi}{|k|}$.

Part 4: The Graphs of $y = a \sin k(\theta + b) + d$ and $y = a \cos k(\theta + b) + d$

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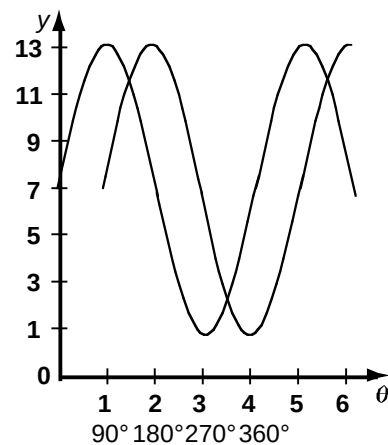
1. Sarah and Billy ride the Ferris wheel at a local fair. The lowest point on the Ferris wheel is 1 m above the ground. The highest point is 13 m above the ground. The wheel is divided into quarters labelled 0, 1, 2, and 3. Extend the pattern and complete each table. Graph each set of coordinate pairs on the same axes in the x-y plane.

(a)

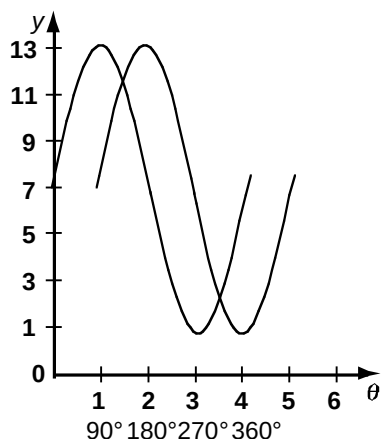
Position on Wheel	0	1	2	3	4	5	6
Height (m)	7	13	7	1	7	13	7

(b)

Position on Wheel	0	1	2	3	4	5	6
Height (m)	1	7	13	7	1	7	13



2. (a) Highlight the first complete cycle for each curve.



- (b) Find the equation of the axis for each curve. Compare the equations. What is the amplitude of each curve?

For each curve, the equation for the axis of the curve is $h = 7$. The amplitude of each curve is 6.

- (c) Compare the maximum and minimum values of each curve.

The maximum and minimum values of each curve are the same, 13 and 1, respectively.

- (d) Compare the periods of the curves.

The period of each curve is the same. The period is 360° , because there are four quarters and the angle in each quarter is 90° .

- (e) How are the curves different?

The base curve for each sine function starts at a different position of the wheel, or a different value of θ .

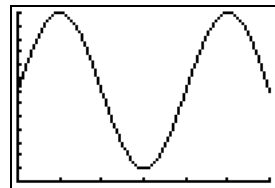
- (f) By how many degrees is each curve separated? Explain the difference.

Both curves appear to be sine functions. The curves are separated by 90° . Each quarter on the wheel has an angle of 90° . In the diagram in step 1(b), the circle has rotated one-quarter or 90° clockwise. In the graph above, the corresponding curve is translated to the right by one unit or 90° .

- (g) State the equation of the curve in step 1(a) in terms of the angle of rotation, θ . Verify that the graph of this equation matches your graph.

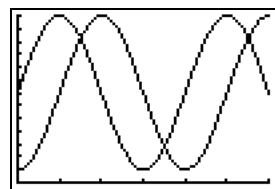
$$f(\theta) = 6 \sin \theta + 7$$

Enter the equation into the equation editor of the graphing calculator. Set **Xmin** to 0 and **Xmax** to 640.



- (h) Replace θ with $(\theta - 90^\circ)$ in the equation. Use a graphing calculator to graph the equation and compare it to the corresponding hand-drawn graph in step 1(b).

The new function becomes
 $f(\theta) = 6 \sin (\theta - 90^\circ) + 7$.



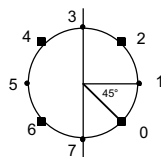
This graph and the corresponding hand-drawn graph in step 1(b) are the same.

- (i) Describe the transformation of the original graph in step 1(a) to the graph in step 1(b). Explain the change in terms of the starting position.

The first graph is translated horizontally or shifted to the right by 90° . This transformation corresponds to a one-quarter turn in a clockwise direction from the starting position.

3. Suppose the function for the height, h , in metres on the Ferris wheel is modelled by
 $h(\theta) = 6 \sin (\theta - 45^\circ) + 7$.

