

7.6U The Parabola

Specific Expectations

- LC1.05U determine equations for conics from their locus definitions, by hand for simple particular cases [e.g., determine the equation of the locus of points the sum of whose distances from $(-3, 0)$ and $(3, 0)$ is 10]
- LC2.01U identify the standard forms for the equations of parabolas, circles, ellipses, and hyperbolas having centres at $(0, 0)$ and at (h, k)
- LC2.02U identify the type of conic, given its equation in the form $ax^2 + by^2 + 2gx + 2fy + c = 0$
- LC2.03U determine the key features (e.g., the centre or the vertex, the focus or foci, the asymptotes, the lengths of the axes) of a conic whose equation is given in the form $ax^2 + by^2 + 2gx + 2fy + c = 0$, by hand in simple cases (e.g., $x^2 + 9y^2 - 6x + 36y - 36 = 0$)
- LC2.04U sketch the graph of a conic whose equation is given in the form $ax^2 + by^2 + 2gx + 2fy + c = 0$
- LC2.05U illustrate the conics as intersections of planes with cones, using concrete materials or technology
- LC3.01U describe the importance, within applications, of the focus of a parabola, an ellipse, or a hyperbola (e.g., all incoming rays parallel to the axis of a parabolic antenna are reflected through the focus; the planets move in elliptical orbits with the sun at one of the foci)
- LC3.02U pose and solve problems drawn from a variety of applications involving conics, and communicate the solutions with clarity and justification (Sample problem: A parabolic antenna is 320 m wide at a distance of 50 m above its vertex. Determine the distance above the vertex of the focus of the antenna)

Evidence of Learning

Students should be able to

- use the locus definition of a parabola to find the equation in a specific case
- determine the focus and directrix of a parabola from its equation when the vertex is $(0, 0)$
- determine the equation of a parabola with vertex $(0, 0)$ when the focus and directrix are given
- determine the equation of a parabola in standard form with vertex $(0, 0)$ or (h, k)
- determine the vertex and sketch the graph of a parabola for the equation is either in standard or general form
- solve problems involving the application of parabolas

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Time Required Functions and Relations:
75 min

Materials and Equipment

- graph paper
- graphing calculator or graphing software

Overview

This section introduces students to the equation of a parabola by starting with a locus definition of the parabola and then introducing the focus and directrix, which will extend students' knowledge of parabolas from grade 10.

Management Options

Groups – Students in pairs or small groups could work through Part 1 using large graph paper. The same groups could work on Part 2, a and they would find using graphing technology very helpful for this part.

EXTENSION

1. After Part 1, give each group of students another focus and directrix and ask them to determine the equation using the same steps. One person in the group could use the shortcut to find the equation and then the group could compare answers to check their work.

Teacher Demonstration – Lead the class through Part 1 of the lesson and demonstrate Part 2 using graphing technology. It might be helpful to have students working with graphing technology throughout the discussion of Part 2.

Independent Work – Students could read Part 1 and work out the development and the examples in their notebooks and then answer the *Think, Do, Discuss* questions in Part 2, with the aid of graphing technology, if available.

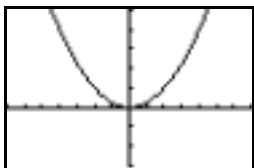
Part 1: Using the Locus Definition to Find the Equation of a Parabola

This section develops the locus definition of a parabola — the distance from any point on the curve to the directrix, a fixed line, and from this point to the focus, a fixed point, are equal. To find the equation for a parabola, start with a specific focus and directrix and then apply the distance formula. Students should then be able to find an equation given the focus and directrix, without having to go back to the original locus definition.

Part 2: Translating the Parabola

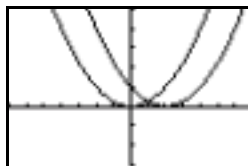
Think, Do, Discuss, p. 599

1. Begin with the equation $x^2 = 4y$. Sketch this graph.



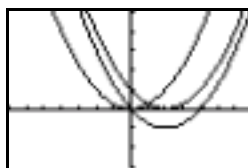
2. Using what you know about transformations, predict the shape and features of the graph of $(x - 2)^2 = 4y$. Use a sketch or graphing technology to check your prediction. Compare your sketch with the one in step 1.

The shape of the graph should be the same as the shape of the graph in step 1. But the new graph will be translated horizontally 2 units right.



3. Predict the shape and features of the graph of $(x - 2)^2 = 4(y + 1)$. Check your prediction.

This graph should be congruent to the graphs in steps 1 and 2. However, the graph will be vertically translated down 1 unit from the graph in step 2.

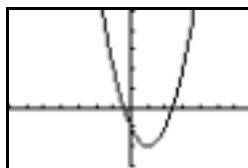


4. What is the vertex of $(x - 2)^2 = 4(y + 1)$? Describe the translation of the vertex from $x^2 = 4y$ to $(x - 2)^2 = 4(y + 1)$.

The new vertex is (2, -1). The original vertex, (0, 0), is translated 2 units right and 1 unit down.

5. Graph $(x - 1)^2 = y + 2$, using this new method.

The new vertex is (1, -2). This graph is congruent to $x^2 = y$, but it will be translated horizontally right 1 unit and vertically down 2 units.



