

6.3 Skill Builder: Using Special Triangles to Determine Exact Values

Specific Expectations

The following expectations are common to both the Functions and Relations and the Functions courses.

- TF2.03 represent, in applications, radian measure in exact form as an expression involving π (e.g., $\frac{\pi}{3}$, 2π) and in approximate form as a real number (e.g., 1.05)
- TF2.04 determine the exact values of the sine, cosine, and tangent of the special angles 0 , $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$, $\frac{\pi}{2}$ and their multiples less than or equal to 2π

Evidence of Learning

Students should be able to

- draw both special triangles
- use the special triangles to determine the exact values of trigonometric ratios of related angles in a given domain
- use the special triangles to solve linear trigonometric equations
- decide whether to use special triangles or a calculator

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Time Required Functions and Relations or Functions: 75 min

Overview

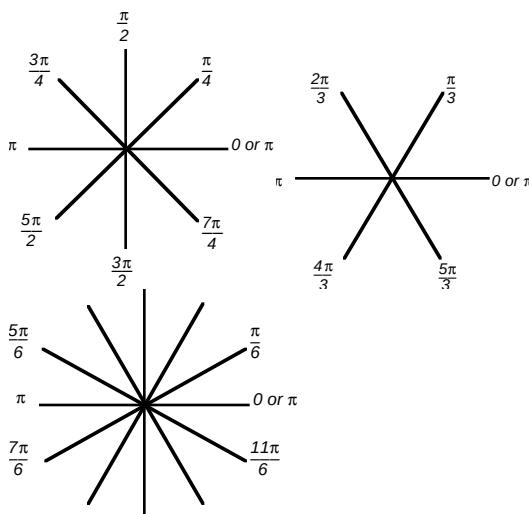
- Introduce the use of special triangles to calculate exact values of trigonometric ratios for 30° $\left(\frac{\pi}{6}\right)$, 60° $\left(\frac{\pi}{3}\right)$, or 45° $\left(\frac{\pi}{4}\right)$.
- Extend the use of these special triangles to the related angles.
- Use exact values to solve linear trigonometric equations.
- Hint at the concept of an identity.

Special Triangles

Exact values are easier to manipulate than decimal approximations that have many digits. The first special triangle is introduced as a right isosceles triangle, which appeared in the unit circle (see chapter 5, section 5.3, page 429). The second special triangle is introduced as an equilateral triangle.

Suggest that students visualize radian measure rather than convert radians to degrees. The special angles are $\frac{\pi}{6}$, $\frac{\pi}{4}$, and $\frac{\pi}{3}$.

Display and discuss the following diagrams. After discussion, students should record these diagrams in their journals.



TEACHER INTERVENTION

Students often think in error that a complete rotation is π radians. You might need to remind them frequently that π radians is only half a 360° or 2π rotation.

Management Options

Groups – Pairs could read this lesson together, ensuring each other's understanding. Circulate and ask questions to check students' understanding.

Teacher Demonstration – If time is limited, then demonstrate this section.

Independent Work – Individuals can practise reading a math text in a supportive environment. Circulate and ask questions to check students' understanding.

TEACHER INTERVENTION

Guide the first student(s) who has questions. Then this student could help another student who has the same question, and so on. Students clarify and consolidate their learning when they teach and explain to peers.

EXTENSION SUGGESTION

As partial fulfillment of an independent study requirement, a student could present this section. You might meet with the student prior to presentation to ensure comprehensive coverage of the topic and arrange any extras. While you may need to provide support for the student during the lesson, students will cooperate with a classmate.

ASSESSMENT SUGGESTION

Artistic or creative students could design posters, showing the special triangles or the graphs of the special angles, by hand drawing or using a computer.

MANAGEMENT SUGGESTION

Example 2

Remind students to also consider the quadrant in which the terminal arm lies and to use the CAST rule to confirm the sign.

TEACHER INTERVENTION

Example 5

Remind students that there are two possible solutions to most questions of this type, because each trigonometric ratio is positive in two quadrants or it is negative in two other quadrants. Sometimes the context of the question eliminates one solution.

Additional Example: Functions

Solve.

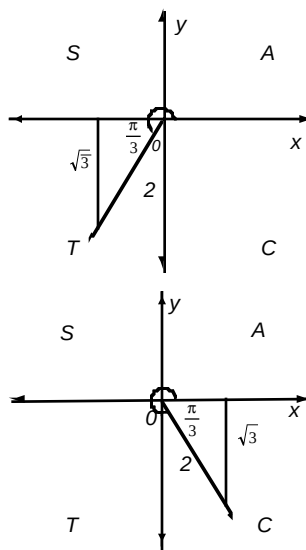
(a) $\sin \theta = -\frac{\sqrt{3}}{2}, 0 \leq \theta \leq 2\pi$

(b) $\cos \alpha = \frac{1}{\sqrt{2}}, 0 \leq \alpha \leq 360^\circ$

Solution

(a) $\sin \theta$ is negative in quadrants III and IV.

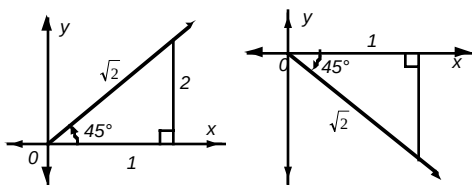
θ is related to $\frac{\pi}{3}$. $y = \sqrt{3}$ and $r = 2$.



Therefore, in quadrant III, $\theta = \frac{4\pi}{3}$,

and, in quadrant IV, $\theta = \frac{5\pi}{3}$.

- (b) $\cos \alpha$ is positive in quadrants I and IV. α is related to 45° . $x = 1$ and $r = \sqrt{2}$.



Therefore, in quadrant I, $\alpha = 45^\circ$ and, in quadrant IV, $\alpha = 315^\circ$.

Practise, Apply, Solve 6.3

Questions with a Functions and Relations Focus:

Focus:

1–11, 12 (every other part), 15

Questions with a Functions Focus:

1–9, 12 (every other part), 15

6. Remind students to find the acute related angle.

10. Point out that $(\sin \theta)^2$ is usually written as $\sin^2 \theta$ angle.

15. Check Your Understanding, p. 533

Management Suggestion

Partners could exchange solutions and discuss differences. Students should understand that several solutions to the same problem are valid.

Challenge 2

Students should not create a table or use a calculator for this challenge. Remind students that they should be able to draw and label the required special triangles, but they will learn how to prove identities in section 6.5.

The Challenge Answers are on p. 6-36 of the *Teacher Resource*.