

5.1 Periodic Phenomena

Specific Expectations

The following expectations are common to both the Functions and Relations course and the Functions course.

- TF4.01 determine, through investigation, the periodic properties of various models (e.g., the table of values, the graph, the equation) of sinusoidal functions drawn from a variety of applications
- TF4.04 pose and solve problems related to models of sinusoidal functions drawn from a variety of applications, and communicate the solutions with clarity and justification, using appropriate mathematical forms

Evidence of Learning

Students should be able to

- identify data or a graph as periodic
- from a graph of a function, determine the period, the maximum and minimum values, the equation of the axis of the curve, and the amplitude of the function

Text Page 408

Time Required Functions and Relations or Functions: 75 min

Materials and Equipment

- graph paper, ruler

Overview

- Graph data and identify the graph as linear, exponential, quadratic, or other nonlinear.
- Examine data that is self-repeating, including sinusoidal data.
- Determine properties of the graph of sinusoidal data.

Periodic Phenomena

The approach for this introductory section should be the same for either a Functions and Relations class or a Functions class.

Students begin by examining different sets of data. Then they draw the corresponding graphs and identify mathematical models. The graphs move from known nonlinear (exponential and quadratic) to self-replicating or periodic, and eventually to sinusoidal.

Students identify properties of periodic graphs. These properties include period, shape, maximum or minimum values, amplitude, and axis of the curve.

Management Options

Pairs – Students can work in pairs for the *Think, Do, Discuss* questions in Part 1. Select students to sketch their graphs at the board or overhead and to identify the type of model. Have each pair of students verify their answers to the *Think, Do, Discuss* questions in Part 1 with another pair. Pairs can also answer the *Think, Do, Discuss* questions in Part 2. Take up the answers to these questions orally. Summarize and consolidate at the board or overhead and lead the class through the *Think, Do, Discuss* questions in Part 3 and also the *Key Ideas*. Ask students to read Example 1 and to discuss, as pairs, the periodic nature and properties of each situation. Ask students what could be done to the vibrating metre stick to make the graph periodic. Lead the class through Example 2 and have pairs read Example 3.

Teacher Demonstration – Work through parts 1, 2, and 3 at the board or overhead, having various students plot the points and interpolate as required. Discuss the meaning of full moon and new moon and help students see the progression from one to the other in terms of illumination. Summarize and consolidate using the *Key Ideas*. Have students read examples 1 and 2. Ask students to explain in their own words. Do Example 3 at the board or overhead.

Independent Work – Have students answer the *Think Do Discuss* questions in Part 1 and, after about ten minutes, take up the answers orally. Repeat for Part 2. Do Part 3 as a whole group activity. Have students read examples 1, 2, and 3 and have them select one example that best summarizes the *Key Ideas*. Have each student write that example in his or her notebook.

EXTENSION

Ask students to consider the following question:

1. In Example 1, how could the graph of the motion of the vibrating metre stick be made periodic?

Part 1: Investigating Periodic and Non-Periodic Models

Think, Do, Discuss, text page 408

1. (a) Draw and label a separate scatter plot for each situation.
 - (b) Which situation can be modelled with an exponential relation? Justify your answer. Draw the curve of best fit and comment on how well the model fits the data.
 - (c) Which situation can be modelled with a quadratic relation? Justify your answer. Draw the curve of best fit and comment on how well the model fits the data.
 - (d) Which situation is neither exponential nor quadratic but is still nonlinear? Draw a curve of best fit to match the data points. How well does the curve fit the model?

The height of the moving model rocket can be modelled by a quadratic relation. The bacterial growth can be modelled by an exponential relation. The gymnast's breathing can be modelled by a nonlinear relation, but this relation is neither exponential nor quadratic. All curves closely fit their respective models.
2. Use your graphs from step 1 to answer these questions.
 - (a) How high is the rocket after 4.5 s?

After 4.5 s, the rocket is about 35 m high.

- (b) About how many bacteria should there be after 8 h?

After 8 h, there would be about 400 000 bacteria.

- (c) What is the velocity of the air after 6.75 s? Is the air being inhaled or exhaled at this time? Explain.

After 6.75 s, the air is exhaled at about 1.5 L/s.

3. (a) How long does it take for one complete cycle of inhaling and exhaling?

One cycle is 4 s.

- (b) What is the maximum velocity of the air? What is the minimum velocity?

The maximum velocity is 1.75 L/s. The minimum velocity is -1.75 L/s.