6. Graph $y = (x - 1)^2 - 2$, using techniques learned in grade 10.

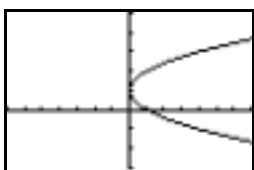
For the parabola, $y = a(x - p)^2 + q$, (p, q) is the vertex. Therefore, the vertex for this parabola is $(1, -2)$. The graph would be the same as the one in step 5.

7. Compare and explain the results of steps 5 and 6.

The graphs are the same because the equations are the same, if you rearrange them. Subtract 2 from both sides of the first equation to get the second equation.

8. Predict the shape and features of the graph of $(y - 1)^2 = x$. Confirm your prediction by graphing $(y - 1)^2 = x$.

Because y is squared, not x , this graph will open to the right. Also, the vertex will be $(0, 1)$.



9. Equations have many different forms. The **standard form** of the parabola is $(x - h)^2 = 4p(y - k)$. The **general form** of the parabola is $x^2 + 2gx + 2fy + C = 0$. Change $(x - 3)^2 = 8y$ to its general form.

The general form of $(x - 3)^2 = 8y$ is $x^2 - 6x - 8y + 9 = 0$.

Additional Example: Functions and Relations

Find the equation of a parabola whose vertex is $(5, 3)$ and whose horizontal directrix is 4 units above the focus.

Solution

The equation of a parabola with vertex $(5, 3)$ and a horizontal directrix is $(x - 5)^2 = 4p(y - 3)$. Now determine p . Since the directrix is 4 units above the focus, the focal length, $|p|$, is 2 units. Since the directrix is above the focus, the graph opens down and p is negative. Therefore, the equation of the parabola is $(x - 5)^2 = -8(y - 3)$.

Practise, Apply, Solve 7.6U

Questions with a Functions and Relations Focus:

1–10, 12–14, 16, 17, 19, 21

2. Knowledge and Understanding, p. 602

Management Suggestion

Ask students to answer this question orally before you assign other questions for written response. In this way, you can check that students recognize the vertex and direction of opening for different parabolas before they proceed with more complex questions.

Achievement Chart Connection

Students must demonstrate they recognize the vertex and the direction of opening for a parabola in standard form.

Level 3 Performance

As well as identifying the vertex easily and stating the direction of opening, students should explain how to find the vertex and what clues indicate the direction of opening.

ASSESSMENT SUGGESTION

Knowledge and Understanding — Informal Diagnostic Assessment

Diagnostic assessment of students' oral responses to this question will tell you whether students understand the basic features of a parabola as shown in its equation and whether they are ready to work on other questions independently.

5. Communication, p. 602

Management Suggestion

Students could work in pairs to share ideas and write a response to this question.

Achievement Chart Connection

Students should use correct mathematical language and conventions as well as clearly justify their reasoning.

Level 3 Performance

In their responses, students should incorporate the correct use of the general form and the standard form for the equation of an ellipse, indicating that the vertex and the direction of opening may be read from the standard form.

10. Application, p. 603

Management Suggestion

Assign this question as homework or to pairs or groups. Invite a pair or group to present their solution to the class.

Achievement Chart Connection

Students are expected to apply their knowledge of parabolas in a new context and in a different way.

Level 3 Performance

Solutions should be accurate and show a range of values for each part.

16. Check Your Understanding, p. 603

Management Suggestion

Most of the parabolas that students draw will be different. Post the sets of parabolas on the wall or board and, as a class, discuss similarities and differences. In the discussion, students should refer to the location of the focus and the directrix, the equations of the parabolas, the direction of opening, and so on.

19. Thinking, Problem Solving, and Inquiry, p. 604

Management Suggestion

Small groups could discuss this question and then present their solutions to the class.

Achievement Chart Connection

Students are expected to demonstrate their reasoning abilities.

Level 3 Performance

Students might

- manipulate (expand) the equations to compare them
- discuss finding the vertex using each form
- clearly state the advantages and disadvantages of each equation in the context of finding the stretch factor, the vertex, the direction of opening, the focus, and the directrix

ASSESSMENT SUGGESTION

Thinking, Inquiry, Problem Solving

By assessing the responses to this question, you may see how well students justify their reasoning. The last two rows of the generic rubric for Thinking, Inquiry, and Problem Solving make a good tool for assessing the responses to this question.

The Chapter Problem—Designing a Bridge

By the end of this section, students should explore equations of parabolas that may satisfy the criteria given in the Chapter Problem. Students should test the equations for many different parabolas.

The Chapter Problem Answers are on p. 7-48 of the *Teacher Resource*.

Challenge 1

Remind students that they should be able to

- see if the measurements of the flashlight's reflector fit the equation of a parabola. If so, what is the focal point?
- find the equation of the parabola that was created by shining the flashlight on the wall

Challenge 2

Remind students that they should be able to

- test whether the curve they have chosen is a parabola
- determine which model fits the data best – a circle, an ellipse, or a parabola

The Challenge Answers are on pp. 7-51 of the *Teacher Resource*.