

7.11U When Lines Meet Conics

Specific Expectations

- LC2.04U sketch the graph of a conic whose equation is given in the form $ax^2 + by^2 + 2gx + 2fy + c = 0$
- LC3.01U describe the importance, within applications, of the focus of a parabola, an ellipse, or a hyperbola (e.g., all incoming rays parallel to the axis of a parabolic antenna are reflected through the focus; the planets move in elliptical orbits with the sun at one of the foci)
- LC3.02U pose and solve problems drawn from a variety of applications involving conics, and communicate the solutions with clarity and justification (Sample problem: A parabolic antenna is 320 m wide at a distance of 50 m above its vertex. Determine the distance above the vertex of the focus of the antenna)
- LC3.03U solve problems involving the intersections of lines and conics

Evidence of Learning

Students should be able to

- find the intersection of a line and conic, both algebraically and graphically
- apply the solution to the intersection of lines and conics in problem-solving situations

Text Page 623

Time Required Functions and Relations:

75 min

Materials and Equipment

- graphing paper or graphing software

Overview

The focus of this section is solving linear and second-degree systems algebraically and graphically. Students will also model a problem using a conic and a line and then solve the resulting system of equations.

Management Options

Groups – Students could read the first paragraph and the text up to, but not including, the *Consolidate Your Understanding*. Then pairs could discuss the answers to the *Consolidate Your Understanding* questions and work through the examples. Students could test one another using the first question in the *Practise, Apply, Solve*.

Teacher Demonstration – Work through this section with students, using a Socratic style of teaching. Discuss the *Consolidate Your Understanding* questions. Students could then study the examples in pairs, individually, or with the teacher.

Independent Work – Individuals could read the lesson and record their answers to the *Consolidate Your Understanding* questions in their journals or notebooks. Then students can work through the examples in their notebooks.

EXTENSION

Ask students to consider the following questions:

1. Create a problem involving the intersection of a conic and a line. Be sure that you can solve the problem.
2. Trade questions with your partner and solve.

Consolidate Your Understanding, p. 624

1. *Why does drawing a diagram or a sketch of the graph help you solve an intersection problem?*

A sketch will help you determine whether there are no solutions, one solution, or more solutions.

2. *Why is it important to check your answers for both equations in the system of equations?*

Checking helps verify answers and the points of intersection.

3. *List the algebraic methods that you know for solving systems of equations.*

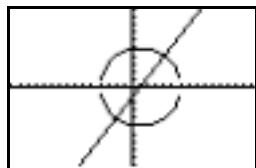
Substitution, elimination, or comparison. (Students might use other words to describe the same methods or refer to only two methods since comparison is a form of substitution.)

Additional Example: Functions and Relations

Show that the intersection points of the circle $(x - 1)^2 + y^2 = 25$ and the line $-4x + 3y + 4 = 0$ are the endpoints of a diameter of the given circle.

Solution

Graph the system.



The graph shows two intersection points.

Rearrange the equation of the line so that

$y = \frac{4}{3}x - \frac{4}{3}$ and substitute in the equation of the circle.

$$(x - 1)^2 + \left(\frac{4}{3}x - \frac{4}{3}\right)^2 = 25$$

$$x^2 - 2x + 1 + \frac{16}{9}x^2 - \frac{32}{9}x + \frac{16}{9} = 25$$

Multiply by 9 to get

$$9x^2 - 18x + 9 + 16x^2 - 32x + 16 = 225$$

Collect like terms and then factor.

$$25x^2 - 50x - 200 = 0$$

$$x^2 - 2x - 8 = 0$$

$$(x - 4)(x + 2) = 0$$

Therefore, $x = 4$ and $x = -2$.

Substituting the values for x and then solving for y gives the solutions $(4, 4)$ and $(-2, -4)$.

To determine if the line is a diameter of the circle, verify that the line segment joining the two points passes through the centre of the circle, $(1, 0)$. The midpoint of the line segment between the two points is

$$\left[\frac{4 + (-2)}{2}, \frac{4 + (-4)}{2} \right], \text{ or } (1, 0), \text{ which is}$$

the centre of the circle. Therefore, the solution represents the endpoints of a diameter of the circle.

Practise, Apply, Solve 7.11U

Questions with a Functions and Relations Focus:

1–9, 11, 13, 15, 16, 18, 19

6. Knowledge and Understanding, p. 626

Management Suggestion

Assign this question as homework but let students start it in class. Before the end of class, write the question on the board and take up the solution to ensure that all students understand the question. This approach will help students adapt the method for subsequent questions.

Achievement Chart Connection

This question checks students' understanding of both the algebraic and graphical solutions to a system of a line and a conic.

Level 3 Performance

Students should draw graphs that show the intersection points and verify their graphs algebraically.

8. Application, p. 627

Management Suggestion

The students could use a think-pair-share strategy for this question. After they have attempted this question on their own, pair them up to share and discuss their work.

Achievement Chart Connection

Students must apply their skills in solving a system of a conic and a line in a familiar context. This problem is similar to the opening problem in this section.

Level 3 Performance

Students should draw a diagram and solve the problem algebraically.

ASSESSMENT SUGGESTION

Application — Informal Diagnostic Assessment

As students share their solutions in pairs, circulate and observe if students apply their knowledge of solving systems to this familiar context. If needed, refer them to the opening problem in this section.

16. Check Your Understanding, p. 628

Management Suggestion

Students could work on this question in groups. Then divide the class into four groups and assign one part to each group. Ask each group to make a poster, including diagrams of different solutions and one example of a system for each possible scenario.

18. Thinking, Inquiry, Problem Solving, p. 628

Management Suggestion

Students might solve this problem using assorted tools—drawing a graph, using dynamic geometry software, or analytic geometry. Have students work in groups and present various solutions. Perhaps ask “How many different ways can we solve this problem?”

Achievement Chart Connection

Students select and use various tools and strategies to solve this problem.

Level 3 Performance

Students should find the intersection points of the system, and then determine the equation of the perpendicular bisector. Once students have rearranged the equation in standard form, they can determine the centre of the circle and verify that it lies on the perpendicular bisector. Students may also choose to use *The Geometer's Sketchpad* to solve this problem, rather than take the analytic approach outlined above, where they would construct the same objects rather than finding the equations of them.

ASSESSMENT SUGGESTION

Teamwork

As students work on question 18 in groups, assess their abilities to work as a team using the related generic rubric for learning skills. (See Assessment and Evaluation in the *Teacher Resource*.)