

7.10U The General Form of Conics

Specific Expectations

- LC2.02U identify the type of conic, given its equation in the form $ax^2 + by^2 + 2gx + 2fy + c = 0$
- LC2.03U determine the key features (e.g., the centre or the vertex, the focus or foci, the asymptotes, the lengths of the axes) of a conic whose equation is given in the form $ax^2 + by^2 + 2gx + 2fy + c = 0$, by hand in simple cases (e.g., $x^2 + 9y^2 - 6x + 36y - 36 = 0$)
- LC2.04U sketch the graph of a conic whose equation is given in the form $ax^2 + by^2 + 2gx + 2fy + c = 0$
- LC3.02U pose and solve problems drawn from a variety of applications involving conics, and communicate the solutions with clarity and justification (Sample problem: A parabolic antenna is 320 m wide at a distance of 50 m above its vertex. Determine the distance above the vertex of the focus of the antenna)

Evidence of Learning

Students should be able to

- determine the type of conic given the equation in either standard form or general form
- move between general form and standard form
- solve problems related to conics

Text Page 618

Time Required Functions and Relations:

75 min

Overview

The focus of this section is the general form of the equation of the conic. Students are expected to determine the type of conic from the values of a and b in the equation $ax^2 + by^2 + 2gx + 2fy + c = 0$. As well, students will rearrange equations of conics, changing between standard equations and general equations.

MANAGEMENT SUGGESTION

Review graphing techniques for linear relations—creating a table, plotting x - and y -intercepts, and using the slope- y -intercept method—before taking up this lesson.

Management Options

Groups – Students could work in pairs on the *Think, Do, Discuss* questions in Part 1. For Part 2, partners can take turns explaining to one another the solution for each part of Example 1.

Both partners can work on Example 2 and compare answers.

MANAGEMENT SUGGESTION

Ask students to consider the following questions as they work through the *Think, Do, Discuss* questions:

1. Can you illustrate your results for steps 4, 5, and 6 with an example of a specific conic that meets the criteria?
2. What other information do f and g give you about the conic?

Teacher Demonstration – You might have the whole class discuss the *Think, Do, Discuss* questions in Part 1 and summarize the findings in a table, such as the one in Part 2, on the board. Then work through the examples in Part 2, using a Socratic method of teaching.

Independent Work – Students could work through the *Think, Do, Discuss* questions in Part 1 on their own and then summarize their findings in a table in their notebooks, using the same format as in Part 2. Then

they can work through the examples in Part 2 on their own.

Part 1: Recognizing Conics in General Form

Think, Do, Discuss, p. 618

1. What type of conic is $9x^2 + 4y^2 = 36$? State the values of a , b , g , f , and C for the general form of this equation.

The conic is an ellipse. $a = 9$, $b = 4$, $g = 0$, $f = 0$, and $C = -36$

2. Change $\frac{(x-1)^2}{4} - \frac{y^2}{25} = 1$ to the general form and state the values of a , b , g , f , and C . Name the conic.

The equation in standard form is $25x^2 - 4y^2 - 50x - 75 = 0$. $a = 25$, $b = -4$, $g = -25$, $f = 0$, and $C = -75$. This conic is a hyperbola.

3. Predict the conic represented by $x^2 - 4x - 8y - 28 = 0$. Then rearrange the equation in standard form and determine if your prediction is correct.

There is no y^2 term, so the equation represents a parabola. The equation is $(x - 2)^2 = 8(y + 4)$ in standard form.

4. If $ax^2 + by^2 + 2gx + 2fy + C = 0$ and $a = b = 0$ or $ab = 0$, then what is the conic?

If $a = b = 0$, the equation represents a straight line, not a conic. If $ab = 0$ and either a or b is 0, then the conic is a parabola.

5. If $ax^2 + by^2 + 2gx + 2fy + C = 0$ and $a = b$, then what is the conic?

In this case, the conic is a circle, unless both a and b are 0.

6. Consider the case of $ab > 0$. What is the conic? What is the conic if $ab < 0$?

For $ab > 0$, the conic is an ellipse. For $ab < 0$, the conic is a hyperbola.