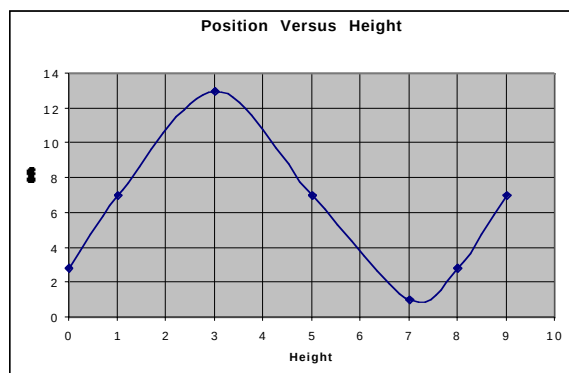
- (a) Sketch the starting point on the Ferris wheel.



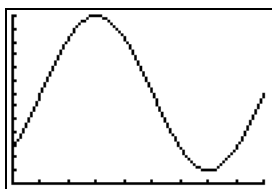
(b) Graph the equation by hand.

Look at the triangle, which has angles 45° , 45° , and 90° . The hypotenuse, or the radius, is 6, so $2a^2 = 6^2$ or $a = 4.2$, to one decimal place. The height of the wheel at this starting point is $7 \text{ m} - 4.2 \text{ m}$, or 2.8 m .

Position	0	1	3	5	7	8	9
Height	2.8	7	13	7	1	2.8	7

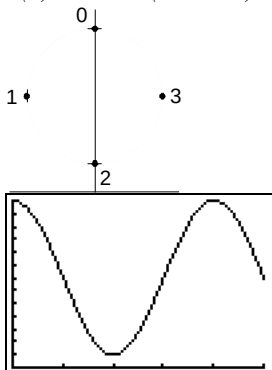


(c) Verify your hand-drawn graph using a graphing calculator.

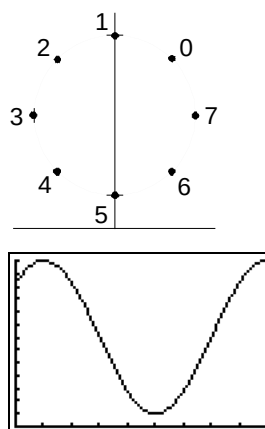


4. Sketch the starting point on a circle and graph each function.

(a) $h(\theta) = 6 \sin(\theta + 90^\circ) + 7$



(b) $h(\theta) = 6 \sin(\theta + 45^\circ) + 7$

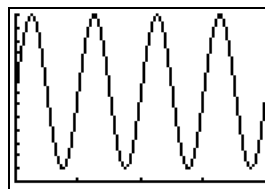


5. (a) The Ferris wheel's speed in step 1(a) is one revolution every 1.5 min. Write the equation to represent the height if the angle at the boarding point is in the standard position. Graph the equation.

One revolution every 1.5 min is $\frac{360^\circ}{90 \text{ s}}$

$4^\circ/\text{s}$.

Then $f(t) = 6 \sin(4t)^\circ + 7$, where t is the time in seconds.



(b) Replace θ with $(\theta - 45^\circ)$ in the equation. Graph the new equation. Describe the horizontal shift of the graph of the equation with θ to the graph of the equation with $(\theta - 45^\circ)$. What is the starting position on the Ferris wheel? Describe this point in terms of the angle in standard position.

$f(\theta) = 6 \sin(\theta - 45^\circ) + 7$

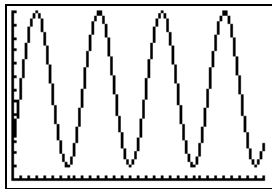
But $\theta = (4t)^\circ$. Then

$f(t) = 6 \sin[(4t)^\circ - 45^\circ] + 7$, where t is the time in seconds.

Simplifying gives

$f(t) = 6 \sin 4(t - 11.25)^\circ + 7$. The base graph is horizontally translated or shifted right by 11.25° .

Help students see that there are four cycles of the sine function in an interval of 360° . The 45° angle must be divided by 4 to meet the initial condition that there is one revolution every 1.5 min.



The starting position on the Ferris wheel is 45° below the horizontal. The angle in standard position is 45° .

