

7.7U Exploration: Reflective Properties of Conics

Specific Expectations

- LC1.04U construct geometric models to represent the locus definitions of the conics
- LC3.01U describe the importance, within applications, of the focus of a parabola, an ellipse, or a hyperbola (e.g., all incoming rays parallel to the axis of a parabolic antenna are reflected through the focus; the planets move in elliptical orbits with the sun at one of the foci)

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Time Required Functions and Relations:

30 min

Materials and Equipment

- cardboard
- large graph paper
- marbles

Overview

In this section, students explore the reflective properties of conics using concrete materials.

Management Options

Groups – If groups do this exploration, have each group use a different equation for creating their geometric model, for example, $x^2 = 4y$. After students have tested their models in step 2, discuss, as a class, step 3 and the diagram of the car headlights. Then the same groups can complete the remaining *Think, Do, Discuss* questions.

Teacher Demonstration – Create a physical model of the parabola, as described in the text, and demonstrate rolling the marble. Then use the *Think, Do, Discuss* questions to begin a class discussion.

Independent Work – Assign this section as independent study to the whole class or to students who would benefit from independent work or an extension activity. Let students do the exploration outside of class time. Students could either hand in their work for assessment, or present to the class.

EXTENSION

Students might independently research other examples of the reflective properties of conics. For instance,

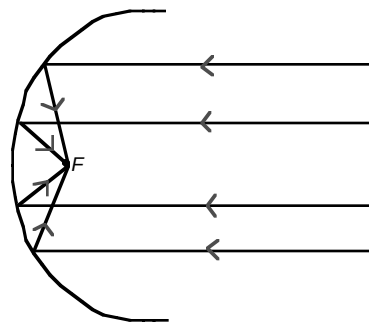
- Many science centres or museums have whispering galleries.
- Parts of buildings, namely, interior ceilings, in students' communities may be elliptical domes.
- An Internet search will yield other examples of reflectors that are parabolic, elliptical, or hyperbolic.

Think, Do, Discuss, p. 605

1. Does the marble's path cross the focus? Explain.

Yes, the marble's path should cross the focus if the marble follows a straight path parallel to the axis of symmetry.

2. Draw a diagram of this model. Using the diagram, demonstrate why the marble's path crosses the focus.



Each line that is parallel to the axis of symmetry is reflected in the curve of the parabola such that the angle of reflection is equal to the angle of deflection. The result appears to be that each path of the marble is deflected through the focus.

3. *The reflective properties of the parabola appear in the design of flashlights, car headlights, and satellite dishes. Can you find other instances of parabolic reflectors?*

Other instances include spotlights, solar-cooking devices, and parabolic reflectors that gather sound at large sporting events. There are many other instances as well.

4. *Try to create a similar model of an ellipse that is similar to the model of the parabolic reflector. This time, start with one marble at one focal point and “shoot” it toward any point on the ellipse. Shoot the marble from this focal point at several different points along the ellipse. What do you notice?*

The marble seems to always cross the other focal point.

5. *Draw a diagram of this model. Discuss why you think that, if you were to roll a marble from one focal point to a point on the ellipse, the marble would roll back through the other focal point.*

According to the diagram in the text, the angle of reflection appears to be the same as the angle of deflection at each point. (A tangent is assumed at each point.)

6. *If you wanted to build an elliptical pool table with a length of 2 m and a width of 1.8 m, then where would you place the “pocket”?*

Ideally, place a pocket at each focus. According to the dimensions of the table, $a = 1$ and $b = 0.9$. Then, by $a^2 + b^2 = c^2$, c is about 0.436 m. So place each pocket 0.436 m from the centre of the pool table.