7. Name each conic.

$$4y^2 - 8y + 9x^2 - 54x + 49 = 0$$

$$x^2 - 4x - y^2 - 5 - 4y = 0$$

For the first conic, the value of ab is 36, which means the conic is an ellipse. For the second equation, the value of ab is -1 , which means the conic is a hyperbola.

8. If $f = g = 0$, then describe the conic.

The conic is centred at the origin.

Part 2: Organizing What You Know

Consolidate Your Understanding, p. 620

1. Explain how you can determine what type of conic an equation in general form represents.

The signs of a and b indicate the type of conic. The *Key Ideas* summarize the conics and the corresponding values of a and b .

2. Explain how you can tell whether a conic is centred at the origin given the general form of the equation.

A circle, an ellipse, or a hyperbola is centred about the origin if f and g are both 0. A parabola is centred about the origin if either f or g is 0.

Additional Example: Functions and Relations

For the conic $4y^2 - 8y + 9x^2 - 54x + 49 = 0$, find the coordinates of the centre, the foci, and the vertices. Then graph the conic.

Solution

Since the value of ab is 36, the conic is an ellipse. First rewrite the equation of the conic in standard form.

$$4y^2 - 8y + 9x^2 - 54x + 49 = 0$$

$$4(y^2 - 2y) + 9(x^2 - 6x) = -49$$

$$4(y^2 - 2y + 1) + 9(x^2 - 6x + 9) = -49 + 81 + 4$$

$$4(y - 1)^2 + 9(x - 3)^2 = 36$$

Therefore, the equation in standard form is

$$\frac{(x-3)^2}{4} + \frac{(y-1)^2}{9} = 1.$$

The centre is (3, 1). If the conic were centred at the origin, the vertices would be (0, 3) and (0, -3). Since a translation of (3, 1) is applied, the vertices are (3, 4) and (3, -2).

To find the foci, first find the foci of the ellipse if it were centred about the origin. Since $a = 3$, $b = 2$, and $a^2 = b^2 + c^2$, then $c = \sqrt{5}$. Therefore, if the conic were centred at the origin, the foci would be $(0, \sqrt{5})$ and $(0, -\sqrt{5})$. Translate the foci to get $(3, 1 + \sqrt{5})$ and $(3, 1 - \sqrt{5})$.

Practise, Apply, Solve 7.10U

Questions with a Functions and Relations Focus:

1-10, 13-15, 17-20

2. Knowledge and Understanding, p. 621

Management Suggestion

Students could respond to this question orally in class before turning to other homework questions.

Achievement Chart Connection

Students must be able to recognize the type of conic given the equation in general form.

Level 3 Performance

Students should identify the values of a and b and then name the conic based on a combination of these values.

9. Communication, p. 622

Management Suggestion

Students could write their solutions in their journals.

Achievement Chart Connection

Students need to explain how to determine the type of conic.

Level 3 Performance

In this case, students should recognize that, since $ab > 0$ and $a = b$, the conic is a circle. They might divide the equation by 2 to show that the coefficients of x^2 and y^2 are both 1 or rearrange the equation in standard form.

14. Application, p. 622

Management Suggestion

Students could work on this question in groups of two or three. Pick one group to present their solution to the class. Discussing different strategies is important. Did groups use graphical or algebraic models? Did they work first with the locus definition or the standard form of the equation of the parabola?

Achievement Chart Connection

Students are expected to select the appropriate conic and the strategy for finding its equation.

Level 3 Performance

Students should choose a method of solution that will lead to the standard form of the equation of the parabola.

17. Thinking, Inquiry, Problem Solving, p. 622

Management Suggestion

Students may want to solve this problem using various tools – drawing a graph, using dynamic geometry software, or analytic geometry. Ask students to present various solutions — “In how many different ways can we solve this problem?”

Achievement Chart Connection

This question requires students to select different tools to solve the problem.

Level 3 Performance

Students should find the midpoint of the diameter, which is the centre of the circle, and the length of the diameter, which they can use to find the radius.

18. Check Your Understanding, p. 622

Management Suggestion

Direct students to write their responses to this question and then partners can share responses. Through refining their own work, they will create descriptions of the relationship between a and b for each type of conic, including specific restrictions.