6. State the horizontal shift for $y = 2 \sin 3(\theta + 90^\circ) + 5$ and compare it to the horizontal shift for $y = 2 \sin (3\theta + 90^\circ) + 5$.

Draw students' attention to the position of the 3 in each equation.

For $y = 2 \sin 3(\theta + 90^\circ) + 5$, the shift is 90° to the left. The second equation is not in the general form. Rewrite as

$y = 2 \sin 3(\theta + 30^\circ) + 5$. Then the horizontal shift is 30° to the left.

7. Explain how the value of b affects $y = a \sin k(\theta + b) + d$ and $y = a \cos k(\theta + b) + d$.

The value of b determines the horizontal translation (shift). When $b > 0$, the curve shifts left from the base curve. When $b < 0$, the curve shifts right from the base curve.

ASSESSING LEARNING SKILLS

Work Habits

Observe whether students follow through by independently reading the examples and making appropriate notes. Do they complete assigned tasks? Use the Generic Rubric for Work Habits to assess the work habits of each student.

Additional Example: Functions and Relations:

Determine the amplitude, period, phase shift, the equation of the axis of the curve, and the maximum

and minimum values for $y = 3 - 5 \cos \left(\frac{\theta}{2} - 30^\circ \right)$.

Solution

Rewrite the equation in the form

$$y = a \cos k(\theta + b) + d.$$

$$y = -5 \cos \frac{1}{2}(\theta - 60^\circ) + 3$$

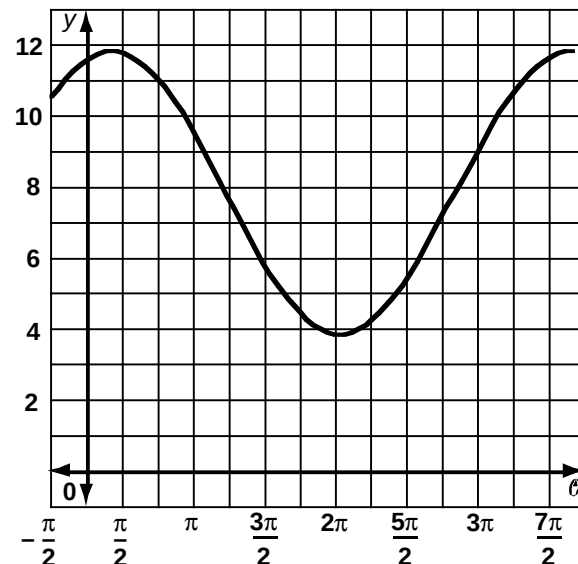
The amplitude, a , is 5.

$k = \frac{1}{2}$. Then $2 \times 360^\circ = 720^\circ$, and the period is 720° . The phase shift, b , is -60° . The curve has been translated 60° to the right of the base curve. $d = 3$. The vertical translation raises the axis of the curve to $y = 3$.

The minimum is $-5(1) + 3 = -2$ and the maximum is $-5(-1) + 3 = 8$.

Additional Example: Functions

Determine the equation represented by the graph.



Solution

$y = a \cos k(\theta + b) + d$ is an appropriate model.

The amplitude, a , is $\frac{(12 - 4)}{2} = 4$. The period is

$$\frac{7\pi}{2} - \left(-\frac{\pi}{2} \right) = \frac{8\pi}{2} \text{ or } 4\pi. \text{ Then}$$

$$\frac{2\pi}{k} = 4\pi$$

$$k = \frac{2\pi}{4\pi}$$

$$= \frac{1}{2}$$

The phase shift is $\frac{\pi}{2}$ to the left of the base curve.

Then $b = \frac{\pi}{2}$. The axis of the curve is

$y = 8$. Then $d = 8$.

The equation is $y = 4 \cos \frac{1}{2} \left(\theta + \frac{\pi}{2} \right) + 8$.

EXTENSION TO THE ADDITIONAL EXAMPLE: FUNCTIONS

Ask students to consider these variations to the question:

1. What is the equation if the graph is translated down 10 units?
2. What is the equation if the graph is reflected about its axis?
3. What is the equation if the graph is reflected about the θ -axis?

Practise, Apply, Solve 5.6

7. Knowledge and Understanding, p. 456

Management Suggestion

Students could work in pairs and take turns asking, “Is the equation in the required form?” and “How do you know this form is correct?” Students should respond to the question individually.

