

7.1U Introducing Locus Definitions

Specific Expectations

- LC1.01U construct a geometric model (e.g., a diagram created by hand, a diagram created by using dynamic geometry software) to represent a described locus of points; determine the properties of the geometric model; and use the properties to interpret the locus (e.g., the locus of points equidistant from two fixed points is the right bisector of the line segment joining the two fixed points)
- LC1.02U explain the process used in constructing a geometric model of a described locus;
- LC1.03U determine an equation to represent a described locus [e.g., determine the equation of the locus of points equidistant from $(-2, 7)$ and $(5, 4)$]

Evidence of Learning

Students should be able to

- demonstrate that relationships can be represented in many different ways – geometrically, algebraically, or through a locus definition
- create a geometric model given the locus definition
- create an equation for a locus definition of a familiar relationship

Text Page 570

Time Required Functions and Relations: 75 min

Materials and Equipment

- *The Geometer's Sketchpad* would be helpful for demonstration or exploration purposes.
- string
- compasses and rulers
- graph paper (large sheets for groups)

Overview

This section introduces students to the locus definition. Students first write locus definitions for familiar figures such as lines, circles, and spheres and then create graphical and algebraic models based on locus definitions.

MANAGEMENT SUGGESTION

Students should have access to *The Geometer's Sketchpad* for the *Think, Do, Discuss* questions of Part 1. Ideally, a combination of individual and group exploration should be followed by whole-class discussion.

Management Options

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Groups – Divide the class into pairs and have them work through the *Think, Do, Discuss* questions in Part 1. It would be helpful if each pair has access to *The Geometer's Sketchpad* for these questions. Then have groups of four compare answers. These groups, using large sheets of graph paper, can next work on the *Think, Do, Discuss* questions of Part 2. Post the solutions to these questions and compare them in a whole-class discussion.

Teacher-Led Discussion – Use *The Geometer's Sketchpad* for demonstration and, with the class, work through the *Think, Do, Discuss* questions of Part 1. Ideally, students should work at individual computers to explore with the teacher. If this is not possible, have a student demonstrate a specific construction for one of the questions at a computer. For Part 2, work through the first few *Think, Do, Discuss* questions and then have students do the rest of the questions. Compare solutions in a whole-class discussion.

TECHNOLOGY SUGGESTION

If *The Geometer's Sketchpad* is not available, students could use diagrams, string, and geometry construction tools.

Independent Work – Direct each student to work independently on the *Think, Do, Discuss* questions of Part 1 using *The Geometer's Sketchpad*. Take up the solutions to these questions in a whole-class discussion. Students can then work on Part 2 independently and also away from the computer. Take up the solutions to these questions in a whole-class discussion.

EXTENSION

1. Students could experiment with a Spirograph™, making simple designs that they could define using a locus definition.
2. Students can go to the Nelson Mathematics student Web site at www.math.nelson.com to explore other designs created with *The Geometer's Sketchpad*, which are related to Spirograph™.

Part 1: Locus Definitions

In Part 1 of this lesson, the locus definition is introduced. Students are expected to connect the locus definition with a geometric model. Using *The Geometer's Sketchpad* to explore these questions is very helpful.

Think, Do, Discuss, p. 570

1. *If you created a circle with a Spirograph™, then you would certainly recognize that this is not the usual way of drawing a circle. You usually draw a circle with compasses, by placing one tip of a compasses' arm at a fixed point and then rotating the other arm at a fixed radius about the fixed point. Construct a circle using compasses and then define the circle using a locus definition. You might begin the definition with, "A circle is the set of points that are"*

A circle is the set of points that are a fixed distance (or equidistant) from a fixed point.

2. *Different definitions can describe the same geometric figure. What figure would be described by the locus of points that are equidistant from the centre and edge of a circle? What specific characteristics would this figure have?*

This figure would also be a circle; it is the same figure described in the Spirograph™ example.

3. *The typical locus definition of a circle is the set of points that are equidistant from a given point. In the three-dimensional plane, what geometric figure is the locus of points that are the same distance from a given point?*

The geometric figure is a sphere.

