

6.4 Exploration: Investigating Identical Expressions

Specific Expectations

The following expectation is common to both the Functions and Relations and the Functions courses.

- TF2.05 prove simple identities, using the Pythagorean identity, $\sin^2 x + \cos^2 x = 1$, and the quotient

$$\text{relation, } \tan x = \frac{\sin x}{\cos x}$$

Evidence of Learning

Students should be able to

- define an identity
- recognize the restrictions as the only values that do not satisfy an identity

Text Page 534

Time Required Functions and Relations or Functions: 30 min

Materials and Equipment

- graphing calculator or graph paper

Overview

- Introduce a simple trigonometric identity.
- Prove, graphically and algebraically, that an expression is an identity.
- Show that both sides of an equation must be equal for the equation to be an identity.

Management Options

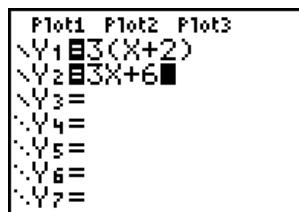
Groups – Have one partner graph the first function and the other partner graph the second function. Then partners should compare the two graphs.

Teacher Demonstration – Invite one student to draw the graphs using a TI-83 Plus and the view screen while other students answer the *Think, Do, Discuss* questions orally.

Defining an Identity

Think, Do, Discuss, text page 534

- With or without graphing technology, graph $y = 3(x + 2)$. On the same set of axes, graph $y = 3x + 6$.



- How many points of intersection exist between the two relationships?

These graphs have an infinite number of points of intersection.

- Solve $3(x + 2) = 3x + 6$.

$$3(x + 2) = 3x + 6$$

$$3x + 6 = 3x + 6 \text{ Subtract 6 from both sides.}$$

$$3x = 3x \quad \text{Divide both sides by 3.}$$

$$x = x$$

- What happens when you try to solve this equation?

The solution is a true statement, but does not give a unique answer.

- How many solutions are possible for this equation?

Any value in the domain would solve this equation. There are an infinite number of solutions.

- Are there any real numbers that do not satisfy the equation? Explain.

All real numbers satisfy the equation, because there are no restrictions.

TEACHER INTERVENTION

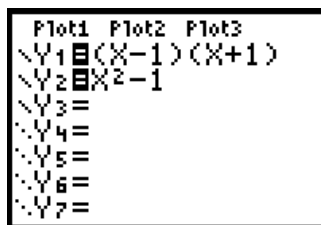
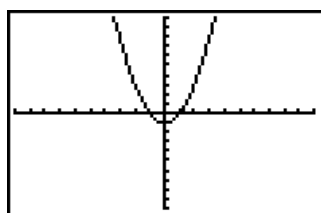
You might show another example involving restricted values. For example, the domain of $\frac{\sin \theta}{\cos \theta} = \tan \theta$ must include the restriction for the individual trigonometric functions. In this case, $\cos \theta \neq 0$ and $\tan \theta$ is not defined for 90° , 270° , 450° , and so on.

7. Compare the left side and right side of the original equation. How would you describe the relation between these expressions?

The expressions are identical.

8. Repeat steps 1 to 7 for $(x - 1)(x + 1) = x^2 - 1$.

Step 1



Step 2: These graphs have infinite number of points of intersection.

Step 3:

$$\begin{aligned} (x - 1)(x + 1) &= x^2 - 1 && \text{Expand.} \\ x^2 - x + x - 1 &= x^2 - 1 && \text{Simplify.} \\ x^2 - 1 &= x^2 - 1 && \text{Add 1 to both sides.} \\ x^2 &= x^2 && \text{Take the square root of each side.} \\ \pm x &= \pm x \end{aligned}$$

Step 4: The solution is a true statement, but does not give a unique answer.

Step 5: Any value in the domain would solve this equation. There is an infinite number of solutions.

Step 6: All real numbers satisfy the equation, because there are no restrictions.

Step 7: The expressions in the original equation are identical.

9. Equations like the ones in steps 3 and 8 are called **identities**. Explain why this name is appropriate.

Both an equation and an identity are statements of equality. However, an identity is true for all values of the variables. An equation is true for only certain values of the variable, for example, $x + 2 = 5$ is only true for $x = 3$. The name identity is appropriate because the sides of each equation are identical.

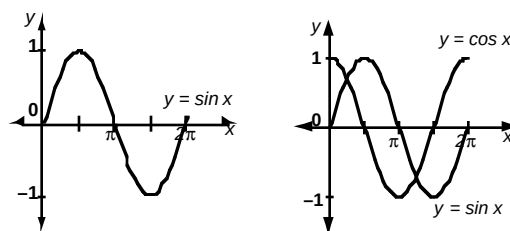
10. Are $4x + 6x = 10x$, $(x + 3)(x + 2) = x^2 + 5x + 6$, and $\sin x = \cos\left(x - \frac{\pi}{2}\right)$ identities?

Explain.

Yes. If the left side of $4x + 6x = 10x$ is simplified, both sides are the same. If the right side of $(x + 3)(x + 2) = x^2 + 5x + 6$ is factored, both sides are the same. Recall section 5.6. Shifting the base cosine curve right by $\frac{\pi}{2}$ gives the base sine curve.

Therefore, $\sin x = \cos\left(x - \frac{\pi}{2}\right)$ is a true equation.

Since the sides of each equation are identical and true for every value of the domain, all of the equations are identities.



11. Explain how to show graphically that a given equation is an identity.

Graph each side of the equation separately. The graphs will be the same if the equation is an identity.

Challenge 2

Remind students that they will learn how to prove identities in section 6.5. The Challenge Answers are on pp. 6-45 of the *Teacher Resource*.