4. (a) What is the velocity of air for the gymnast after 13 s? after 16 s?

After 13 s, the velocity is 1.75 L/s. After 16 s, the velocity is 0 L/s.

- (b) Explain how the graph was extended to extrapolate the required information.

The cycle repeats itself every 4 s.

5. (a) Describe the shape of the graph of the gymnast's breathing.

The graph is shaped like the letter S, turned 90° clockwise. The shape describes the pattern of breathing.

- (b) Explain why the shape of the curve is reasonable for this case.

The velocity of air reaches a maximum when the gymnast's lungs are full (or somewhat close to being) of air. As the gymnast exhales, the volume of air decreases to a minimum. The speed of the gymnast's breathing does not change, but the volume of air in the lungs does change over time. One exhale and one inhale form a cycle.

6. *As time goes on, the gymnast will relax. What changes would you expect to see in the graph?*

The gymnast will need to breathe less deeply, so the maximum and minimum values will decrease over time.

Part 2: Repeating Functions

Think, Do, Discuss, text page 409

1. (a) *Draw and label a scatter plot of the data.*

Label the independent variable as Day of the Year and label the dependent variable as Fraction of Moon Visible.

- (b) *Draw the curve of best fit.*

The curve of best fit will be sinusoidal.

2. (a) *Starting with day 1, how many days does it take for the shortest complete pattern of the graph to repeat?*

- (b) *Starting with day 6, how many days does the graph take to repeat?*

- (c) *On what other day could the graph begin and still repeat?*

The pattern repeats itself regardless of the starting point every 29 days to 30 days.

3. (a) *Extend the pattern of the graph to include the 95th day of the new millennium. Was the phase of the moon closer to a full moon or a new moon? Explain.*

The cycle is 29 days to 30 days. Then day 95 is day 65, plus 30 days. On day 95, as on day 65, the fraction of the moon that is visible is close to 0. Therefore, on day 65, there is a new moon.

- (b) *Extend the graph to predict the fraction of the moon that was visible on the summer solstice, June 21. Was the moon waxing or waning? Explain.*

There are 173 days from January 1, 2000 to June 21, 2000 (note the leap year).

June 21 is at, or close to, the maximum of the sixth cycle. So, on June 21, the moon was full, or had been waning for just a few days.

Part 3: Properties of Repeating Functions

Think, Do, Discuss, text page 410

1. *Redraw the graph of the phases of the moon for the first three months of the year.*

The graph should have three cycles and is an extension of the graph that students drew for *Think, Do, Discuss* question 1, Part 1.

2. *What is the maximum value of the curve? the minimum value? Explain the meaning of maximum and minimum in this case.*

The maximum value is 1.00 and the minimum value is 0. These values represent the full moon and the new moon, respectively.

3. (a) *Draw a straight, horizontal line through the graph that is halfway between the maximum and minimum values. This line is called the axis of the curve or axis of the shape. Write the equation of the axis of the curve using the maximum and minimum values.*

The axis of the curve should be horizontal, and the equation of the axis is $y = \frac{(1.00 + 0.00)}{2}$ or $y = 0.5$.

- (b) *The amplitude of a periodic function is the magnitude of the distance from this line to either the maximum or minimum value. How can the maximum and minimum values be used to calculate the amplitude?*

The amplitude is half the difference between the maximum and minimum values.

- (c) Draw one complete cycle of the graph that begins on the line and ends on the line. Draw a different cycle that also begins and ends on the line. Explain how the two cycles are alike and not alike.

Both cycles begin on the line and end on the line. The two cycles are congruent.

The locations of the two cycles along the horizontal axis are different.

ASSESSING LEARNING SKILLS

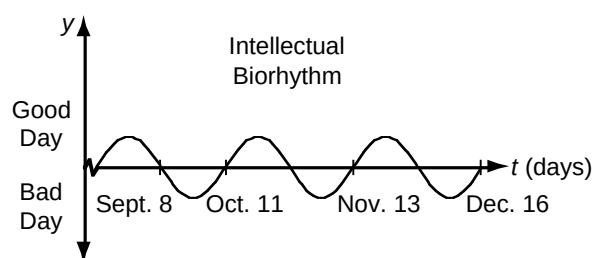
Work Habits

After students have completed the *Think Do, Discuss* questions in Part 3 and are working at their desks, circulate to verify that students are using class time appropriately and are working on the assignment. Use the Generic Rubric for Work Habits to assist you in their assessment.