4. *How would you construct a set of points, in the two-dimensional plane, that are equidistant from two given points? Use The Geometer's Sketchpad to explore this question. Where would these points lie? Describe the set of points geometrically.*

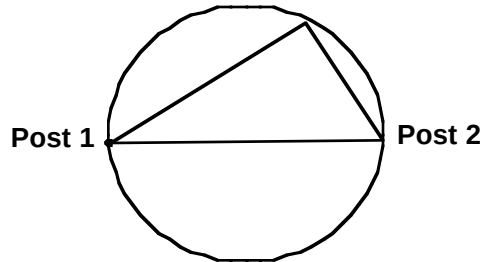
A set of points that are equidistant from two given points is the perpendicular bisector of the line segment joining the two points. This activity is best explored using *The Geometer's Sketchpad*.

5. *Consider the locus of points that are equidistant from two lines. How would you describe this locus.*

This locus is the angle bisector of the angle formed between the two lines. If the lines were parallel, the locus would be a line that is parallel to both lines and lies halfway between them.

6. *The locus definition may also describe a set of points that fall within a region or area. Imagine that you are with your class on a nature hike. You approach a swampy area, but you cannot tell what is swamp and what is solid ground. There are two posts in the ground. Your teacher tells you that, if the angle between you and the two posts is more than 90° , then you will walk on swampy ground. If the angle is less than 90° , then you will not walk on swampy ground. Draw a diagram or use The Geometer's Sketchpad to explore the problem and determine the edges of the swamp. Explore what would happen for an angle other than 90° .*

Most of the swampy ground forms a circular area. (The ground enclosed by the triangle is not swampy.) The diameter of the circle is the line segment between the two posts.



Part 2: Locus Definitions That Lead to Algebraic Expressions

Think, Do, Discuss, p. 571

1. The adjacent diagram shows the locus of points that are 3 units to the right of the y-axis. What is the equation of this locus?

The equation of the locus is $x = 3$.

2. Describe the locus of points that are 5 units from the x-axis. Draw a diagram to help you.

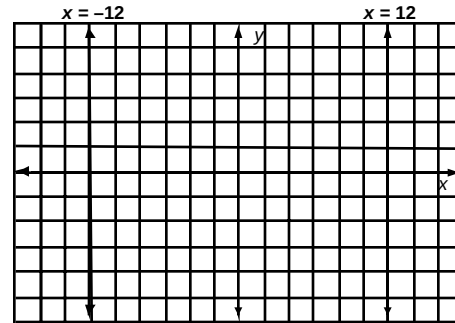
The locus of points that are 5 units from the x-axis could be a horizontal line 5 units above the x-axis or the locus could be a horizontal line 5 units below the x-axis.

3. What is the equation, or equations, for this locus of points?

There are two equations. The line above the x-axis is $y = 5$ and the line below the x-axis is $y = -5$.

4. Draw a sketch of the locus of points that are 12 units from the y-axis. Describe this locus geometrically and algebraically.

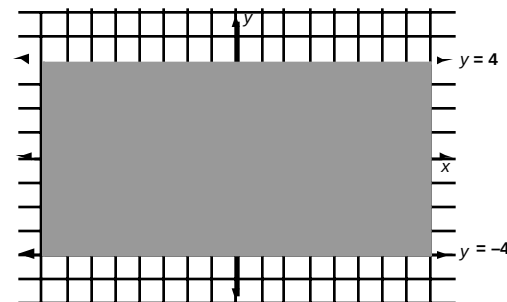
The locus is two vertical lines. One line is $x = 12$ and the other is $x = -12$.



5. Describe all the points that are less than 4 units from the x-axis. Create a geometric model and an algebraic model.

All points that are less than 4 units from the x-axis form a horizontal band bounded by the lines $y = 4$ and $y = -4$. However, these lines are not included in the locus, so $-4 < y < 4$.

In the graph, there are no left or right boundaries.



6. Describe the locus of points that are 4 units from the origin. If possible, write the equation for the locus.

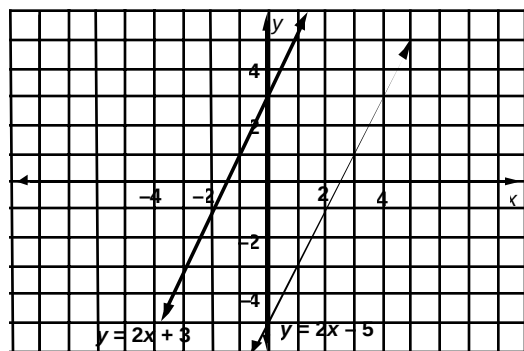
The locus of points that are 4 units from the origin is a circle. The equation for the circle is $x^2 + y^2 = 16$.

