

5.7 Modelling Periodic Phenomena

Specific Expectations

The following expectations are common to both the Functions and Relations course and the Functions course.

- TF3.01 sketch the graphs of $y = \sin x$ and $y = \cos x$, and describe their periodic properties
- TF3.03 determine the amplitude, period, phase shift, domain, and range of sinusoidal functions whose equations are given in the form $y = a \sin(kx + d) + c$ or $y = a \cos(kx + d) + c$
- TF3.04 sketch the graphs of simple sinusoidal functions [e.g., $y = a \sin x$, $y = \cos kx$, $y = \sin(x + d)$, $y = a \cos kx + c$]
- TF3.05 write the equation of a sinusoidal function, given its graph and given its properties
- TF4.01 determine, through investigation, the periodic properties of various models (e.g., the table of values, the graph, the equation) of sinusoidal functions drawn from a variety of applications.
- TF4.02 explain the relationship between the properties of a sinusoidal function and the parameters of its equation, within the context of an application, and over a restricted domain
- TF4.04 pose and solve problems related to models of sinusoidal functions drawn from a variety of applications, and communicate the solutions with clarity and justification, using appropriate mathematical form

Evidence of Learning

Students should be able to

- identify periodic data
- determine the specific equation that represents given data using the general form of a sinusoidal curve
- model and solve real-life problems using sinusoidal equations

Text Page 460

Time Required Functions and Relations:

105 min

Time Required Functions: 150 min

Materials and Equipment

- TI-83 Plus calculator, graph paper

Overview

- Review the properties of periodic phenomena: period, maximum and minimum values, axis of the curve, and amplitude.
- Review the general form of a sinusoidal curve, $y = a \sin k(\theta + b) + d$ and $y = a \cos k(\theta + b) + d$, and determine a , k , b , and d .

The approach for students in either course is the same, start with a set of data and develop a model.

Students return to the waxing and waning of the moon first introduced in Section 5.1. They review that the data is periodic, the model is sinusoidal, and the properties of the model. The algebraic models are $y = a \sin k(\theta + b) + d$ or $y = a \cos k(\theta + b) + d$. Their task is to write a specific equation for this data by determining a , k , b , and d and then do the same for other data.

Management Options

Pairs or Groups – Divide the class into small groups. Ask students to review their notes from section 5.1. How can they tell whether data is periodic and sinusoidal? Specifically, ask them to study the waxing and waning of the moon as shown on text page 460, including the properties of the graph. After about five minutes, have one group sketch the graph at the board or the overhead and indicate the properties. Ask each group to discuss the general form of the sinusoidal model and how they could determine the specific equation. Have one group verbally report to the class and write key points on the board or an overhead transparency.

Review the cosine model in Example 1. Complete the introduction by having students graph both models using graphing calculators. Half the groups can graph the sine model and the other half can graph the cosine model. Select one group from each half to sketch the graphs at the board or at the overhead.

Have groups work through Example 2 and discuss the *Consolidate Your Understanding*. Then various students can report to the class about the *Consolidate Your Understanding*. Ask each student to record a summary of the discussion and presentation.

Teacher-Led Discussion Put the table for the waxing and waning of the moon on the board or on the overhead and draw the scatter plot and the curve of best fit. Focus on the periodic and sinusoidal nature of the graph and the properties: the period is 29.5 d, the maximum value is 1, the minimum value is 0, the axis of the curve is $y = 0.5$, and the amplitude is 0.5.

Indicate the general form of a sinusoidal equation and determine the values of a , k , b , and d . Verify that the equation represents the hand-drawn graph using a graphing calculator and TI-83 view screen.

Lead students through Example 2 and build a board or overhead summary of the *Consolidate Your Understanding*.

Independent Work – Have students read the introduction, the information about graphing with a graphing calculator, and Example 1. Students should consolidate by doing Example 2 in their notebooks. Help students think through the *Consolidate Your Understanding* and ask them to write a summary of their thoughts.

ASSESSING LEARNING SKILLS

Teamwork

If students are working in pairs or small groups, assess teamwork. Observe the sharing of ideas between students and the overall contribution made by each student in the group. Observe if students are working on the task and if they show respect for each other. Use the Generic Rubric for Teamwork for assessment.

Consolidate Your Understanding, text page 462

1. *How do you decide that a trigonometric model best represents the data?*

Use a graphing calculator to draw a scatter plot or draw the graph of the data. Observe if the graph is trigonometric.

