

7.5U Exploration: Waxed-Paper Models

Specific Expectations

- LC1.01U construct a geometric model (e.g., a diagram created by hand, a diagram created by using dynamic geometry software) to represent a described locus of points; determine the properties of the geometric model; and use the properties to interpret the locus (e.g., the locus of points equidistant from two fixed points is the right bisector of the line segment joining the two fixed points)
- LC1.02U explain the process used in constructing a geometric model of a described locus
- LC1.04U construct geometric models to represent the locus definitions of the conics

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Time Required Functions and Relations:

30 min

Materials and Equipment

- rectangles of waxed paper (about 10 cm by 15 cm)
- circles of waxed paper (about 10 cm to 15 cm in diameter. **Hint:** trace around a roll of masking tape)
- *The Geometer's Sketchpad*

Overview

For this exploration, students

- create geometric models of conics by paper folding
- use *The Geometer's Sketchpad* to model the paper-folding constructions
- explore the properties of conics

Management Options

Groups – This activity is excellent for group work. Students often work in isolation at a computer, but this activity will encourage groups of two or three students to work together. Each group member should do the waxed-paper construction. Then each group can determine how to recreate the model using *The Geometer's Sketchpad*.

Teacher Demonstration – It is best for students to do this exploration on their own. However, you might review the geometric construction once some students have finished step 7.

ASSESSMENT SUGGESTION

Teamwork, Initiative

This activity provides an excellent opportunity for students to solve problems as a group and also to demonstrate initiative. For assessment, use the self-reflection sheets for teamwork and initiative. (See Assessment and Evaluation in the *Teacher Resource*.)

Independent Work – Individuals can do this exploration, but they should periodically compare their discoveries.

TEACHER INTERVENTION

The teacher may need to offer hints — perhaps in the form of cards — to students who are having difficulty using *The Geometer's Sketchpad*.

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1. What pattern emerges?

The outline of the shape that emerges appears to be a curve.

2. Compare your shape with the shapes that other students in the class have made. How are the shapes similar? How are they different? What conic does your shape resemble?

Although all the shapes are similar, some shapes are wider than others. The shape seems to be a parabola.

3. How does changing the position of point P affect the curve?

The closer P is to the bottom edge of the paper, the narrower the curve. The curve widens the further that P is from the edge of the paper.

4. Explain why you could describe each line you made as a **tangent**. (A line is tangent to a curve if it touches the curve at exactly one point.)

Each line seems to touch the curve in just one place.

5. How could you construct the tangent lines using what you know about geometry?

Choose a point on the edge of the paper that touches point P . Label this point A . If you connect point A to point P with a line, you can see that the crease line is the perpendicular bisector of AP .

6. Use The Geometer's Sketchpad to recreate this waxed-paper model of your curve. Begin with a line and a point, P .

Use the line and point tools to make a line, representing the edge of the paper, and a point (call it P) that is not on the line.

7. How will you construct the crease lines? (**Hint:** Choose a point on the line and connect it to P . What is the next step?)

Construct a point on the line and call it A . Construct a line segment from A to P . Construct the midpoint of AP and then select the midpoint and the segment and construct the perpendicular bisector of AP . This perpendicular bisector is the crease line.

8. Once you have drawn the first crease line, choose **Trace Line** from the **Display** menu to create other crease lines. Is the figure you created similar to your waxed-paper model?

Yes, it should be.

9. What happens if you move the original point or line?

The curve will become narrower or wider.

10. This shape appears to be a conic. But how can you be sure that the shape is a conic? What do you need to do?

Trace the curve on a set of x - y axes with the vertex at the origin. Find the coordinates of some points and see if you can find the equation of a parabola that will fit these points.

11. You can make similar models of the conics, using waxed paper and The Geometer's Sketchpad. Start by cutting out a circle, but not at the centre. Fold the paper so that the edge of the circle just touches the point. Continue creasing the paper by folding the edge at different angles.

12. What type of conic appears to be forming?

an ellipse

13. Try recreating the waxed-paper construction by using The Geometer's Sketchpad. Can you confirm your conjecture? What happens if the dot on the inside of the circle is near to the centre? very close to the edge?

The shape also appears to be an ellipse, using *The Geometer's Sketchpad*. If the dot is near the centre, the ellipse looks more like a circle. If the dot is very close to the edge, the ellipse is longer.

14. You can create another design by starting with a point outside the circle, rather than inside the circle. What shape do you construct?

The shape appears to be a hyperbola.

EXTENSION

Ask students to consider the following questions:

- (a) How can you use the locus definition of a parabola to determine if *The Geometer's Sketchpad* construction is truly a parabola?
- (b) What dimensions do you need to measure? Remember that the distance from a point to a line should be measured by making a perpendicular from the point to the line.
- (b) Using constructions and measurements, show that the parabola created is a parabola and that it satisfies the locus definition that a parabola is a locus of points that are equidistant from a fixed point and a fixed line.