Additional Example: Functions and Relations

Find the equation of the locus of points that are equidistant from the lines $y = 2x - 5$ and $y = 2x + 3$.

Solution

First draw a diagram.



Since the two lines are parallel, an equidistant line would also have to be parallel. The slopes of both lines are the same, that is, 2, so the slope of the equidistant line is also 2. The y-intercept of this line is halfway between the y-intercepts of the other two lines, (0, -5) and (0, 3). The midpoint of the equidistant line is then (0, -1) and the equation of this line is $y = 2x - 1$.

Practise, Apply, Solve 7.1U

Questions with a Functions and Relations Focus:

1–5, 6–8 (every other part), 9, 11, 13, 16, 17–20

4. Knowledge and Understanding, p. 574

Management Suggestion

Students might begin solving this question in class. Take up the solutions together before the end of the class. Students might explore the problem using *The Geometer's Sketchpad*.

Achievement Chart Connection

This question tests students' knowledge of a locus definition but more importantly, in (b), their understanding of connecting a written description with geometric and algebraic models.

Level 3 Performance

Solutions should include

- an accurate equation for (a)
- a well-labelled diagram for (b)
- an accurate equation for the locus of points for the vertex A
- an accurate description of the trace of points

9. Communication, p. 575

Management Suggestion

Have students write each locus definition on a card or strip of paper. Direct pairs to exchange definitions. Each partner should read the locus definition and determine if it defines one of the equations in the question. Some locus definitions may need to be refined.

Achievement Chart Connection

Students are expected to provide locus definitions for various algebraic expressions, which means that they will have to communicate clearly and concisely using mathematical language.

Level 3 Performance

Each description should result in the given relation. Students may make the occasional error, but solutions should show corrections of errors.

ASSESSMENT SUGGESTION

Communication

Assess students' mathematical communication skills using the appropriate components of the related generic rubric for communication, for example, "appropriate use of mathematical vocabulary." (See Assessment and Evaluation in the *Teacher Resource*.) Students could also peer assess on the same basis as they check one another's work.

16. Application, p. 576

Management Suggestion

Include this question for homework. Before taking up the solutions in the next class, have small groups of students compare and discuss their responses. Ask each group to share a unique solution with the class.

Achievement Chart Connection

Students will need to choose a tool or strategy for the problem and apply it appropriately. A student might draw a diagram, use *The Geometer's Sketchpad*, or work with manipulatives to solve the problem.

Level 3 Performance

Diagrams should show the regions that satisfy each condition.

17. Check Your Understanding, p. 576

Management Suggestion

Suggest that students choose a different method — draw a diagram, solve algebraically, graph, or use technology — for solving each part of the question.

19. Thinking, Inquiry, Problem Solving, p. 576

Management Suggestion

Pairs can work on this problem in class. First have students make a conjecture and then explore the task. Have several different “tools” available – cardboard, graph paper, *The Geometer’s Sketchpad*, or string. Each pair could then join another pair for comparing solutions before sharing them with the class.

Achievement Chart Connection

This question is hard for students to visualize with simply the given diagram. They might make conjectures and then explore different strategies to verify their conjectures.

Level 3 Performance

Responses might include

- a diagram
- a sketch created with *The Geometer’s Sketchpad*
- a written explanation
- an algebraic verification

The Chapter Problem—Designing a Bridge

By the end of this section, students might begin designing a semicircular bridge by drawing a diagram and using what they know about circles from grade 10. In the next section, they will learn more about circles.

The Chapter Problem Answers are on p. 7-48 of the *Teacher Resource*.

Challenge 1

Remind students that they should be able to

- draw diagrams and measure parts of the flashlight’s reflector
- trace (on paper) the various reflections of the flashlight on the wall. (The next step is exploring equations.)

Challenge 2

Remind students that they should begin looking for pictures in magazines that show the curves they would see if they were to “flip through” this chapter.

The Challenge Answers are on p. 7-51 of the *Teacher Resource*.