2. *How do you decide which trigonometric model to use?*

For sinusoidal data, use either the sine function or the cosine function, but the resulting equation should be the simpler of the two, that is, the one with the fewer terms. Remember that the sine and cosine curves are similar; the difference between them is a phase shift. Determine the values of a , k , b , and d for one of the functions. The values for k and d will be the same for the other function, so focus on the phase shift. Will a translation left or right make one curve more like the model? Will reflecting one curve, that is, changing the sign of a , make one curve more like the model?

For a graph with vertical asymptotes, the data might be modelled using a trigonometric function.

3. *How can you confirm that the model represents the data?*

Draw the scatter plot of the data and then graph the equation of the model on the same axes. The graph of the equation should be a good fit for the points of the scatter plot.

Additional Example: Functions and Relations:

The sales of a seasonal product seem to rise and fall, following the pattern of a sinusoidal curve. Maximum sales are about 150 000 units and occur in April. Minimum sales are about 50 000 units and occur in October. This pattern repeats itself on a yearly basis. Determine an equation that models the monthly sales.

Solution

The axis of a sinusoidal curve occurs halfway between the maximum and minimum values.

Then the average is $\frac{(150\,000 + 50\,000)}{2} = 100\,000$. Therefore, the vertical translation is 100 000.

The amplitude is half the difference between the maximum and minimum values.

Then $\frac{(150\,000 - 50\,000)}{2} = 50\,000$.

The period is 12 months.

$$12k = 2\pi$$

$$k = \frac{\pi}{6}$$

$$k \doteq 0.5236$$

January represents month 0. But the maximum sales are in April, so the curve shifts left one month (examine each quarter of the curve).

Then the number of units, S , sold per month, t , where t at 0 is January, is

$$S(t) = 50\,000 \sin 0.5236(t - 1) + 100\,000$$

Additional Example: Functions

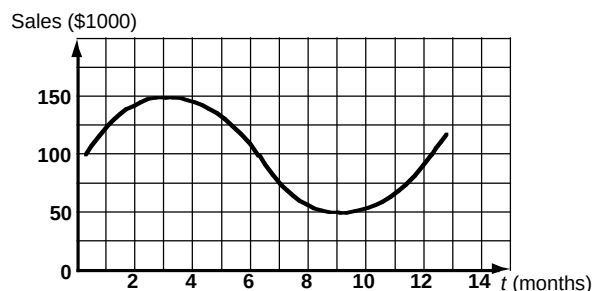
The data for sales of a seasonal product are collected. The results are shown in the table.

Month	Sales (thousands of dollars)
Feb	100
Apr	150
Aug	99
Nov	50
Feb	100
Apr	150

Determine an equation that models the data.

Solution

Draw a scatter plot of the data. The period is 12 months, the maximum value is 150 000, the minimum is 50 000. Use this information to determine the values of a , k , b , and d in the equation $y = a \sin k(\theta + b) + d$.



$$a = \frac{(150\,000 - 50\,000)}{2} = 50\,000$$

The axis of the curve is $y = \frac{150\,000 + 50\,000}{2}$ or $y = 100\,000$. Then $d = 100\,000$.

The shape of the sine curve starts at month 1, which is a shift to the right from the base curve. Then $b = -1$.

$$12k = 2\pi$$

$$k = \frac{\pi}{6}$$

$$k \doteq 0.5236$$

$$S(t) = 50\,000 \sin 0.5236(t - 1) + 100\,000$$

ASSESSING LEARNING SKILLS

Independence

If students are working independently, assess how much they need the teacher's assistance. Use the Generic Rubric for Independence Rubric for assessment.

Practise, Apply, Solve 5.7

8. Knowledge and Understanding, p. 464

Management Suggestion

Ask students to sketch each graph and highlight one cycle of the sine function, beginning at a point that is closest to the θ -axis. They use this cycle to determine the value of b .

Achievement Chart Connection

Students must demonstrate their understanding of the shape of the base sine graph and of amplitude, period, maximum and minimum values, and axis of the curve. As well, they should recognize vertical and horizontal translations and transformations and be able to determine the values of a , k , b , and d .

Level 3 Performance

Students might sketch each curve and label the properties. The values of a , k , b , and d should be correct.

ASSESSMENT SUGGESTION

Knowledge and Application, Understanding

Ask students to select their responses to two assigned questions from any *Practise, Apply, Solve* in this chapter such that one question is a *Knowledge and Understanding* and the other is an *Application*. Students' responses to these questions should show their best work. Collect and evaluate these responses.