Additional Example: Functions and Relations and Functions

A biorhythm graph is said to predict the ups and downs of daily intellectual life, which begins at birth. The graph represents John's intellectual biorhythm over several months.

- Explain why the graph is periodic.
- Extend the graph for one more cycle.
- What is the equation of the axis of the curve?
- If John could write his math test on a day of his choosing, what days would be best? What days would be the worst?



Solution

- The graph repeats itself every 33 days. From September 8 to October 11 is 33 days. From October 11 to November 13 is 33 days. From November 13 to December 16 is 33 days.
- Extend the graph to January 18.
- The x -axis is the axis of the curve. Therefore, $y = 0$.

- The best days occur at the maximum value of each cycle, on or about September 16, October 19, and November 21. The worst days occur at the minimum value of each cycle, on or about October 3, November 5, and December 8.

Practise, Apply, Solve 5.1

2. Communication, p. 414

Management Suggestion

In pairs or small groups, students could discuss each situation. During discussion, remind students to jot down key points or ideas.

Achievement Chart Connection

Students must explain orally or in writing how various real-life situations are periodic. Their explanations should be clear and justify why each situation is self-replicating.

Level 3 Performance

Students should explain orally or in writing, with justification, why the model for each situation is periodic. The notions of self-replicating, cycle, and period should be communicated clearly.

9. Application, p. 416

Management Suggestion

After students have had a few minutes to think about the question as individuals, arrange them into small groups. Sharing information and experiences is part of the learning process.

Achievement Chart Connection

Students need to understand cycles in a realistic situation.

Level 3 Performance

Students should recognize and apply the concepts of period, cycle, and amplitude to interpret the graph and understand the laundry cycle.

10. Knowledge and Understanding, p. 416

Management Suggestion

Suggest students draw the Ferris wheel and locate points on the wheel that correspond to points on the graph. Students will do this in Section 5.3.

Achievement Chart Connection

Students must identify the maximum and minimum values, locate the axis of the curve, and determine the amplitude and period of the curve.

Level 3 Performance

Students must accurately determine the maximum and minimum heights, the equation of the axis of the curve, and the amplitude and period of the curve. They must also demonstrate understanding of how the graph and the position of any point on the Ferris wheel are related.

**12. Thinking, Inquiry, Problem Solving, p. 416
Management Suggestion**

Consider approaching this question as a whole-class activity. Have one student be the driver and walk around an outline of the track, slowing down and speeding up as she walks around each corner. While this student is walking around the track, have several other students draw the graph of what they see on the board.

Achievement Chart Connection

Students need to understand the relation between the speed of the race car and its position at the corners of the track.

Level 3 Performance

Students may need more information to understand the context and should demonstrate their ability to collect more information. Students should model the relation in a graph.

**14. Check Your Understanding, p. 417
Management Suggestion**

Have students share their definitions orally. Note the similarities and differences of correct responses.

Ask a few student to write their examples of periodic functions on large sheets of chart paper. Post these displays around the classroom. Compare these periodic functions to others that arise as students work through the chapter. For a periodic function, the dependent variable follows a repetitive pattern as the independent variable increases or decreases.

The height above the road of the valve on a bicycle tire in relation to its rotation about the centre axle is an example of a periodic function. The height is the independent variable. The angle of rotation is the dependent variable. The valve is at its lowest height at the bottom of the rotation.

It rises to its greatest height at the top of the rotation and then continues to move to its lowest position. This cycle is repeated with each revolution of the wheel.

Questions with a Functions and Relations:**Focus:**

1–6, 9, 10, 12–14

Questions with a Functions Focus:

1–6, 9, 10, 12–14

The Chapter Problem—How Much Daylight?

By the end of this section, students should be able to draw the scatter plot of the data and the curve of best fit. They should also be able to determine that the data is periodic and to state the period. Finally, they should identify the maximum and minimum values of the curve, determine the axis of the curve, and calculate the amplitude.

The Chapter Problem Answers are on p. 5-61 of the *Teacher Resource*.

Challenge 1

Remind students that they should be able to

- find a Web site that shows average monthly temperature for a place close to where they live (for example, www.worldclimate.com)
- draw the scatter plot and the curve of best fit of the data and decide if the graph is periodic

Challenge 2

Remind students that they should be able to

- find information about wave action pools at the library or on the Internet

The Challenge Answers are on p. 5-68 of the *Teacher Resource*.