Achievement Chart Connection

Students need to recognize the values of a , k , b , and d and relate them to amplitude, period, phase shift, and vertical translation.

Level 3 Performance

Students must demonstrate understanding of the roles of a , k , b , and d in transforming the base sinusoidal functions. In particular, they need to correctly factor k when the equations are given as $y = a \sin (k\theta + kb) + d$ or $y = a \cos (k\theta + kb) + d$.

14. Communication, p. 457

Management Suggestion

Do this question as a whole class activity. For (a), ask some students go to the board or overhead and list one similarity or one difference. Continue until the lists are complete.

Then direct all students to write their answers to (b) and (c). Select several students to orally and visually present to the class.

Achievement Chart Connection

Students should communicate their reasoning orally, in writing, and graphically, using correct terminology.

Level 3 Performance

Students should justify each similarity and difference and refer to the properties of each graph (shape, period, maximum and minimum values, and the axis of each curve). As well, students should include the concept of phase shift in their explanations and visuals.

21. Application, p. 458

Management Suggestion

Assign the question for homework and tell students that you will collect their responses the next day for evaluation. Remind students to answer all parts of the question. They can use a graphing calculator to draw the graph, but they must either print the graph (TI-GRAPH LINK) or sketch the graph and label it. Ask students to discuss blood pressure with their families and to find out how a doctor or nurse takes blood pressure.

Achievement Chart Connection

Students apply the concepts of sinusoidal functions to a health application. They need to correctly identify the parameters of the equation and understand the meaning of the values.

Level 3 Performance

The concept of blood pressure may be familiar to some students. Students need to determine the values of a , k , and d and understand them in the context of blood pressure. As well, they need to see the parallel between the maximum and minimum values of a function and high and low blood pressures.

25. Check Your Understanding, p. 459

Management Suggestion

Have students discuss the question in pairs. Take up the answers to this question with the class as an opportunity to reinforce the main ideas of this section.

Students review transformations and sketching graphs. Remind students to apply transformations in the order given on text page 452.

26. Thinking, Inquiry, Problem Solving, p. 459 Management Suggestion

Arrange students into small groups and ask them to spend about ten minutes talking about the question and how to answer it. Ask students to prepare a group response as if they were presenting it to a local conservation authority.

Achievement Chart Connection

Students need to apply the steps of an inquiry or problem-solving process.

Level 3 Performance

Students must explain how the populations of rabbits and fox are related and how the increase or decrease in one population is followed by a decrease or increase in the other. Their arguments should refer to sinusoidal functions and the application of transformations to graphing.

ASSESSMENT SUGGESTION

Communication, Inquiry, Problem Solving

Evaluate several group presentations for question 26.

Use the Generic Rubric for Thinking, Inquiry, and Problem Solving and the Generic Rubric for Communication for assessment. Discuss key aspects of the rubrics so that students know what is expected. Select the level that best describes each group as a whole.

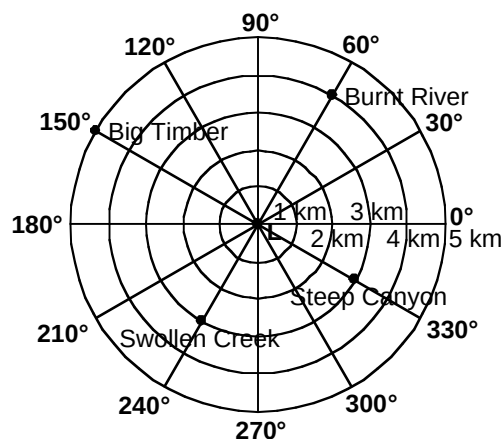
Extension for both Functions and Relations and Functions

Polar Coordinates

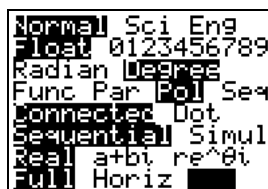
One way to name a point in two dimensions is with Cartesian, or rectangular, coordinates. The Cartesian coordinates for point P are $P(x, y)$ or $P(r \cos \theta, r \sin \theta)$. Another way to name the same point is with polar coordinates. The ordered pair (r, θ) describes a point where r is the polar distance or radius from the centre of a circle to the point and θ is the polar angle, which is an angle in standard position whose vertex is at the centre of the circle.