14. Application, p. 465

Management Suggestion

This question is a fairly straightforward application. Have each student complete the question independently. Take up the answers to the question with the class to help consolidate the section.

Achievement Chart Connection

Students must apply the concepts of periodic phenomena to identify the model generated by the data. They should develop an equation that models the situation and use that equation to solve both the dependent and independent variables.

Level 3 Performance

Once students graph the data, they should recognize that they can use a sinusoidal model to solve the problem. Responses should include a sinusoidal model with values for a , k , b , and d .

18. Communication, p. 466

Management Suggestion

Have students work in pairs to discuss how to graph all the data on the same axes. Once students have graphed the data, they should determine the individual functions from the graph. When you take up the answers to this question, have several students describe orally the meaning of the amplitude and the vertical translation.

Achievement Chart Connection

Students must communicate with clarity and justification of reasoning and use mathematical language correctly most of the time.

Level 3 Performance

Responses should include correctly labelled hand-drawn graphs and equations in the correct form. Each curve may appear in a different colour. The differences in amplitude and the vertical translation for each city could be explained by the location of each city in relation to the equator, the variation in temperature from its average yearly temperature, the moderating effects of oceans, to name a few factors.

20. Check Your Understanding, p. 467

The amplitude is half the difference between the maximum and minimum values and is always positive, but the value of a can be positive or negative. A negative value implies the curve is reflected about its axis, so keep in mind the curve may be a reflection of the sine or cosine curve.

The value of k is determined from the period of the curve. It tells how many cycles there are for each full cycle of $y = \sin \theta$, 360° or 2π . Then

$$k = \frac{2\pi}{\text{period}} \text{ or } k = \frac{360^\circ}{\text{period}}.$$

So look for one rotated S shape in the graph and compare it to one complete cycle of the base sine curve.

Read the value of the phase or horizontal shift, b , from the graph. First decide where the typical sine or cosine curve begins. Look for a point close to the horizontal axis (x or θ), if not on it. Then find a corresponding point on the graph of the function. The difference is b , or $-b$, if the graph is to the right of the base sine or cosine curve.

The value of d is the axis of the curve. The axis of the curve is the average of the maximum and minimum values, which can be read from the graph.

21. Thinking, Inquiry, Problem Solving, p. 467 Management Suggestion

Have students meet in small groups when about ten minutes remain in the period. Ask them to answer the following questions.

What must be answered in the question? What kind of diagram would you draw? What can you conclude or interpret before solving the problem?

Tell students to write a summary of the above discussion for next class.

For the next class, reconvene the groups, and have individuals share data within each group.

Each group should find a solution to the problem. Tell students that one solution will be collected and marked. Select a student from each group and ask this student to orally respond to any part of the question. The mark for each student will be divided between marks for the group response and marks for the individual's response.

Achievement Chart Connection

This problem is complex, because it combines circular motion and horizontal motion occurring at the same time, and students must interpret the facts correctly.

Level 3 Performance

Students must note that both circular and horizontal motion begin at the bottom of the wheel where the nail is first picked up by the tire. They must also convert between centimetres and kilometres when determining height and distance. Students should also state assumptions about the speed of the car and other factors: the speed is assumed to be constant, the tires do not spin, and the car does not skid to a stop at any point.

Questions with a Functions and Relations:

Focus:

8–10, 13–22

Questions with a Functions Focus:

1–4, 8–10, 13–18, 20, 21

The Chapter Problem—How Much Daylight?

By the end of this section, students should be able to use the given data to develop an equation in the form $y = a \sin k(\theta + b) + d$. They should be able to enter the equation into a graphing calculator, graph the equation, and then compare this graph to a hand-drawn graph.

The Chapter Problem Answers are on p. 5-61 of the *Teacher Resource*.

Challenge 1

Remind students that they should be able to

- use the given data to develop an equation in the form $y = a \sin k(t + b) + d$ and to verify that the model is a reasonable representation of the original data

Challenge 2

Remind students that they should be able to

- use the given data to develop an equation in the form $y = a \sin k(t + b) + d$ for the initial part of the wave. They could then experiment with a graphing calculator to find an equation that models the motion of the waves as they reach the “shore”
- finish constructing their working model of a wave action pool

The Challenge Answers are on pp. 5-68 of the *Teacher Resource*.