

6.6 Skill Builder: Solving Quadratic Trigonometric Equations

Specific Expectations

The following expectations are common to both the Functions and Relations and the Functions courses.

- **TF2.06** solve linear and quadratic trigonometric equations (e.g., $6 \cos^2 x - \sin x - 4 = 0$) on the interval $0 \leq x \leq 2\pi$
- **TF2.07** demonstrate facility in the use of radian measure in solving equations and in graphing

Evidence of Learning

Students should be able to

- solve quadratic trigonometric equations by factoring, using special triangles, or using a calculator
- use graphing technology to solve for the zeros
- select the best method to use
- determine all possible solutions for the given period and domain

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Time Required Functions and Relations:
75 min

Time Required Functions: 105 min

Materials and Equipment

- scientific calculator
- graphing calculator

Overview

- Define a quadratic trigonometric equation.
- Solve a quadratic trigonometric equation by factoring and by using graphing technology.
- Solve a quadratic trigonometric equation using special triangles or a scientific calculator.

Quadratic Trigonometric Equations

This section outlines several methods for solving quadratic trigonometric equations. Students must learn to choose the appropriate method for the given period and domain.

Management Options

Groups – Pairs could read examples 1 to 4 together, ensuring each other's understanding. For Example 5, pair a student who has experience using a graphing calculator with a student who has less experience; both students will benefit. Circulate and ask questions to check students' understanding.

Teacher Demonstration – If time is short, you might demonstrate this lesson.

Independent Work – This lesson provides an opportunity for individuals to practise reading a math text in a supportive environment. Circulate and ask questions to check students' understanding. Also, since students used graphing calculators to calculate zeros of functions in section 5.8, invite a student to present Example 5 on the overhead, as part of an independent study project.

MANAGEMENT SUGGESTION

Example 2

Emphasize that the solutions are visible in the graph.

Example 3

Alternatively, suggest replacing $\cos 2\theta$ with x . Then $2 \cos^2 2\theta + \cos 2\theta - 1$ (the second line in the solution) becomes $2x^2 + x - 1$, which can be factored as $(2x - 1)(x + 1)$. Solve for x . Then replace x with $\cos 2\theta$ to get $\cos 2\theta = \frac{1}{2}$ and $\cos 2\theta = -1$.

Remind students that, if the domain is in radian measure, the answer should be in radian measure as well.

Example 4

Just as with an identity, it is good practice to express the equations in terms of one trigonometric function, which, in this case, is sine.

Additional Example: Functions and Relations

Solve $\sin^2 \alpha - 5 \sin \alpha + 3 = 0$, $0 \leq \alpha \leq 360^\circ$.

Solution

Let $x = \sin \alpha$. Then $\sin^2 \alpha - 5 \sin \alpha + 3 = 0$ becomes $x^2 - 5x + 3 = 0$. Use the quadratic

formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where $a = 1$, $b = -5$, and $c = 3$.

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(3)}}{2(1)}$$

$$x = \frac{5 \pm \sqrt{13}}{2}$$

$$x \doteq 4.302\,775\,638 \text{ or } x \doteq 0.697\,224\,362\,3$$

But $x = \sin \alpha$, so $\sin \alpha \doteq 4.3$ and $\sin \alpha \doteq 0.7$.

Since $-1 \leq \sin \alpha \leq 1$, $\sin \alpha \neq 4.3$. Therefore, $\sin \alpha \doteq 0.7$. Use the CAST rule and related angles to find α . The sine function is positive in quadrants I and II. Therefore, $\alpha \doteq 44^\circ$ in quadrant I and $\alpha \doteq 136^\circ$ in quadrant II.

Additional Example: Functions

Solve $\tan^2 \beta = 3$, $0 \leq \beta \leq 2\pi$.

Solution

$$\tan^2 \beta = 3$$

$$\tan \beta = \pm\sqrt{3}$$

Use the CAST rule and related angles to find β . The tangent function is positive in quadrants I and III. The tangent function is negative in quadrants II and IV.

$$\tan \beta = +\sqrt{3} \text{ in I and III, and}$$

$$\tan \beta = -\sqrt{3} \text{ in II and IV}$$

By related angles,

$$\beta = \frac{\pi}{3} \text{ and } \beta = \frac{4\pi}{3}$$

$$\beta = \frac{2\pi}{3} \text{ and } \beta = \frac{5\pi}{3}$$

Practise, Apply, Solve 6.6

Questions with a Functions and Relations Focus:

2, 4–16

Questions with a Functions Focus:

1–15

15. Check Your Understanding, p. 547

Management Suggestion

Have teams compare factoring to graphing technology for solving quadratic trigonometric equations. Consolidate ideas in a class discussion. Students should record the results of this discussion and the answer to question 15 (c) in their journals.

Solving a quadratic trigonometric equation algebraically is frequently easier, but determining all the possible answers for the given domain and period of the function is more difficult.

The Chapter Problem—What Time Is It?

Students should use graphing technology to solve the seemingly complex trigonometric equations in question CP7 (d), which requires them to consider the domain and related angles.

The Chapter Problem Answers are on p. 6-36 of the *Teacher Resource*.