- Draw a series of concentric circles with radii 1, 2, 3, and 4 units on polar graph paper. Mark angles in standard position from 0° to 360° in steps of 15° .
- The logging camp, $L(0, 0^\circ)$ is at the centre of

the map. State the polar coordinates, (r, θ) , for each logging site shown on the map.



- List three other ways to locate the Steep Canyon site using polar coordinates. What type of angles are in your list?
- How would you find polar coordinates on a map that does not have a polar grid?
- Convert each point $P(x, y)$ to the polar coordinates, $P(r, \theta)$. Round r to one decimal place and round θ to the nearest degree. Graph the polar coordinates on polar graph paper.
 - $(3, 4)$
 - $(-2, 4)$
 - $(1, 3)$
- Convert each pair of polar coordinates, (r, θ) , to rectangular coordinates, (x, y) . Round to one decimal.
 - $(5, 60^\circ)$
 - $(3, -45^\circ)$
 - $(2, 240^\circ)$
- You can use a graphing calculator to graph the polar equation $r = 5 \sin 2\theta$. To graph the equation, first set the mode to degrees and polar.



Then set the window for the domain $0^\circ \leq \theta \leq 360^\circ$ in steps of 15° . Extend the θ - and y -axes by 5 units in each direction to accommodate the radius of 5 units.

```

WINDOW
θmin=0
θmax=360
θstep=15
Xmin=-5
Xmax=5
Xscl=1
Ymin=-5

```

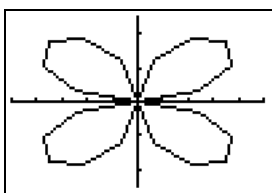
Enter the equation into the equation editor.

```

Plot1 Plot2 Plot3
Y1=5sin(2θ)
Y2=
Y3=
Y4=
Y5=
Y6=

```

Press **GRAPH**.



- (h) Predict the graph of $r = 5 \cos 2\theta$. Then graph this function and verify your prediction.
- (i) Graph $r = 4 \sin k\theta$ and $r = 4 \cos k\theta$ for $k = 4$ and $k = 6$ and then do the same for $k = 3$ and $k = 5$. Comment on the shape of the curves when k is an even number and when k is an odd number.
- (j) An ambulance is sent to an accident site at the same time that a helicopter is dispatched. Who will be more likely to use a rectangular grid to locate the accident site, the ambulance driver or the helicopter pilot? Who will be more likely to use a polar grid, the ambulance driver or the helicopter pilot? Explain your reasoning.

Extension for Functions and Relations Students

Polar coordinates can be tied to earlier work on complex numbers through De Moivre's theorem. Ask students to research De Moivre's theorem, to state it, and to give several examples.

De Moivre's Theorem

This theorem is a rule for raising a complex number to a power when the number is expressed in polar form.

$r(\cos \theta + i \sin \theta)$ is a complex number and n is a positive integer. Then

$$[r(\cos \theta + i \sin \theta)]^n = r^n(\cos n\theta + i \sin n\theta)$$

For example,

$$\begin{aligned}
 (\sqrt{2} + i\sqrt{2})^2 &= [2(\cos 45^\circ + i \sin 45^\circ)]^2 \\
 &= 4(\cos 90^\circ + i \sin 90^\circ) \\
 &= 4i
 \end{aligned}$$

Questions with a Functions and Relations:

Focus:

1–14, 18, 20, 21, 23–26

Questions with a Functions Focus:

1–11, 14, 18, 21, 22, 25, 26

The Chapter Problem—How Much Daylight?

By the end of this section, students should be able to relate the axis of the graph to the vertical translation and describe the axis of the graph in terms of hours of daylight. They should recognize that sinusoidal data can be expressed as either a sine function or a cosine function. The value of b determines whether the function is sine or cosine.

The Chapter Problem Answers are on p. 5-61 of the *Teacher Resource*.

Challenge 1

Remind students that they should be able to

- describe the transformations of the graph using the values of a , k , b , and d

Challenge 2

Remind students that they should be able to

- describe the transformations of the initial part of the graph using the values of a , k , b , and d
- add or continue building the model of a wave action pool

The Challenge Answers are on p. 5-68 of the *Teacher Resource*.